

# Second-Best Income Taxation with Endogenous Human Capital and Borrowing Constraints

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## Abstract

We formulate a two-period life-cycle model of saving, labor supply, and human capital investments when individuals differ in ability and initial wealth. Borrowing constraints prevent individuals to optimally smooth consumption over the life-cycle and to optimally invest in human capital. We show that the optimal linear income tax is positive – even in the absence of any redistributive concerns. A progressive income tax is efficient because it relaxes borrowing constraints by redistributing resources from the unconstrained to the borrowing constrained stages of the life-cycle. Hence, consumption is smoothed better and investments in human capital increase. The progressive income tax is a second-best instrument to correct the non-tax distortion in the capital market. The equity-efficiency trade-off is therefore less severe when progressive income taxes mitigate capital market imperfections.

*JEL classification:* H21, I2, J2

*Keywords:* labor taxation, human capital investment, credit constraints

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# 1 Introduction

This paper examines optimal income taxation and human capital formation in an economy where individuals are subject to binding credit constraints. Empirical evidence for credit constraints is presented in two strands of the literature. Firstly, (poor) individuals can experience difficulties financing their higher education as shown in Kane (1996), Keane and Wolpin (2001), Plug and Vijverberg (2005), Belley and Lochner (2007), Stinebrickner and Stinebrickner (2008) and Lochner and Monge-Naranjo (2008).<sup>1</sup> Secondly, ample empirical evidence for binding borrowing constraints is found when empirically testing the life-cycle hypothesis in consumption. See Attanasio and Weber (2010) for an excellent overview of this literature. Binding credit constraints preventing individuals to invest optimally in human capital could contribute to persistence in income mobility, result in larger inequality, strengthen segregation of neighborhoods, and decrease economic growth (Loury, 1981; Galor and Zeira, 1993; Durlauf, 1996, and Benabou, 1996a,b; De Gregorio, 1996; Mookherjee and Ray, 2003; Galor and Moav, 2004).

The purpose of this paper is to analyze optimal redistributive tax policies when individuals cannot borrow the funds to smooth consumption and to finance human capital investments. To that end, we develop a two-period life-cycle model, where individuals make educational investments in first period and they work in the second period. In doing so, we make four important assumptions: 1) exogenous constraints restrict the amount of borrowing that can be made by individuals in period one; 2) educational investments are not verifiable to the government; 3) individualized lump-sum taxes are not feasible; 4) age-specific tax instruments are ruled out. The first assumption originates from the fact that human capital is poor collateral for borrowing. Poor individuals (or their parents) cannot borrow to finance investments in education (of their children). In a non-slave state legal restrictions prevent individuals engaging in a contract that employs future income as collateral (see also Stiglitz, 1994; Palacios, 2002; Jacobs and van Wijnbergen, 2007). The second assumption assumes that all costs are non-verifiable. This ensures that the government cannot directly off-set underinvestment in human capital with education subsidies. Our main results would be insensitive to including some verifiable educational investments, which can be subsidized, as long as some non-verifiable costs remain.<sup>2</sup> The

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<sup>1</sup>Consistent with the presence of credit constraints Kane (1995) and van der Klaauw (2002) identify large impacts of financial aid on college enrollment. Acemoglu and Pischke (2001) show that family income plays an important role in determining educational attainment. Caneiro and Heckman (2002) point out that credit constraints are relevant for about 8% of the youth in the US.

<sup>2</sup>It can be argued that a substantial part of educational costs is verifiable, e.g. opportunity costs and institutional, direct costs of education, and therefore subsidizable. Nevertheless, some part of the time and study effort invested in education is non-verifiable and, therefore, difficult to disentangle from working or leisure time. Similarly, it can be difficult for the government to distinguish direct costs of education (books, computers, etc) from pure consumption. Hence, subsidies on direct costs could be difficult to target efficiently and some underinvestment due to capital market imperfections would remain.

third assumption implies that individualized lump-sum taxes are not possible due to informational constraints that prevent the government from verifying individual abilities and/or individual initial wealth. The last assumption ensures that the tax system cannot discriminate between individuals by their age.

We demonstrate that the optimal income tax is progressive even in representative agent settings where distributional concerns are absent. That is, we provide a case for distortionary taxation on grounds of efficiency only. The intuition is that, as long as incomes are increasing over the life-cycle, progressive tax systems redistribute resources from later stages to earlier stages in the life-cycle. By taxing later incomes at higher average rates than current incomes, while redistributing the revenue through age-invariant lump-sum transfers, the progressive income tax alleviates credit constraints. Hence, not only consumption is smoothed better, but also investments in human capital increase. The labor tax trades off the welfare gains of alleviating credit constraints against the tax distortions in labor supply and human capital formation. The extent to which individuals are credit constrained, and the tax elasticities of labour supply and educational investment determine the optimal tax rate. In an extension of the model with heterogeneous agents, we demonstrate that all the results derived under homogeneous agents carry over to the more general case with heterogeneous agents. We show that with credit constraints the trade-off between equity and efficiency is less severe, since redistribution generates not only equity gains, but also efficiency gains. Hence, when distributional concerns are allowed for, the case for progressive income taxation is strengthened further.

Our paper relates in a number of ways to the existing literature. The paper being conceptually closest to ours is Hubbard and Judd (1986). They simulate a life-cycle model to demonstrate that progressive income taxation compared to proportional income taxation is welfare-improving when credit constraints are binding. The intuition is the same as ours: the progressive income tax redistributes resources over the life-cycle and allows for better consumption smoothing. Our paper, however, provides a formal proof for their finding as a special case of our model in which educational investment is kept exogenous. Hoff and Lyon (1995) also show that redistributive income taxation improves welfare by mitigating adverse-selection in the capital market. Taxing labor income progressively and rebating the tax revenue through lump-sum transfers increases collateralizable wealth. Progressive taxes thereby moderate inefficient overinvestment in education. Our model in contrast emphasizes underinvestment in human capital, which results from binding credit constraints.

Earlier work by Loury (1981), Glomm and Ravikumar (1992), Benabou (1996a, 1996b), and Fernandez and Rogerson (1996; 1998) demonstrates that when credit constraints are binding public provision of education or equalizing expenditure on education among communities can increase income equality, reduce segregation, promote income mobility, and boost economic growth. Tobin (1980) also points out that government policy should help

credit constrained individual to move resources from the future to the present. However, this literature has not yet conducted an analysis of *optimal* redistributive policies when individuals face binding credit constraints.

Our paper also contributes to an extensive literature, which emphasizes the potentially efficiency-enhancing effects of distortionary taxes in second-best settings. See also Van der Ploeg (2006a) for an overview. We show that the introduction of a distortionary tax instrument, can reduce a pre-existing non-tax distortion in the economy, i.e., the credit constraint. For example, Akerlof (1976) shows that the introduction of a distortionary income tax helps to tame the ‘rate race’ and reduces the individuals’ excessive incentives to work. Related is Layard (1980, 2005) who argues that progressive taxation is welfare-improving because individuals are involved in status races (‘keeping up with the Joneses’) and exhibit habit persistence, both giving excessive incentives to work. Labor-market imperfections arising from trade unions, efficiency wages and search frictions also provide second-best arguments for progressive taxes (see Koskela and Vilmunen, 1996; Pissarides, 1998; and Sørensen, 1999; Boone and Bovenberg, 2002; Van der Ploeg, 2006b; and Bovenberg 2006). Unions set wages above market clearing levels when unemployment benefits improve the outside options of workers. Also firms pay too high efficiency wages in order to recruit, to retain and to motivate workers when workers face attractive outside options. Progressive taxes punish both unions and firms to bid up wages, so that wages are moderated, and unemployment decreases.<sup>3</sup> Progressive taxation could also correct search frictions in labor market. Progressive taxation lowers the wage demands by workers, which increases vacancies and expands employment. This is optimal if workers have too much bargaining power compared to firms, i.e., when the Hosios (1990) condition is not met. In the presence of missing insurance markets, progressive taxation redistributes income across different states of nature and improves upon efficiency by partially replacing the missing insurance market (Eaton and Rosen, 1980; Varian, 1980; Jacobs et al., 2010).

