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# Lose a Fly to Catch a Trout?

## On Dual Growth Accounting in a Dynamic Economy[1]

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## Abstract

# Lose a Fly to Catch a Trout?

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## 1 Introduction

In this paper we address the observation that measured productivity growth did not increase with the introduction of ICT in the early 1980s. This puzzling observation has been expressed by Solow (1987) as: "You can see the computer age everywhere but in the productivity statistics". A more refined statement of the puzzle is presented in Table 1.1: why did it take so long before ICT delivers measurable contributions to growth?

A number of explanations for this paradoxical finding have been put forward. Oliner and Sichel (1994) claim that the stock of ICT capital was so small that it obviously had a negligible influence on productivity. Another explanation of the paradox is that productivity increases are difficult to measure because it is hard to capture quality improvements due to increased use of computers that are not reflected by higher prices (see Griliches, 1994). Griliches argues that an ever increasing part of the economy is 'unmeasurable'.

**Table 1.1 ICT Contributions to US growth, 1974-2000 (percentage points per year)**

| | 1974-90 | 1991-95 | 1996-2000 |
|---|---|---|---|
| Computer Hardware Capital Growth | 28.8 | 17.5 | 35.9 |
| Income Share[a] | 1.0 | 1.4 | 1.8 |
| Computer Software Capital Growth | 14.7 | 12.8 | 22.2 |
| Income Share[a] | 0.8 | 2.0 | 2.5 |
| Communication Equipment Capital Growth | 7.7 | 3.6 | 7.9 |
| Income Share[a] | 1.5 | 1.9 | 2.0 |
| **ICT Capital Contribution** | **0.52** | **0.57** | **1.36** |
| Computer Sector TFP Growth | 11.4 | 11.3 | 14.2 |
| Output Share[a] | 1.1 | 1.1 | 1.6 |
| Semi-conductor TFP Growth | 30.9 | 22.3 | 49.4 |
| Output Share[a] | 0.3 | 0.5 | 0.9 |
| **ICT TFP Contribution[b]** | **0.17** | **0.24** | **0.50** |
| **Total ICT Contribution (as % GDP/Person Growth)** | **0.69** | **0.79** | **1.86** |
| | **(30.4)** | **(54.6)** | **(56.3)** |

[a] Percentage
[b] Based on output-weighted contribution of computers plus 60 per cent of output-weighted contribution of semi-conductors
Source: Derived from Oliner and Sichel (2000).

This paper argues that the ICT general purpose technology leads to investment and learning within firms and that in growth-accounting exercises these investments are not appropriately taken into account. We show that, if in-house investment to accommodate the new general purpose technology is important, standard measures register growth too late. The intuition, opposite to the common sense, is that a general purpose technology bias in technological change leads to a slowdown in measured output growth.[4] The reason is that technology has to be produced and implemented by scarce resources ('lose a fly') before it is going to pay off ('catch a trout'). Thus, a technological revolution distracts resources from directly productive activities. This explains the paradox!

This paper contributes more generally to the literature on measurement in the 'knowledge-based economy'. We do so by explicitly spelling out the different fallacies in growth-accounting and by providing a solution to the difficulties. Our main contribution is that a dual approach to growth-accounting provides a superior indication of productivity growth. If the economy is in *a stationary state* the dual is more accurate if productivity growth in learning or research activity differs from that in final goods production. In an economy *in transition* (meaning not in a stationary state) the dual is accurate, whereas the primal ignores changes in allocation between learning time and production time and it ignores the (implied) changes in relative productivity of learning and production. The important advantage of the dual, of the common production function-based growth accounting, is that it takes the cost function as starting point. In an economy where unobservable knowledge production is important the dual approach to growth-accounting overcomes the problem that knowledge *investment* is *unobservable* whereas the *cost* of investment in knowledge is *observable*.

Finally, we illustrate that the dual account for growth was indeed above the primal in the early days of the general purpose technology whereas from the late 1980s the reverse was true. This illustrates that our explanation of the Solow paradox is consistent with the facts.

