Appendix – Optimal Redistributive Tax and Education Policies in General Equilibrium

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Appendix

For later reference, we provide the main model equations, i.e., the utility function, the production function for human capital, the household budget constraint, and the first-order conditions for labor and education, respectively:

$$u_n(c_n, l_n) \equiv c_n - \frac{l_n^{1+1/\varepsilon_n}}{1+1/\varepsilon_n}, \quad n = 1, 2, \tag{1}$$

$$z_n \equiv w_n h_n l_n = w_n a_n \phi_n(e_n) l_n, \quad \phi'(e_n) > 0, \quad \phi''(e_n) < 0, \quad n = 1, 2, \tag{2}$$

$$c_n = (1-t)\left(w_n a_n \phi(e_n) l_n - (1-s) p e_n\right) + b, \quad n = 1, 2. \tag{3}$$

$$l_n = \left((1-t) w_n a_n \phi(e_n)\right)^{\varepsilon_n}, \quad n = 1, 2. \tag{4}$$

$$w_n a_n \phi'(e_n) l_n = (1-s)p, \quad n = 1, 2. \tag{5}$$

Second-order conditions of individual optimization

By substituting the household budget constraint (3) into the utility function (1) to eliminate c_n, we arrive at the following unconstrained maximization problem

$$\max_{\{l_n, e_n\}} U_n = (1-t)\left(w_n a_n \phi(e_n) l_n - (1-s) p e_n\right) + b - \frac{l_n^{1+1/\varepsilon_n}}{1+1/\varepsilon_n}. \tag{6}$$

The first-order conditions are

$$\frac{\partial U_n}{\partial l_n} = (1-t) w_n a_n \phi(e_n) - l_n^{1/\varepsilon_n} = 0, \tag{7}$$

$$\frac{\partial U_n}{\partial e_n} = (1-t)\big(w_n a_n \phi'(e_n) l_n - (1-s)p\big) = 0. \tag{8}$$

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The second-order partial derivatives are ordered in the Hessian matrix H:

$$H \equiv \begin{bmatrix} -\frac{1}{\varepsilon_n} l_n^{1/\varepsilon_n - 1} & (1-t) w_n a_n \phi'(e_n) \\ (1-t) w_n a_n \phi'(e_n) & (1-t) w_n a_n l_n \phi''(e_n) \end{bmatrix}. \tag{9}$$

For utility to reach a maximum, the Hessian matrix should be negative definite. This is the case if the leading principal minors of H switch signs. The first principal minor is negative. Therefore, the second leading principal minor must be positive, i.e., $-\frac{1}{\varepsilon_n} l_n^{1/\varepsilon_n}(1-t) w_n a_n \phi''(e_n) - ((1-t) w_n a_n \phi'(e_n))^2 > 0$. Using (4) to eliminate l_n and substituting (2), this inequality can be written as

$$\mu_n \equiv 1 - \beta(1 + \varepsilon_n) > 0. \tag{10}$$

Elasticities of individual behavior

Log-linearizing (5) (using $\phi(e_n) = e_n^\beta$) gives

$$\tilde{l}_n + (\beta - 1)\tilde{e}_n = -\tilde{s}. \tag{11}$$

A tilde stands for a relative change (i.e., $\tilde{l}_n \equiv dl_n/l_n$, $\tilde{e}_n \equiv de_n/e_n$, et cetera), except for the tax rate and the subsidy rates, where $\tilde{t} \equiv dt/(1-t)$, and $\tilde{s} \equiv ds/(1-s)$.

Expression (4) implies that labor supply depends only on the after-tax wage rate $(1 - t) w_n a_n \phi(e_n)$ so that

$$\tilde{l}_n = \varepsilon_n(\beta \tilde{e}_n - \tilde{t}). \tag{12}$$

Substituting (12) into (11) to eliminate $\tilde{l}_n$, an expression for $\tilde{e}_n$ is found

$$\tilde{e}_n = \frac{1}{\mu_n}\tilde{s} - \frac{\varepsilon_n}{\mu_n}\tilde{t}. \tag{13}$$

Substitution of (13) into (12), gives a solution for $\tilde{l}_n$

$$\tilde{l}_n = \frac{\beta \varepsilon_n}{\mu_n}\tilde{s} - \frac{\varepsilon_n(1-\beta)}{\mu}\tilde{t}. \tag{14}$$

Therefore, the following elasticities of l_n and $e_n$with respect to the policy parameters are obtained

$$\varepsilon_n^{lt} \equiv -\frac{\partial l_n}{\partial t}\frac{(1-t)}{l_n} = \frac{\varepsilon_n(1-\beta)}{\mu_n}, \tag{15}$$

$$\varepsilon_n^{et} \equiv -\frac{\partial e_n}{\partial t}\frac{(1-t)}{e_n} = \frac{\varepsilon_n}{\mu_n}, \tag{16}$$

$$\varepsilon_n^{ls} \equiv \frac{\partial l_n}{\partial s}\frac{(1-s)}{l_n} = \frac{\beta \varepsilon_n}{\mu_n}, \tag{17}$$

$$\varepsilon_n^{es} \equiv \frac{\partial e_n}{\partial s}\frac{(1-s)}{e_n} = \frac{1}{\mu_n}. \tag{18}$$