The remainder of this paper is organized as follows. Section 2 presents our life-cycle model with imperfect credit markets and human capital investment. Optimal tax policies are analyzed in section 3 in an economy with representative individuals, which focuses on optimal efficient taxation to relax borrowing constraints. In section 4 we extend the model to a heterogeneous agent setting and show how optimal redistribution and alleviating credit constraints determine optimal income taxes. The last section concludes.

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<sup>3</sup>Van Ewijk and Tang (2007) show that education subsidies are optimal in order to off-set the disincentives on human capital investments when the government uses progressive taxes to lower union’s wage demands.

## 2 Model

The economy is populated by a continuum of individuals living for two periods. The mass of all individuals is normalized to one. Individuals differ in their ability  $n$  and initial wealth  $\omega$ . Ability and wealth have a cumulative joint distribution  $F(n, \omega)$ , which has supports  $[\underline{n}, \infty)$  and  $[\underline{\omega}, \infty)$ . We use a subscript to denote the type of individual by its ability and initial wealth, while a superscript is used to label the period in the life-cycle.

We consider a two-period life-cycle model of educational investment, labor supply, saving and borrowing constraints. In the first period the individual does not work, but invests in education and consumes. In the second period, the individual supplies labor and consumes all its wealth.

The resource costs of human capital investment  $e_{n\omega}$  are non-verifiable. We normalize the unit cost of education and consumption goods to one. Besides educational investment, the individual decides on its consumption in first period  $c_{n\omega}^1$  and saving  $a_{n\omega}$ . Consequently, the first-period budget constraint is

$$a_{n\omega} = -e_{n\omega} + \omega + g - c_{n\omega}^1, \quad (1)$$

where  $g$  is the time-invariant lump-sum transfer.

Individuals are only allowed to borrow a maximum of  $a_o$  at the capital market, implying the following borrowing constraint:

$$a_{n\omega} + a_o \geq 0. \quad (2)$$

This assumption reflects the fact that individuals have limited access to loans to finance consumption and educational investments. The (exogenous) interest rate equals  $r$  and is the same for saving and borrowing.

In the second period, the individual chooses labor supply  $l_{n\omega}$ . Gross labor income  $z_{n\omega}$  depends on educational investment  $e_{n\omega}$ , labor supply  $l_{n\omega}$  and ability  $n$ :

$$z_{n\omega} \equiv nl_{n\omega}\phi(e_{n\omega}), \quad \phi' > 0, \quad \phi'' < 0, \quad (3)$$

where  $\phi(e_{n\omega})$  is the production function for human capital with positive but diminishing marginal returns to human capital investment.

Second-period consumption equals after-tax labor income, saving plus interest income and the lump-sum transfer:

$$c_{n\omega}^2 = (1 - t)nl_{n\omega}\phi(e_{n\omega}) + (1 + r)a_{n\omega} + g, \quad (4)$$

where  $t$  denotes the labor tax rate. We rule out taxes on saving, since we will focus

mainly on credit-constrained individuals. Taxes on saving would not yield any revenues when savings are zero.<sup>4</sup>

The individual characteristics ( $n$  and  $\omega$ ) and individual decisions ( $e_{n\omega}$ ,  $a_{n\omega}$  and  $l_{n\omega}$ ) are assumed to be private information. In line with Mirrlees (1971) only total labor income is verifiable to the government. Consequently, government has to rely on distortionary labor taxes to redistribute income. With a flat tax rate and positive non-individualized lump-sum transfers the income tax is progressive. Age-specific lump-sum transfers would be available if the transfers could be conditioned on age. However, we rule out age-specific transfers, since most legal systems do not allow for age-discrimination. Therefore, the transfers must be identical in both periods. The non-verifiability of  $\omega$  implies that the government can neither levy taxes on initial wealth nor condition transfers upon initial wealth of each individual. The informational requirement for levying a flat tax is that the government only needs to verify aggregate labour income.

Individuals derive utility from consumption in both periods and disutility from labor. The utility function is assumed to be separable in consumption and labor:

$$U = u(c_{n\omega}^1, c_{n\omega}^2) - v(l_{n\omega}), \quad u_1, u_2, v' > 0, \quad u_{11}, u_{22}, -v'' < 0, \quad u_{12} \geq 0. \quad (5)$$

Without loss of generality we assume that the subutility function  $u$  is homogenous of degree one and concave in both arguments. The subscripts refer to the derivatives with respect to the first and the second argument of the utility function, respectively. The disutility of labor  $v(\cdot)$  is increasing and convex in  $l_{n\omega}$ . The individual chooses educational investment  $e_{n\omega}$ , saving  $a_{n\omega}$  and labor supply  $l_{n\omega}$  to maximize utility (5) subject to the budget constraints (1), (4), and the credit constraint (2).

After substituting budget constraints for  $c_{n\omega}^1$  and  $c_{n\omega}^2$  we can formulate the following Lagrangian  $\mathcal{L}$  for the individual's maximization problem

$$\begin{aligned} \max_{\{a_{n\omega}, e_{n\omega}, l_{n\omega}\}} \mathcal{L} \equiv & u(-e_{n\omega} + \omega + g - a_{n\omega}; (1-t)nl_{n\omega}\phi(e_{n\omega}) + (1+r)a_{n\omega} + g) \\ & - v(l_{n\omega}) + \mu_{n\omega}(a_{n\omega} + a_o), \end{aligned} \quad (6)$$

$\mu_{n\omega}$  is the Kuhn-Tucker multiplier on the credit constraint (2). The multiplier  $\mu_{n\omega}$  is the shadow price for borrowing more than the borrowing limit, i.e., it measures the marginal increase in individual utility if the individuals' borrowing limit  $a_o$  increases with one unit. We should note that the shadow price  $\mu_{n\omega}$  is different for individuals with different  $n$  and

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<sup>4</sup>Hubbard and Judd (1986) and Aiyagari (1995) show that capital taxation is welfare-improving with binding credit constraints because capital taxation results in redistribution from non-credit constrained individuals (who do save) to credit constrained individuals (who do not save). Consequently, credit constraints are alleviated, but this comes at a price of distorting the saving decisions of the non-constrained individuals.

$\omega$ . The first-order conditions for utility maximization are given by

$$\frac{\partial \mathcal{L}}{\partial a_{n\omega}} = -u_1(c_{n\omega}^1, c_{n\omega}^2) + (1+r)u_2(c_{n\omega}^1, c_{n\omega}^2) + \mu_{n\omega} = 0, \quad (7)$$

$$\mu_{n\omega} \geq 0, \quad \mu_{n\omega} = 0 \text{ if } a_{n\omega} + a_o > 0, \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial e_{n\omega}} = -u_1(c_{n\omega}^1, c_{n\omega}^2) + u_2(c_{n\omega}^1, c_{n\omega}^2)(1-t)nl_{n\omega}\phi'(e_{n\omega}) = 0, \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial l_{n\omega}} = u_2(c_{n\omega}^1, c_{n\omega}^2)(1-t)n\phi(e_{n\omega}) - v'(l_{n\omega}) = 0. \quad (10)$$

If individuals are not credit constrained ( $\mu_{n\omega} = 0$ ), the consumption and educational choices of the household can be summarized as

$$\frac{u_1(c_{n\omega}^1, c_{n\omega}^2)}{u_2(c_{n\omega}^1, c_{n\omega}^2)} = (1-t)nl_{n\omega}\phi'(e_{n\omega}) = 1+r. \quad (11)$$

Intertemporal consumption choices are not distorted since the marginal rate of intertemporal substitution in consumption equals one plus the interest rate, which is the marginal rate of intertemporal transformation. The optimality condition for investment in education equates the marginal costs of investing one unit of resources in education ( $1+r$ ) with the marginal benefits of one unit of resources invested in education ( $(1-t)nl_{n\omega}\phi'(e_{n\omega})$ ). Note that the marginal benefits of education increase if individuals supply more labor. Hence, labor and education are complements in generating gross income. As long as the marginal income tax rate is positive, the tax system distorts educational investments, since the marginal benefits are taxed, whereas the marginal costs are not.