One point has to be made still and that is whether in-house or non-market investment is important. We argue this is the case. Cohen et al. (2000) argue that even formal R&D is shifted towards being kept secret in a non-market way. If firms apply for patents it is only to create a patent-trade portfolio. Romer (1995) argues that we ought to think of the economy as whole as using hard-, soft and wetware (the latter denotes skills in brain) to produce hardware (we call this final or tangible output) and software (knowledge or intangible output). He argues knowledge production takes plase everywhere. Even assembly line workers are investing in software or knowledge: "managers....encourage workers on the assembly line to consider alternative assembly sequences, to experiment and to communicate their successes to others. In effect, they have made their production workers in knowledge workers as well."

[4] A technology is biased if it does not affect the productivity of all sectors and/or factors proportionally.

Contributions that are related are Howitt (1998) and Barro (1999). Howitt (1998) discusses measurement problems with respect to knowledge investment and focusses on endogenous depreciation rates. We differ from Howitt-- apart from the modelling approach we use -- in relating the knowledge investment problem explicitly to dual and primal growth accounting. Moreover we focus on R&D with an incremental character that is performed by firms internally. Hence, our interpretation is not necessarily restricted to formal R&D but applies to all types of jobs with learning opportunities.

Barro (1999) explicitly discusses growth-accounting in an R&D-based growth model. Barro ignores the cost of R&D and the fact that the R&D production function differs from the final output production function. His focus on productivity of gross output (including R&D) serves analytical convenience but ignores a serious measurement problem as we demonstrate in this paper. We complement Barro by introducing an R&D production function and by focussing on the measurement practice; that is, on calculating productivity of output, including R&D production that is often not sold in the market (especially the R&D with an incremental character that is performed by firms internally).

The rest of this paper is organized as follows. Section 2, discusses measurement problems in a knowledge-based economy in general. In section 3 we develop an example of an endogenous growth model of technological change to explore the relationship between a GPT-type technology revolution and the severeness of measurement problems. Section 4 illustrates the empirical relevance of our approach and section 5 concludes. The main text provides all the ingredients and results whereas derivations and proofs can be found in the appendices.

## 2 On Growth Accounting in a Knowledge-based economy

The usual way to infer something about technological change is by accounting for growth. To explore how technological change affects average income, consider the following macroeconomic production function relating outputs to inputs:

$$Y = D\, G(K,L) \ , \tag{1}$$

where $K$ and $L$ stand for the aggregate measures of capital and labour respectively.

**Assumption 1** Production technology is assumed to be of the Hicks-neutral type.

The Solow residual, or measured TFP, equals output growth that is not explained by changes in factor inputs. Time differentiating **(1)** and rearranging yields the fundamental relation between growth of outputs and growth of inputs:

$$\hat{D} = \hat{Y} - \chi_K \hat{K} - \chi_L \hat{L} \ , \tag{2}$$

where a hat denotes a relative change, and $\chi$s denote production elasticities, hence $\chi_i \equiv (D \cdot (\partial G / \partial i) \cdot i) / Y, \quad i = K, L$. Once the appropriate output and input concepts are defined, and when they are measured accurately, *D* is the technology indicator of interest.

> **Definition 1** $\hat{D}$ is the *measured* productivity growth; that is the growth rate delivered by standard growth accounting.

In a knowledge-based economy several pitfalls are to be taken into account to avoid that technological change turns up in the productivity statistics incorrectly. To explore the pitfalls of standard growth accounting in a knowledge-based economy we explicitly introduce knowledge and knowledge accumulation. We reformulate the production function as:

$$Y = A(f, T) \, G(K, uL) \ , \tag{3}$$

We assume that technology index *A*, is a function of *f*; the knowledge stock (we introduce *T*, a time dependent exogenous technology level, for completeness). The fraction of workers producing is *u*. The remainder of the workforce, (1-*u*), is active in knowledge accumulation, that is learning. Hence:

$$f_t = \sum_{\tau \to -\infty}^{t} H(f_\tau, (1 - u_\tau) L_\tau, Q_\tau) \ . \tag{4}$$

where function *H* is some unknown knowledge-accumulation function (we set the depreciation rate of knowledge capital for simplicity equal to zero). *Q* is a productivity factor in learning (a simplified expression for a GPT). Just writing down the expression that formalises the assumption that knowledge is accumulated intentionally allows to pin down the main issues in defining output and input concepts. We make the following assumption:

> **Assumption 2** The fraction of (time) workers spend on learning is unobservable.