Table 1: Optimal non-linear tax and education policies – $\sigma = 0.5$

ε_1		0.3	0.25	0.2	0.15	0.1
$T_1^{'}$		-14.2%	-5.8%	-2.3%	-0.9%	-0.1%
$T_2^{'}$		43.1%	37.5%	35.3%	34.0%	33.4%
$S_1^{'}$		12.5%	5.5%	2.6%	0.9%	0.1%
$S_2^{'}$		-41.4%	-14.5%	-6.2%	-2.0%	-0.3%
ε_2	0.35	0.3	0.25	0.2	0.15	0.1
$T_1^{'}$	-6.4%	-4.7%	-3.5%	-2.7%	-2.0%	-1.5%
$T_2^{'}$	37.3%	36.4%	35.8%	35.3%	34.9%	34.6%
$S_1^{'}$	6.0%	4.5%	3.4%	2.6%	2.0%	1.5%
$S_2^{'}$	-13.6%	-10.2%	-7.9%	-6.2%	-4.9%	-3.8%
β			0.25	0.2	0.15	0.1
$T_1^{'}$			-4.8%	-2.7%	-1.6%	-1.0%
$T_2^{'}$			36.7%	35.3%	34.6%	34.1%
$S_1^{'}$			4.6%	2.6%	1.6%	1.0%
$S_2^{'}$			-11.2%	-6.2%	-3.8%	-2.4%
ω_1	0.5	0.4	0.3	0.2	0.1	
$T_1^{'}$	0.0%	-0.8%	-1.9%	-3.6%	-6.5%	
$T_2^{'}$	0.0%	17.4%	30.1%	39.9%	48.2%	
$S_1^{'}$	0.0%	0.8%	1.9%	3.5%	6.1%	
$S_2^{'}$	0.0%	-1.9%	-4.5%	-8.4%	-15.6%	

Robustness analysis

Tables 1 and 2 demonstrate that the results are completely robust to varying the elasticity of substitution over intervals that are considered empirically relevant.[1]

[1] Some empty cells appear in table 1 because the parameters of the model could not be too widely varied for values of $\sigma = 0.5$. The reason is that it must be ensured that the high-skilled worker has higher earnings than the low-skilled worker, so that a well-defined distribution problem results.

Table 2: Optimal non-linear tax and education policies – $\sigma = 2.5$

ε_1	0.75	0.5	0.35	0.3	0.25	0.2	0.15
$T_1^{'}$	-9.2%	-5.6%	-3.1%	-2.2%	-1.4%	-0.8%	-0.3%
$T_2^{'}$	17.8%	23.2%	27.5%	29.1%	30.5%	31.8%	32.8%
$S_1^{'}$	8.4%	5.3%	3.0%	2.2%	1.4%	0.7%	0.3%
$S_2^{'}$	-18.5%	-11.2%	-6.2%	-4.5%	-2.9%	-1.5%	-0.5%

ε_2	0.75	0.5	0.35	0.3	0.25	0.2	0.15
$T_1^{'}$	-0.1%	-0.2%	-0.4%	-0.5%	-0.6%	-0.8%	-1.0%
$T_2^{'}$	33.1%	32.8%	32.5%	32.3%	32.1%	31.8%	31.5%
$S_1^{'}$	0.1%	0.2%	0.4%	0.5%	0.6%	0.7%	0.9%
$S_2^{'}$	-0.2%	-0.5%	-0.9%	-1.0%	-1.3%	-1.5%	-1.8%

β	0.4	0.3	0.25	0.2	0.15	0.1
$T_1^{'}$	-1.1%	-0.9%	-0.8%	-0.8%	-0.7%	-0.6%
$T_2^{'}$	30.7%	31.4%	31.7%	31.8%	32.0%	32.1%
$S_1^{'}$	1.1%	0.9%	0.8%	0.7%	0.7%	0.6%
$S_2^{'}$	-2.7%	-2.0%	-1.7%	-1.5%	-1.4%	-1.2%

ω_1	0.5	0.4	0.3	0.2	0.1	0
$T_1^{'}$	0.0%	-0.3%	-0.6%	-0.9%	-1.1%	-1.3%
$T_2^{'}$	0.0%	15.9%	27.3%	35.9%	42.6%	48.0%
$S_1^{'}$	0.0%	0.3%	0.6%	0.9%	1.1%	1.3%
$S_2^{'}$	0.0%	-0.7%	-1.2%	-1.8%	-2.3%	-2.7%