For credit constrained individuals ( $\mu_{n\omega} > 0$ ) we have  $a_{n\omega} = -a_o$ , and we obtain

$$\frac{u_1(c_{n\omega}^1, c_{n\omega}^2)}{u_2(c_{n\omega}^1, c_{n\omega}^2)} = (1-t)nl_{n\omega}\phi'(e_{n\omega}) > 1+r. \quad (12)$$

The credit constraint creates a wedge in intertemporal consumption choices, i.e., a difference between marginal rate of intertemporal transformation ( $1+r$ ) and marginal rate of intertemporal substitution ( $\frac{u_1(\cdot)}{u_2(\cdot)}$ ), implying that individuals would like to transfer more consumption from the second to the first period if they could. Thus, a binding credit constraint renders income in the first period relatively more valuable to the agent than in the second period. Investment in education of credit-constrained individuals is distorted by the borrowing constraint, since the marginal returns to investment in human capital ( $(1-t)nl_{n\omega}\phi'(e_{n\omega})$ ) are larger than the marginal returns to financial saving ( $1+r$ ).

We can define the implicit tax  $\pi_{n\omega}$  on human capital investment arising from the credit constraint as:

$$\pi_{n\omega} \equiv 1 - (1+r)\frac{u_2(\cdot)}{u_1(\cdot)}. \quad (13)$$

$\pi_{n\omega}$  measures to which extent the intertemporal consumption choices are distorted. An

intertemporal consumption wedge implies that  $\pi_{n\omega} > 0$ , and  $\frac{u_1(\cdot)}{u_2(\cdot)} > 1 + r$ . If the credit constraint is slack, there is no distortion caused by imperfect capital markets:  $\pi_{n\omega} = 0$ , and the standard Euler-equation applies.

Using the definition of  $\pi_{n\omega}$ , the first-order condition for educational investment can be rewritten as

$$(1 - \pi_{n\omega})(1 - t)nl_{n\omega}\phi'(e_{n\omega}) = 1 + r. \quad (14)$$

From this equation we can see that human capital investment is reduced, because the binding credit constraint acts as an additional, implicit tax on the return from human capital investment. Nevertheless, the value of  $\pi_{n\omega}$  is different for individuals differing in both  $n$  and  $\omega$ . In particular, it decreases with initial wealth until it becomes zero when individuals are not credit constrained. It increases with ability  $n$  – for given levels of initial wealth  $\omega$  –, because more able agents have a higher marginal return to education ( $nl_{n\omega}\phi'(\cdot)$ ) and, consequently, would like to borrow more in order to finance larger investment in education.

First-order conditions are necessary, but not sufficient due to the positive feedback between learning and labor supply. The second-order condition requires that  $\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0$ , where  $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$ ,  $\alpha_{n\omega} \equiv \frac{\phi''(e_{n\omega})e_{n\omega}}{\phi'(e_{n\omega})}$  and  $\varepsilon_{n\omega} \equiv \left(\frac{v''(l_{n\omega})l_{n\omega}}{v'(l_{n\omega})}\right)^{-1}$  denote the elasticity of the human capital production function, the elasticity of the marginal return in human capital production function, and the elasticity of labor supply, respectively (see Appendix A.1). A sufficiently low elasticity of labor supply  $\varepsilon_{n\omega}$ , a sufficiently low elasticity of the human capital production function  $\beta_{n\omega}$ , and a sufficiently high elasticity of the marginal return to human capital investment (in absolute value) ensure that the feedback between labor supply and education dampens out and interior solutions are obtained. We assume in the remainder that the second-order conditions are always respected.

The first-order conditions and the household budget constraints jointly determine optimal investment in education, labor supply, and consumption choices as functions of the policy parameters, ability  $n$  and initial wealth  $\omega$ . By indicating the optimized values with an asterisk, we can write the indirect utility function as

$$V(g, t; n, \omega) \equiv u(c_{n\omega}^{1*}, c_{n\omega}^{2*}) - v(l_{n\omega}^*), \quad (15)$$

Applying Roy's lemma yields the following derivatives with respect to the policy instruments:  $\frac{\partial V_{n\omega}}{\partial g} = u_1(\cdot) + u_2(\cdot)$ , and  $\frac{\partial V_{n\omega}}{\partial t} = -u_2(\cdot)nl_{n\omega}\phi'(e_{n\omega})$ .

For later reference, we also derive the Slutsky-equations for education and labor supply (see Appendix A.2). With capital market failures, deriving the compensated demand and supply functions is not trivial, because the exact timing of the compensation to keep utility fixed matters. If the credit constraint is slack, one unit of compensation given in first period is the same as the discounted value of one unit of compensation given in second period. However, if the credit constraint is binding, the value of one unit of

compensation given in first period is higher than the discounted value of one unit of compensation in the second-period. We derive the Slutsky-equations where a uniform income compensation is given in both periods, e.g. by a higher lump-sum transfer:

$$\frac{\partial e_{n\omega}}{\partial t} = \frac{\partial e_{n\omega}^c}{\partial t} - \frac{u_2(\cdot)}{u_1(\cdot) + u_2(\cdot)} n l_{n\omega} \phi(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial g}, \quad (16)$$

$$\frac{\partial l_{n\omega}}{\partial t} = \frac{\partial l_{n\omega}^c}{\partial t} - \frac{u_2(\cdot)}{u_1(\cdot) + u_2(\cdot)} n l_{n\omega} \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial g}, \quad (17)$$

where  $e_{n\omega}^c$  denotes the compensated demand for education, and  $l_{n\omega}^c$  denotes the compensated supply of labor.

### 3 Optimal taxation without redistribution

This and the next sections derive optimal tax policies with and without redistributive concerns. We assume that the government is benevolent and has full commitment. That is, the government announces the tax schedule before individuals make their decisions and fully commits to it.<sup>5</sup> In this section we discuss optimal taxation when individuals are all identical and there are, consequently, no redistributive concerns. We therefore suppress the subscripts  $n$  and  $\omega$ . Moreover, we assume that the initial wealth of the representative individual is not sufficient to finance the optimal level of human capital investment. Consequently, the credit constraint is binding and educational investment is inefficiently low. The case with a slack credit constraint is very straightforward. In particular, first-best would then be obtained, since all individual choices would be efficient and the government would have access to lump-sum taxes.

We focus on optimal tax policy when age-specific lump-sum transfers are not available to the government. If age-specific lump-sum transfers would be available in a setting with a representative agent, it would follow trivially that the credit constraint could be perfectly removed without any efficiency costs. In particular, a policy with age-specific transfers can be viewed as a government loan where the government provides an amount of lump-sum income to each young individual and requires them to pay it back, including interest costs, with a lump-sum tax when they are old. Consequently, government can act perfectly as a lender to replace the missing capital market without resorting to distortionary taxes on labor income. In heterogeneous agent settings, which we will analyze in the next section, a first-best optimum would require both age-specific *and* individualized lump-sum transfers, which are not feasible due to the informational constraints we have imposed on  $n$  and  $\omega$ .