This assumption is not hard to defend once you start thinking about learning on the job. For example learning to work with new software costs time, but it is of course hard to find out how much time that is.

## 2.1 The knowledge-input problem

What is the proper input concept? This is a hard issue, but what is clear is that the input concept is related to the output concept. That is, if produced knowledge is not taken into account in the output statistics, shouldn't inputs be adjusted accordingly? We argue that, though it is true that successful knowledge creation raises GDP in the long run, resources should be adjusted for those employed in the accumulation of knowledge. The remainder refers to this issue as the *knowledge input problem*.

To measure productivity appropriately, our model stresses that an adjustment for learning time should be made. Recall the production function: $Y = A(.)G(K, uL)$. To calculate the actual TFP growth rate adjusting for inputs in production to the actual level in necessary, hence:

$$\hat{A} = \hat{Y} - \chi_K \hat{K} - \chi_L \hat{L} - \chi_u \hat{u} \ , \tag{5}$$

where $\chi_i \equiv (A(\partial F/\partial i)i)/Y, \quad i = u, L, K$.

**Definition 2** $\hat{A}$ is the *input-adjusted* growth rate.

The difference between *input* adjusted productivity growth, $\hat{A}$, and measured productivity growth, $\hat{D}$, is $-\chi_u \hat{u}$ (see equation (2)). For expositional ease we ignore the exogenous factor, hence $\hat{L} = 0$. Using the results so far we can derive the following proposition.

**Proposition 1** The difference between input-adjusted (*A*) and measured (*D*) productivity growth is: $\hat{A} = \hat{D} - \breve{\chi}_u \hat{u}$ .

The proposition follows directly from the comparison two measures. From proposition one we derive an important second result

**Proposition 2** Productivity *growth* is measured accurately if the time spend learning is constant, that is if $\hat{u} = 0$.

However if $\hat{u}$ is negative, e.g. due to that the technology shock workers increase learning time, we underestimate total factor productivity if it is measured in the naive way. A computer revolution induces increased learning time which leads to an underestimation of productivity growth. As $u$ is unobserved (by assumption 2) , the estimation bias cannot be inferred indirectly.

## 2.2 The knowledge-output problem

What is the proper output concept? *Output*[5] of knowledge investments, that is for example knowledge-capital stocks and human capital due to on the job learning, are not measured in national accounts (that is, the change in $f$ is not measured). There is no obvious reason to treat additions to the knowledge-capital stock any different from those to the physical capital stock. Nevertheless, the production of capital is only measured if it concerns physical capital, but not if it is intangible knowledge capital. The production of both types of capital requires resources that otherwise could be used for current consumption. The difference is due to the fact that the produced item -- knowledge capital -- is not priced on the market, as the product usually is not sold or is unsaleable. The former holds for patents and the latter, for instance, for organisational capital. It would, however, be appropriate is to include -- alongside consumption and investment in physical capital -- investment in knowledge capital in output. Such a broad output measure is not constructed as additions to the knowledge stock are not easily measured. The remainder refers to this issue as the *knowledge output problem*.

An appropriate assessment of the viability of the whole economy requires thus a more comprehensive output concept, broad output, that includes the value of produced knowledge capital, $q_f \Delta f$, where $q_f$ is the (shadow) price of knowledge capital in terms of final output.

**Definition 3** Broad output, $Y^b$, is defined as $Y^b \equiv DG(K, (1 - u)L) + q_f \Delta f$.

Recall that $\Delta f = H(f_t, (1 - u_t) L_t, Q_t)$. We introduce the following assumption:

**Assumption 3** (i)Capital and labour supply are constant over time $\hat{K} = \hat{L} = 0$.
(ii)Learning time is allocated efficiently; that is the marginal products of labour in production and learning are equalised.
(iii) Function $H()$ is homogenous of degree one in both $f$ and $Q$.
(iv) $\Delta f = \Delta A$

The first assumption is purely for notational convenience. Assumption 3 (ii) is necessary but harmless as the assumption of an efficient allocation is necessary in any growth accounting exercise. Assumption 3 (iii) is notationally convenient and makes the knowledge accumulation

[5] Some of these issues are also discussed in Appendix A to Chapter 12 of Aghion and Howitt (1998). The knowledge *output* problem is referred to by Aghion and Howitt as the knowledge investment problem.

process sensible for long run analysis as this assumption allows for endogenous growth Assumption 3 (iv) says that the productivity of final output is driven entirely by knowledge accumulation. We abstain from other (possibly exogenous) changes in *A* as this does not add any new insights.