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<sup>5</sup>However, in view of the sunk character of the educational investment, the optimal policy is generally not time-consistent. Therefore, a benevolent government may want to renege on its announcements and re-optimize taxes after investments have been made, see also Pereira (2009).

The tax system thus consists of a flat tax on labor income and uniform lump-sum transfers in both periods. Without loss of generality we assume that there are no exogenous government expenditures.<sup>6</sup> Tax revenue from labor taxation is used only to finance lump-sum transfers  $g$  in both periods. The government budget constraint is therefore given by

$$tnl\phi(e) - g = (1 + r)g. \quad (18)$$

The net tax payment in the second period should be equal to the transfer in the first period plus interest. Note that we express the government budget constraint in terms of second-period income. We assume that government is not credit-constrained, as opposed to individual households. Intuitively, private markets will make government borrowing available, since the government can effectively collateralize human capital through the tax system. By the government's ability to tax income, the government can secure claims on the future returns from human capital (Jacobs and Van Wijnbergen, 2007). Hence, the government implicitly acts as a lender, but the individual pays back its 'loan' through a labor tax. Consequently, alleviating the credit constraint is costly because labor supply is distorted, and the first-best allocation cannot be obtained any more.

The government chooses  $g$  and  $t$  in order to maximize indirect utility of the representative individual. The Lagrangian  $\mathcal{W}$  for maximizing social welfare is given by

$$\max_{\{g,t\}} \mathcal{W} \equiv V(g, t) + \eta(tnl\phi(e) - g(2 + r)), \quad (19)$$

where  $\eta$  is the shadow value of public resources. The government optimally chooses the lump-sum transfer  $g$  in both periods and the labor tax rate  $t$  to maximize the indirect utility of the representative agent.

The optimal uniform lump-sum transfer  $g$  satisfies (see Appendix A.3):

$$\frac{u_1(\cdot) + u_2(\cdot)}{\eta} + tnl\phi'(e) \frac{\partial e}{\partial g} + tn\phi(e) \frac{\partial l}{\partial g} = 2 + r, \quad (20)$$

where we used Roy's lemma:  $\frac{\partial V}{\partial g} = u_1(\cdot) + u_2(\cdot)$ . Equation (20) states that the marginal social benefit of providing one unit of income in both periods (including the indirect income effects on the tax bases) should be equal to the marginal resource cost of providing one unit of income in both periods (see also Atkinson and Stiglitz, 1980).

The first-order condition for optimal income tax rate can be reformulated as (see Appendix A.3):

$$\frac{t}{1-t} = \frac{\pi(1-\rho)}{\varepsilon_{lt} + \beta\varepsilon_{et}}. \quad (21)$$

where  $\varepsilon_{lt} \equiv -\frac{\partial l^c}{\partial t} \frac{1-t}{l}$  and  $\varepsilon_{et} \equiv -\frac{\partial e^c}{\partial t} \frac{1-t}{e}$  denote the compensated tax elasticities of labor

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<sup>6</sup>An exogenous revenue requirement would not change our main result that progressive income taxes are optimal.

supply and educational investment, respectively,  $\beta \equiv \frac{\phi'(e)e}{\phi(e)}$  is the elasticity of human capital production function, and  $1 - \rho \equiv \frac{1+r}{2+r-\pi} = \left(1 + \frac{u_2(\cdot)}{u_1(\cdot)}\right)^{-1}$ . The optimal tax trades off the welfare gains of alleviating the credit constraint (numerator) against the efficiency costs of doing so (denominator). At the optimum, the welfare gains of alleviating the credit constraint should be equalized to its efficiency costs. The more individuals are credit constrained, as measured by a higher value of  $\pi$ , the larger is the welfare gain of a higher tax rate. The compensated tax elasticities  $\varepsilon_{lt}$  and  $\beta\varepsilon_{et}$  measure the tax distortions on labor supply and educational investment. The more elastic labor supply or educational investments respond to the tax rate  $t$ , the larger are tax distortions, and the lower should be the optimal tax rate.

$\rho \equiv \frac{1-\pi}{2+r-\pi}$  measures the inefficiency of an age-independent tax system with uniform lump-sum transfers in both periods compared to a tax system where the transfer is provided only in the first period. In the latter case we would obtain  $\rho = 0$ . Intuitively, for a given tax rate (and, therefore, for a given level of efficiency costs) the resources available to be transferred to the first period are lower when the same amount has to be transferred to the second period as well. Hence, for one unit of revenue raised in second period by the labor tax only  $\frac{1+r}{2+r}$  can be transferred to the first period. Due to the ‘leak’ of the transfers to the second period, the credit constraint is alleviated to a lesser extent, and the optimal tax rate is lower as a result. The relative share of tax revenue that can be transferred to the first period increases if a higher interest rate ( $r$  higher) or more severe capital market failures ( $\pi$  higher) make intertemporal transfers less costly to the government than to households. The reason is that government faces a lower relative price for first-period consumption, i.e.  $(1+r)$ , than households, i.e.  $\frac{1+r}{1-\pi}$ .

Note that our efficiency case for progressive income taxation does not rely exclusively on the endogeneity of human capital investments. Indeed, the optimal income tax would be progressive even when human capital would be exogenous ( $\beta = 0$ ). Therefore, we formally prove the numerical findings by Hubbard and Judd (1985) that optimal income taxes are progressive when individuals are borrowing constrained.

Even in the absence of redistributive concerns, the optimal labor tax rate is positive. The distortionary income tax helps to reduce a pre-existing non-tax distortion in capital markets. We thus provide second-best argument for employing distortionary income taxation for efficiency reasons. See also the introduction for references to the literature on efficient income taxation in models with rat-races and habit persistence, distorted labor markets, and missing insurance markets.

If investment in education and labor supply would both become perfectly inelastic, the labor tax would become completely non-distortionary and the first-best allocation could be obtained. The labor tax then has become a second-period lump-sum tax, which differs from the first-period lump-sum tax  $g$ . The results of this section are summarized

in the following proposition.

**Proposition 1.** *The optimal labor tax is positive for efficiency reasons when agents are credit constrained. Taxing labor income relaxes credit constraints because it transfers income from non-constrained towards constrained phases in the life-cycle. The optimal tax rate strikes a balance between the welfare gains from alleviating the credit constraints and the efficiency losses of distortionary taxation on labor supply and human capital investment.*

## 4 Optimal taxation with redistribution

Until now we have shown the optimality of positive labor tax assuming identical agents. In this section we allow the individuals to differ in their initial wealth  $\omega$  and their innate ability  $n$ . By doing so, we introduce redistributive concerns in the optimal tax problem. Since we assume that neither  $\omega$  nor  $n$  are observable to the government, individualized lump-sum transfers that are conditioned on either ability or initial wealth are excluded. Consequently, the government has to rely on distortionary labor income taxation to redistribute income between individuals.

Like before, revenues from the labor tax are used to finance the lump-sum transfers in both periods. With heterogeneous agents age-specific lump-sum transfers that are conditioned both on initial wealth and on learning ability would be required to achieve the first-best, which is not feasible due to the presumed non-observability of  $\omega$  and  $n$ .

We can write the government budget constraint as

$$t \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} n l_{n\omega} \phi(e_{n\omega}) dF(n, \omega) = (2 + r) g. \quad (22)$$

The government maximizes a social welfare function, which is a concave sum of individual indirect utilities  $V(g, t; n, \omega)$

$$\int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \Psi(V(g, t; n, \omega)) dF(n, \omega), \quad \Psi' > 0, \quad \Psi'' \leq 0. \quad (23)$$

The social welfare function is utilitarian if  $\Psi' = 1$ , and it is Rawlsian if it features  $\Psi' = 0$  for all agents, except for the individual with the lowest utility.

We follow Diamond (1975) by defining the net social marginal valuation of one unit of income, measured in monetary units, for individuals with ability  $n$  and initial wealth  $\omega$  as:

$$b_{n\omega} = \Psi'(V_{n\omega}) \frac{u_1(\cdot) + u_2(\cdot)}{\eta} + t n l_{n\omega} \phi'(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial g} + t n \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial g}. \quad (24)$$

By using Roy's lemma the optimal lump-sum transfer  $g$  is such that (see Appendix A.4)

$$\overline{b_{n\omega}} = 2 + r, \quad (25)$$

where  $\overline{b_{n\omega}} = \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} b_{n\omega} dF(n, \omega)$  denotes the average social value of income. The lump-sum transfer  $g$  should be set such that the average social marginal valuation of one unit income in both periods equals its resource costs.

In order to characterize the optimal income tax, we define the distributional characteristic of labor income as (see Atkinson and Stiglitz, 1980)

$$\xi = 1 - \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{b_{n\omega} z_{n\omega}}{\overline{b_{n\omega} z_{n\omega}}} dF(n, \omega) > 0. \quad (26)$$

$\xi$  measures the marginal increase in social welfare expressed in monetary units, as a fraction of taxed labor income, of a marginal redistribution through the tax system.  $\xi$  is the negative normalized covariance between  $b_{n\omega}$ , the welfare weight of agent  $n\omega$ , and gross labor income  $z_{n\omega}$ . We (realistically) assume that i) the correlation between endowments and abilities is non-negative, and ii) income effects are sufficiently small, so as to ensure that the distributional characteristic is always positive. That is, a positive value of  $\xi$  implies that agents with a higher gross labor income  $z_{n\omega}$  have a lower social welfare weight  $b_{n\omega}$ .  $\xi$  is therefore a measure for the strength of redistributive concerns implied by the social welfare function (23) given the amount of before-tax inequality.  $\xi$  is zero if the social welfare weights  $b_{n\omega}$  for all individuals are equal, so that the government does not want to redistribute any income, or if there is no inequality in labor earnings  $z_{n\omega}$ , so that taxing labor earnings does not redistribute (income differences can only arise from differences in initial wealth).<sup>7</sup>

By using the definition of  $\xi$ , equations (14) and (25) the optimal labor tax  $t$  can be expressed as (see Appendix A.4)

$$\frac{t}{1-t} = \frac{\xi + \overline{\pi_{n\omega}(1-\rho_{n\omega})}}{\overline{\varepsilon_{lt,n\omega}} + \overline{\beta_{n\omega}\varepsilon_{et,n\omega}}}. \quad (27)$$

where  $\overline{\pi_{n\omega}(1-\rho_{n\omega})} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{b_{n\omega} z_{n\omega}}{b_{n\omega} z_{n\omega}} \pi_{n\omega} (1-\rho_{n\omega}) dF(n, \omega)$  is a weighted average for the marginal welfare gain of relaxing the credit constraint where  $b_{n\omega} z_{n\omega}$  is used as the weight. Similarly,  $\overline{\varepsilon_{lt,n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{z_{n\omega} \varepsilon_{lt,n\omega}}{z_{n\omega}} dF(n, \omega)$  and  $\overline{\beta_{n\omega}\varepsilon_{et,n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{z_{n\omega} \beta_{n\omega} \varepsilon_{et,n\omega}}{z_{n\omega}} dF(n, \omega)$  denote the income-weighted averages of the compensated labor supply and education elasticities, respectively.

Individuals with a different ability  $n$  or initial wealth  $\omega$  face different credit constraints.

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<sup>7</sup>Because of the non-observability of initial wealth, the government cannot tax initial wealth to redistribute income. For individuals with same labor income, but different initial wealth levels, the distributional characteristic of labor income is zero although they feature different welfare weights  $b_{n\omega}$ . In this case, the government will not tax labor income because the labor tax cannot help to reduce inequality.

Therefore, the individual utility gains from relaxing the credit constraints differ across individuals. For agents that are not constrained at all, the welfare gain of alleviating the credit constraint is zero ( $\pi_{n\omega} = 0$ ).  $\overline{\pi_{n\omega}(1 - \rho_{n\omega})}$  gives the average welfare gain – weighted by welfare weights and gross income – from alleviating all potentially relevant credit constraints. The weighted average tax elasticities in labor and education measure the average efficiency costs of taxing labor income. Therefore, the optimal tax rate shows the trade-off between, on the one hand, the welfare gain from redistribution and alleviating the credit constraints and, on the other hand, the efficiency costs of taxing labor income.

In the presence of credit constraints, taxing labor income improves efficiency by encouraging educational investments and reducing the intertemporal distortions in consumption. As a result, the conflict between redistribution and efficiency is less severe, which can be seen by rewriting equation (27) as

$$\frac{t}{1-t} (\overline{\varepsilon_{lt,n\omega}} + \overline{\beta_{n\omega}\varepsilon_{et,n\omega}}) - \overline{\pi_{n\omega}(1 - \rho_{n\omega})} = \xi. \quad (28)$$

The welfare gains from relaxing credit constraints reduce the efficiency costs of redistributive taxation and increases the optimal tax rate – for a given desire to redistribute income  $\xi$ . Even without any desire for income redistribution ( $\xi = 0$ ), there is a positive labor tax rate, implying a case for distortionary taxation only on grounds of efficiency, as we have demonstrated above for homogenous agents. When credit constraints are irrelevant, ( $\overline{\pi_{n\omega}(1 - \rho_{n\omega})} = 0$ ), equation (27) reduces to the standard expression for the optimal linear income tax with endogenous human capital formation (see, for example, Jacobs, 2005; Bovenberg and Jacobs, 2005).

Although the welfare gains from redistribution  $\xi$  and those from alleviating credit constraints enter equation (27) additively, they cannot be separated when determining the optimal tax rate  $t$ . This can be seen by noting that  $b_{n\omega}$  enters the term  $\overline{\pi_{n\omega}(1 - \rho_{n\omega})}$  through the weights for the individual utility gains from a less tight credit constraint. To see this, suppose that initial wealth is equally distributed. Then, higher-ability individuals feature lower social welfare weights  $b_{n\omega}$ , and they would be more credit constrained, because they invest more in education. The weighted average welfare gain  $\overline{\pi_{n\omega}(1 - \rho_{n\omega})}$  is lower than that without any redistributive concerns (where  $b_{n\omega}$  is constant). The reason is that high-ability individuals have a lower welfare weight  $b_{n\omega}$  and therefore the welfare gains for relaxing the credit constraints at high-ability individuals are discounted by lower social welfare weights. Consequently, the optimal tax rate for alleviating the credit constraint is lower than without redistribution. Alternatively, if high-ability individuals remain more credit constrained, they invest less in education, which reduces income inequality and, therefore, mitigates the redistribution problem.

Similarly, if individuals have identical abilities, but differ in initial wealth, individu-

als with lower initial wealth are more credit constrained and also have a larger welfare weight. As long as initial wealth is not observable, the government can achieve some redistribution by alleviating the credit constraint to a larger extent, since alleviating credit constraints helps the individuals with less initial wealth more than the ones with more initial wealth. The optimal tax rate is therefore higher when redistributive concerns are present, compared to the case where they are absent.