We can calculate the productivity growth of broad output, $\hat{A}^*$ following the standard procedure (Appendix A provides a derivation).

**Definition 4** The true measure of productivity for the whole economy is *ideal* TFP: $\hat{A}^*$.

Calculating the measure for ideal TFP gives rise to the following proposition:

**Proposition 3** *Ideal* TFP is:

$$\hat{A}^* = \hat{A} + \frac{Y^B - Y}{Y^B}\left(\hat{q}_f + \hat{Q}\right). \tag{6}$$

Thus *Ideal* TFP is productivity growth in both activities plus a weighted relative price change and a weighted productivity difference between the two sectors. The weights are related to the familiar Domar weights.

The proof is in the appendix. The implication of proposition 4 is that if the value of knowledge capital relative to productivity (-capital) in final output changes, this affects the productivity of the broad economy and thus productivity growth thereof.[6] Moreover if the productivity change in learning activities differs from that in production, the narrow measure of output does not measure this whereas the ideal measure does. From proposition 1 and 3 we can derive the following proposition:

**Proposition 4** *Ideal* TFP is higher (lower) than *input adjusted* TFP (which in turn is higher (lower) that *measured* TFP) if $\hat{u} < (>) 0$.

The proposition is straightforwardly proofed by noting that the weight in proposition 3 is positive and that *u* and $\hat{q}_f + \hat{Q}$ are negatively related. The intuition for the proposition is straightforward: workers in the first place start to learn more intensively on-the-job because the knowledge-investment activity  has become more attractive (for whatever reason). Hence, ignoring the knowledge production part of the economy in growth-accounting -- as the input

[6] Productivity is referred to as productivity capital as the relative price change can be interpreted as indication a capital gain (or loss).

adjustment does -- implies that the part where productivity gains arise is excluded. So far we identified two measurement problems in a learning-based economy. How to solve these problems is a different issue, an issue to which we turn now.

## 2.3 A dual approach to growth accounting

The basic intuition needed to grasp how to solve the measurement problems is to note that the learning activity is chosen optimally by the agents in the economy. Having said this it is fairly intuitive to see that factor rewards necessarily reflect the true state of the economy. This section shows that this intuition is generally correct.

The income-expenditure identity that should hold looks like:

$$rK + wL = Y + q_f \Delta f \,, \tag{7}$$

This says that income should equal broad production. To see that there is no factor reward for *f*, note that most knowledge is accumulated within the firm or on the job (for evidence see Dosi 1988 and Cohen et al., 2000). If *f* referred to patents there would be an explicit price paid for knowledge. Nahuis and Smulders (2002) model patents and in-house knowledge capital accumulation explicitly.

**Assumption 4** Knowledge accumulation is of the in-house type; meaning that knowledge capital is not bought on the market.

This assumption is important as this allows us to measure productivity correctly in a dual way. We could extend the model with market-bought knowledge-capital. This would not change our conclusion substantially (we come back to this later).

Under assumptions 1 to 4, where the latter is only substantial, we can derive the following proposition:

**Proposition 5** The measure for *ideal* TFP is equal to an income share weighted sum of the growth rates of the factor rewards (the *dual* growth account): $\hat{A}^* = \frac{rK}{Y^B}\hat{r} + \frac{wL}{Y^B}\hat{w}$.

The proof is in the appendix. This proposition is our main result and has an important implication: the *ideal* TFP measure can be obtained with observable data. If we extended the model with market-bought knowledge-capital the proposition would be altered as follows: if the reward for market-bought knowledge-capital is observable (e.g. the patent income share and the patent licence fee) proposition 3 would still hold. If this reward is not observable we should restate the proposition as: The measure for *ideal* TFP is **best** approximated by the *dual* growth account.