However, the effect of credit constraints on the optimal tax rate tends to be smaller lower if initial wealth and ability are more (positively) correlated. On the one hand, credit constraints are less severe, since high-ability individuals also have more initial wealth to finance their higher investment in education, i.e.  $\pi_{nw}$  is lower. On the other hand, credit constraints for high-ability individuals are weighted less in view of the redistributive concerns of the government.

We also note that, even without any desire to redistribute income, the optimal tax rate is not the same as that in (21). If the government has no redistributive concerns ( $b_{nw}$  is constant), we have  $\xi = 0$ . The optimal tax rate with heterogeneous agents is different than that with homogeneous agents, since the welfare gains and tax distortions are weighted by gross income. Credit constraints facing high-income agents are more important than credit constraints facing low-income agents, because alleviating credit constraints for them has larger efficiency effects. Similarly, the tax distortions of high-income earners are relatively more important than the tax distortions of low-income earners, since the elasticities are weighted with income. We summarize the results of this section in Proposition 3.

**Proposition 2.** *The optimal tax rate trades-off the welfare gains from alleviating credit constraints and equity against the efficiency costs of labor tax. The efficiency-enhancing effects of redistribution improve the trade-off between redistribution and efficiency. For given levels of initial wealth, the optimal income tax is lower if high-ability individuals are relatively more credit constrained and mitigating credit constraints results in larger inequality. For given levels of ability, redistributive concerns raise the income tax, since credit constraints of poorer individuals are weighted more heavily.*

## 5 Conclusion

This paper has formulated a two-period life-cycle model of saving, labor supply, and human capital investment when individuals differ in their ability and their initial wealth to analyze optimal income taxes. Binding borrowing constraints can prevent individuals to optimally smooth consumption over their life-cycle and to optimally invest in human capital. We have demonstrated that the optimal linear income tax is always positive – even in the absence of any redistributive concerns. A distortionary income tax is

optimal because it relaxes borrowing constraints by redistributing resources from the unconstrained to the borrowing constrained stages of the life-cycle. Hence, redistribution allows for better consumption smoothing and larger investments in human capital. The progressive income tax is thus a second-best instrument to correct the non-tax distortion in the capital market. The equity-efficiency trade-off is therefore less severe when progressive income taxes mitigate capital market imperfections.

In future research we would like to analyze the optimal setting of capital income taxes in our model of saving and learning with heterogeneous agents and borrowing constraints. We expect that capital taxes are positive. Aiyagari (1995) has shown that, with binding credit constraints, capital taxes are optimally positive in infinite horizon Ramsey models. Hubbard and Judd (1986) showed that this is also true in life-cycle models. Jacobs and Bovenberg (2009) demonstrated that capital taxes are optimally positive even in the absence of binding credit constraints, since the capital tax reduces the disincentives of the labor tax on human capital investments.

## A Appendix

### A.1 Second-order conditions

We will first derive the second-order conditions of the household's maximization problem when credit constraints are slack and then when credit constraints are binding.

#### A.1.1 Slack credit constraint

We employ a two-step budgeting procedure to derive the second-order conditions. We assume homothetic sub-utility in consumption  $u$  over  $c^1$  and  $c^2$  and we define the real price-index for consumption  $p_c$  such that  $p_c \equiv \frac{c_{n\omega}^2 + (1+r)c_{n\omega}^1}{u(c_{n\omega}^1, c_{n\omega}^2)}$ . Due to homotheticity the consumption price  $p_c$  is independent of  $n$  and  $\omega$ . Using the budget constraint

$$(1+r)c_{n\omega}^1 + c_{n\omega}^2 = (1-t)n\phi(e_{n\omega})l_{n\omega} + (1+r)(-e_{n\omega} + \omega + g) + g, \quad (29)$$

we can rewrite the individual maximization problem as an unconstrained maximization problem:

$$\max_{\{e_{n\omega}, l_{n\omega}\}} \frac{(1-t)n\phi(e_{n\omega})l_{n\omega}}{p_c} + \frac{(1+r)(-e_{n\omega} + \omega + g) + g}{p_c} - v(l_{n\omega}). \quad (30)$$

The first-order conditions are given by

$$(1-t)n\phi'(e_{n\omega})l_{n\omega} - (1+r) = 0, \quad (31)$$

$$(1-t)n\phi(e_{n\omega}) - v'(l_{n\omega})p_c = 0. \quad (32)$$

Hence, the Hessian matrix with second-order derivatives is

$$H \equiv \begin{bmatrix} (1-t)n\phi''l_{n\omega} & (1-t)n\phi' \\ (1-t)n\phi' & -v''p_c \end{bmatrix}. \quad (33)$$

The first principal minor,  $(1-t)n\phi''l_{n\omega}$  is negative, because by assumption  $\phi''(e_{n\omega}) < 0$ . Therefore, for the Hessian to be negative semi-definite, the second principal minor should be positive:

$$-(1-t)n\phi''l_{n\omega}v''p_c - ((1-t)n\phi')^2 > 0. \quad (34)$$

By defining the wage elasticity of labor supply as  $\varepsilon_{n\omega} \equiv \left(\frac{v''(l_{n\omega})l_{n\omega}}{v'(l_{n\omega})}\right)^{-1}$ , the elasticity of human capital production function as  $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$ , the elasticity of the marginal return to education as  $\alpha_{n\omega} \equiv \frac{\phi''(e_{n\omega})e_{n\omega}}{\phi'(e_{n\omega})}$ , and using the first-order condition for labor supply, we can rewrite the above inequality as

$$\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0. \quad (35)$$

Since human capital production function is concave ( $\phi''(e_{n\omega}) < 0$ ),  $\alpha_{n\omega}$  is negative. The second-order condition thus requires that the elasticity of labor supply and the elasticity of human capital production function should be sufficiently small and the elasticity of the marginal return to education is sufficiently large (in absolute terms) so as to avoid corner solutions. In the second stage of the budgeting procedure, individuals maximize  $u(c_{n\omega}^1, c_{n\omega}^2)$  subject to the constraint  $p_c u(c_{n\omega}^1, c_{n\omega}^2) = (1+r)c_{n\omega}^1 + c_{n\omega}^2$ . The associated second-order condition  $u_{11}(\cdot)u_{22}(\cdot) - u_{12}^2(\cdot) > 0$  is always satisfied since  $u(\cdot)$  is assumed to be strictly concave.

### A.1.2 Binding credit constraint

With a binding credit constraint, savings are zero ( $a = 0$ ). Hence, we can again obtain an unconstrained maximization problem upon substitution of budget constraints in the utility function:

$$\max_{\{e_{n\omega}, l_{n\omega}\}} u(-e_{n\omega} + \omega + g, (1-t)n\phi(e_{n\omega})l_{n\omega} + g) - v(l_{n\omega}). \quad (36)$$

The first-order conditions are given by

$$-u_1(\cdot) + u_2(\cdot)(1-t)n\phi'(e_{n\omega})l_{n\omega} = 0, \quad (37)$$

$$u_2(\cdot)(1-t)n\phi(e_{n\omega}) - v'(l_{n\omega}) = 0. \quad (38)$$

The Hessian matrix  $H$  with second-order partial derivatives is given by

$$H \equiv \begin{bmatrix} u_{11} - 2u_{12}(1-t)n\phi'l_{n\omega} & -u_{12}(1-t)n\phi + u_{22}(1-t)^2n^2\phi\phi'l_{n\omega} \\ +u_{22}((1-t)n\phi'l_{n\omega})^2 + u_2(1-t)n\phi''l_{n\omega} & +u_2(1-t)n\phi' \\ -u_{12}(1-t)n\phi + u_{22}(1-t)^2n^2\phi\phi'l_{n\omega} & u_{22}((1-t)n\phi)^2 - v''(l_{n\omega}) \\ +u_2(1-t)n\phi' & \end{bmatrix}. \quad (39)$$

For the Hessian matrix to be negative semi-definite, the principal minors should switch signs. The first principal minor

$$u_{11} - 2u_{12}(1-t)n\phi'l_{n\omega} + u_{22}((1-t)n\phi'l_{n\omega})^2 + u_2(1-t)n\phi''l_{n\omega} < 0, \quad (40)$$

is negative since all terms of (40) are negative under the assumptions that the consumption utility function is concave in both arguments ( $u_{11} < 0$ ,  $u_{22} < 0$ ), the human capital production function is concave ( $\phi'' < 0$ ), and consumption in two periods are complementary ( $u_{12} \geq 0$ ). The second principal minor should therefore be positive:

$$\begin{aligned} & \left( u_{11} - 2u_{12}(1-t)n\phi'l_{n\omega} + u_{22}((1-t)n\phi'l_{n\omega})^2 + u_2(1-t)n\phi''l_{n\omega} \right) \times \left( u_{22}((1-t)n\phi)^2 - v'' \right) \\ & - \left( -u_{12}(1-t)n\phi + u_{22}(1-t)^2n^2\phi\phi'l_{n\omega} + u_2(1-t)n\phi' \right)^2 > 0. \end{aligned} \quad (41)$$

Use the first-order conditions (37) and (38) and the definitions  $\varepsilon_{n\omega} \equiv \left( \frac{v''(l_{n\omega})l_{n\omega}}{v'(l_{n\omega})} \right)^{-1}$ ,  $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$  and  $\alpha_{n\omega} \equiv \frac{\phi''(e_{n\omega})e_{n\omega}}{\phi'(e_{n\omega})}$  to reformulate the above inequality as

$$\begin{aligned} & u_{11}u_{22} + \frac{\alpha_{n\omega}}{e_{n\omega}}u_{22}u_1 - \frac{u_2}{\varepsilon_{n\omega}}\frac{\beta_{n\omega}}{e_{n\omega}} \left( u_{11}\frac{u_2}{u_1} - 2u_{12} + u_{22}\frac{u_1}{u_2} + u_2\frac{\alpha_{n\omega}}{e_{n\omega}} \right) \\ & > u_{12}^2 + \left( u_2\frac{\beta_{n\omega}}{e_{n\omega}} \right)^2 - 2u_{12}u_2\frac{\beta_{n\omega}}{e_{n\omega}} + 2u_{22}u_1\frac{\beta_{n\omega}}{e_{n\omega}}. \end{aligned} \quad (42)$$

We assume that the utility function is linear homogenous and we use the properties  $u_{11}(\cdot)c_1 = -u_{12}(\cdot)c_2$  and  $u_{12}(\cdot)c_1 = -u_{22}(\cdot)c_2$  to derive

$$u_{11}(\cdot)u_{22}(\cdot) - u_{12}^2(\cdot) = 0. \quad (43)$$

Using (43) we can rewrite (42) as

$$\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < -\frac{u_{22}u_1e_{n\omega}}{u_2^2} \left( 1 - \frac{\alpha_{n\omega}\varepsilon_{n\omega}}{\beta_{n\omega}} + 2\varepsilon_{n\omega} \right) - \frac{u_{11}e_{n\omega}}{u_1} + 2\frac{u_{12}e_{n\omega}}{u_2}(\varepsilon_{n\omega} + 1). \quad (44)$$

Because  $u_{22} < 0$ ,  $u_{11} < 0$  and  $\alpha_{n\omega} < 0$ , the right-hand-side of equation (44) is always positive. Consequently,

$$\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0 \quad (45)$$

is sufficient for (44) to be fulfilled. Therefore,  $\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0$  is the sufficient second-order condition for the households' maximization problem for both the cases of slack and binding credit constraints.

## A.2 Slutsky-equations

In order to derive the expected utility-compensated substitution effects, we calculate how much lump-sum income  $g$  an individual should receive (pay) in order to keep its utility constant ( $dU_{n\omega} = 0$ ) when the tax rate  $t$  changes. This is equivalent to deriving the expenditure function and applying Shephard's lemma. Totally differentiating the utility function (5) and the budget constraints (1) and (4) yields

$$dU_{n\omega} = u_1(\cdot)dc_{n\omega}^1 + u_2(\cdot)dc_{n\omega}^2 - v'(l_{n\omega})dl_{n\omega} = 0, \quad (46)$$

$$dc_{n\omega}^1 = -de_{n\omega} + d\omega + dg - da_{n\omega}, \quad (47)$$

$$dc_{n\omega}^2 = (1-t)nl_{n\omega}\phi'(e_{n\omega})de_{n\omega} + (1-t)n\phi(e_{n\omega})dl_{n\omega} - nl_{n\omega}\phi(e_{n\omega})dt + (1+r)da_{n\omega} + dg. \quad (48)$$

Substitute  $dc_{n\omega}^1$  and  $dc_{n\omega}^2$  in  $dU_{n\omega}$  to find

$$\begin{aligned} dU_{n\omega} &= (u_2(\cdot)(1-t)nl_{n\omega}\phi'(e_{n\omega}) - u_1(\cdot))de_{n\omega} + u_1d\omega \\ &+ (u_1(\cdot) + u_2(\cdot))dg + (u_2(\cdot)(1+r) - u_1(\cdot))da_{n\omega} \\ &+ (u_2(\cdot)(1-t)n\phi(e_{n\omega}) - v'(l_{n\omega}))dl_{n\omega} - u_2(\cdot)nl_{n\omega}\phi(e_{n\omega})dt = 0 \end{aligned} \quad (49)$$

$(u_2(\cdot)(1-t)nl_{n\omega}\phi'(e_{n\omega}) - u_1(\cdot))de_{n\omega}$  and  $(u_2(\cdot)(1-t)n\phi(e_{n\omega}) - v'(l_{n\omega}))dl_{n\omega}$  are both equal to zero from the first-order conditions. The term  $(u_2(\cdot)(1+r) - u_1(\cdot))da_{n\omega}$  is equal to zero as well both in case of a binding credit constraint ( $da_{n\omega} = 0$ ) and a slack credit constraint ( $u_2(\cdot)(1+r) - u_1(\cdot) = 0$ ). Thus, we have

$$dU_{n\omega} = -u_2(\cdot)nl_{n\omega}\phi(e_{n\omega})dt + u_1(\cdot)d\omega + (u_1(\cdot) + u_2(\cdot))dg = 0. \quad (50)$$

For utility compensation through a transfer given in both periods, the Slutsky-equations are thus given by

$$\frac{\partial e_{n\omega}}{\partial t} = \frac{\partial e_{n\omega}^c}{\partial t} - \frac{u_2(\cdot)nl_{n\omega}\phi(e_{n\omega})}{u_1(\cdot) + u_2(\cdot)} \frac{\partial e_{n\omega}}{\partial g}, \quad (51)$$

$$\frac{\partial l_{n\omega}}{\partial t} = \frac{\partial l_{n\omega}^c}{\partial t} - \frac{u_2(\cdot)n\phi(e_{n\omega})}{u_1(\cdot) + u_2(\cdot)} \frac{\partial l_{n\omega}}{\partial g}. \quad (52)$$

### A.3 Optimal taxation without redistribution

This appendix derives the optimal income tax when all agents are identical and age-specific transfers are not available. The Lagrangian for maximization of social welfare is

$$\max_{\{g,t\}} \mathcal{W} \equiv V(g, t) + \eta(tnl\phi(e) - (2+r)g). \quad (53)$$

The first-order conditions for the government's maximization problem are given by

$$\frac{\partial \mathcal{W}}{\partial g} = u_1(\cdot) + u_2(\cdot) - \eta(2+r) + \eta \left( tnl\phi'(e) \frac{\partial e}{\partial g} + tn\phi(e) \frac{\partial l}{\partial g} \right) = 0, \quad (54)$$

$$\frac{\partial \mathcal{W}}{\partial t} = -u_2(\cdot)nl\phi(e) + \eta nl\phi(e) + \eta \left( tnl\phi'(e) \frac{\partial e}{\partial t} + tn\phi(e) \frac{\partial l}{\partial t} \right) = 0. \quad (55)$$

Using the Slutsky-equations (51) and (52) we can derive from (54) that

$$\frac{u_1(\cdot) + u_2(\cdot)}{\eta} + tnl\phi'(e) \frac{\partial e}{\partial g} + tn\phi(e) \frac{\partial l}{\partial g} = 2+r. \quad (56)$$

Using the definitions of the compensated elasticities  $\varepsilon_{lt} \equiv -\frac{\partial l^c}{\partial t} \frac{1-t}{l}$  and  $\varepsilon_{et} \equiv -\frac{\partial e^c}{\partial t} \frac{1-t}{e}$ , (55) can be rewritten as

$$-\frac{u_2(\cdot)}{u_1(\cdot) + u_2(\cdot)} \left( \frac{u_1(\cdot) + u_2(\cdot)}{\eta} + tnl\phi'(e) \frac{\partial e}{\partial g} + tn\phi(e) \frac{\partial l}{\partial g} \right) + 1 - \frac{t}{1-t} \frac{\phi'(e)e}{\phi(e)} \varepsilon_{et} - \frac{t}{1-t} \varepsilon_{lt} = 0. \quad (57)$$

Using (56) and  $\frac{u_1}{u_2} = \frac{1+r}{1-\pi}$  we find

$$\frac{t}{1-t} = \frac{\frac{\pi}{1+\frac{1-\pi}{1+r}}}{\varepsilon_{lt} + \beta \varepsilon_{et}} = \frac{\pi(1-\rho)}{\varepsilon_{lt} + \beta \varepsilon_{et}}. \quad (58)$$

where  $\beta \equiv \frac{\phi'(e)e}{\phi(e)}$  is the elasticity of human capital production function and  $\rho \equiv \frac{1-\pi}{2+r-\pi}$ .

### A.4 Optimal taxation with redistribution

This appendix derives the optimal taxation when agents differ in initial wealth and abilities and when the government does not have access to age-specific transfers. The Lagrangian for the government's maximization problem is:

$$\max_{\{g,t\}} \mathcal{W} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} (\Psi(V(g, t; n, \omega)) + \eta(tnl_{n\omega}\phi(e_{n\omega}) - (2+r)g)) dF(n, \omega). \quad (59)$$

The first-order conditions are

$$\frac{\partial \mathcal{W}}{\partial g} = \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left[ \Psi'(V_{n\omega}) \frac{\partial V_{n\omega}}{\partial g} + \eta t \left( nl_{n\omega} \phi'(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial g} + n \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial g} \right) - \eta(2+r) \right. \\ \left. - \eta(2+r) \right] dF(n, \omega) = 0, \quad (60)$$

$$\frac{\partial \mathcal{W}}{\partial t} = \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left[ \Psi'(V_{n\omega}) \frac{\partial V_{n\omega}}{\partial t} + \eta t \left( nl_{n\omega} \phi'(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial t} + n \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial t} \right) \right. \\ \left. + \eta nl_{n\omega} \phi(e_{n\omega}) \right] dF(n, \omega) = 0. \quad (61)$$

Using Roy's lemma, (60) can be written as

$$\int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left( \Psi'(V_{n\omega}) \frac{u_1(\cdot) + u_2(\cdot)}{\eta} + tnl_{n\omega} \phi'(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial g} + tn \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial g} \right) dF(n, \omega) = 2 + r. \quad (62)$$

Define  $b_{n\omega} \equiv \Psi'(V_{n\omega}) \frac{u_1(\cdot) + u_2(\cdot)}{\eta} + tnl_{n\omega} \phi'(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial g} + tn \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial g}$  as the social valuation of one unit income for individuals with ability  $n$  and initial wealth  $\omega$ . We derive

$$\overline{b_{n\omega}} = 2 + r, \quad (63)$$

where  $\overline{b_{n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} b_{n\omega} dF(n, \omega)$ . Using Roy's lemma  $\frac{\partial V_{n\omega}}{\partial t} = -u_2(\cdot) nl_{n\omega} \phi(e_{n\omega})$ , the Slutsky-equations (51) and (52),  $\overline{b_{n\omega}} = 2 + r$  and  $\frac{u_1(\cdot)}{u_2(\cdot)} = \frac{1+r}{1-\pi_{n\omega}}$  we can rewrite equation (61) as

$$\int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left( -\frac{b_{n\omega} nl_{n\omega} \phi(e_{n\omega})}{\overline{b_{n\omega}}} \left( 1 - \frac{\pi_{n\omega}}{1 + \frac{1-\pi_{n\omega}}{1+r}} \right) - \frac{t}{1-t} nl_{n\omega} \phi(e_{n\omega}) (\beta_{n\omega} \varepsilon_{et,n\omega} + \varepsilon_{lt,n\omega}) \right) dF(n, \omega) \\ + \overline{z_{n\omega}} = 0, \quad (64)$$

where  $\overline{z_{n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} nl_{n\omega} \phi(e_{n\omega}) dF(n, \omega)$  is average labor income in the economy and  $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega}) e_{n\omega}}{\phi(e_{n\omega})}$  denotes the elasticity of the human capital production function for individuals with ability  $n$  and initial wealth  $\omega$ . We define the distributional characteristic as  $\xi \equiv 1 - \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{b_{n\omega} z_{n\omega}}{\overline{b_{n\omega}} \overline{z_{n\omega}}} dF(n, \omega) = -\frac{cov(b_{n\omega}, z_{n\omega})}{\overline{b_{n\omega}} \overline{z_{n\omega}}}$ , which measures the social concern for redistribution. The first-order condition (61) can be further simplified to

$$\xi + \overline{\pi_{n\omega}(1 - \rho_{n\omega})} = \frac{t}{1-t} \overline{\beta_{n\omega} \varepsilon_{et,n\omega} + \varepsilon_{lt,n\omega}}, \quad (65)$$

where  $1 - \rho_{n\omega} \equiv \frac{1+r}{2+r-\pi_{n\omega}}$  and we define  $\overline{\pi_{n\omega}(1 - \rho_{n\omega})} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{b_{n\omega} z_{n\omega}}{\overline{b_{n\omega}} \overline{z_{n\omega}}} \pi_{n\omega} (1 - \rho_{n\omega}) dF(n, \omega)$  as the weighted average of the welfare gain from alleviating the credit constraint and  $\overline{\beta_{n\omega} \varepsilon_{et,n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{z_{n\omega}}{\overline{z_{n\omega}}} (\beta_{n\omega} \varepsilon_{et,n\omega}) dF(n, \omega)$  and  $\overline{\varepsilon_{lt,n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \frac{z_{n\omega}}{\overline{z_{n\omega}}} (\varepsilon_{lt,n\omega}) dF(n, \omega)$  denote the weighted-average elasticities of human capital and labor supply. The optimal tax rate

can be therefore characterized by

$$\frac{t}{1-t} = \frac{\xi + \overline{\pi_{n\omega}(1 - \rho_{n\omega})}}{\beta_{n\omega}\varepsilon_{et,n\omega} + \overline{\varepsilon_{lt,n\omega}}}. \quad (66)$$

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