## 2.4 Some further remarks on measurement

To obtain the prices needed to measure the *ideal* TFP by the dual some caveats need to be taken into account. The price of output is necessary (here normalised to one) and the prices of inputs are required. These prices are available but might not exactly measure what we are after. First, the output price is only measured correctly in a perfectly competitive market, otherwise mark-up pricing interferes (see for an elaboration Roeger (1995)). Second, measured input prices ca be affected by distortions too, for the case of wages for example, efficiency wages might be paid. The measurement of the cost of capital is notoriously difficult, see for example Hsieh (2002) and Young (1998). The latter is also a familiar problem affecting primal growth accounting. However, when these measurement problems are constant over time they do not distort the dual measure for productivity *growth*.

There are two other important measurement issues. First, the measurement accuracy of (narrow) output is problematic due to difficulties with incorporating quality improvements and new goods or varieties. We do not deal explicitly with these issues as this problem, however, is not *specifically* related to knowledge-based growth nor to a GPT. In the remainder we assume that output of goods and services is measured accurately.[7]

Second, the arrival of a new GPT might make existing capital obsolete. Hence, a technology shock could cause capital stocks to decay excessively rapid. Due to discontinuous depreciation, inputs might also not be measured accurately as a consequence. If the national accounts where to account for knowledge as they do for physical capital this problem would even be more pressing. Aghion and Howitt (1998) coin this the *obsolescence* problem. Howitt (1998) deals with this explicitly.

One question remains: are the measurement problems amplified at times of an arrival of a GPT? It turns out convenient to analyse the relations between our two production functions for the knowledge based economy -- **(3)** and **(4)** above -- in a structural model. The next section sets out a minimalist example of an endogenous growth model that relates the arrival of a GPT to observed productivity.

# 3 An example

[7] Thus we assume that observed tangible output equals actual tangible output.

To illustrate the working of the growth accounting procedure developed in the previous section we set up a minimalist example model. For the purpose of illustration, the model should be characterised by transitional dynamics and should have marketable and non-marketable capital. Hence, we set up a model with a few ingredients distinct from the benchmark endogenous growth models (Romer, 1990, Grossman and Helpman, 1991, Aghion and Howitt, 1992). First, to focus on the knowledge output problem, we introduce knowledge capital that is non marketable. That is we introduce non-tradeable knowledge capital alongside a tradeable capital good. Second, to illustrate the knowledge input problem, we introduce non-tradable knowledge for which learning is necessary. Learning is an in-house or firm-specific activity by its non-tradable nature.

Putting the necessary pieces together in a closed-economy general-equilibrium model we have three decision units: first, firms who produce and improve (by producing non-tradable R&D capital) an unique good in a monopolistic environment; second, there are competitive capital-assembly firms producing tradable capital; and finally, households who supply capital and labour. This section explains subsequently the behaviour of these agents and solves for the model. For mathematical convenience the model is continuous in all dimensions.

## 3.1 Production

Let there be one sector of production consisting of firms indexed $j$, with mass 1, engaged in monopolistic competition. Every firm produces one product variety $j$ facing a downward sloping demand function for that variety due to love for variety by consumers. We assume that there is no entry/exit, so the mass of firms is fixed.

Production in firm $j$ of final goods ($x_j$) requires labour ($L_j$), capital goods ($K_j$) and technology-capital or firm-specific knowledge ($f_j$). Firms devote resources to research in order to remain competitive. Time that workers are engaged in production is $u_j L_j$. The fraction of the time endowment, which is normalized to unity, that is spent producing is denoted by $u_j$. The remainder of the time endowment is devoted to research (or learning). Production is designated by a Cobb-Douglas production function:[8]

$$x_j = T f_j^{1-\alpha} K_j^{\alpha} (u_j L_j)^{1-\alpha} , \qquad (8)$$

where $\alpha \in (0,1)$ and $T$ is the exogenously given technology level. This production function features the duplication argument: doubling physical inputs, $K$ and $uL$, leads to a doubling of output. We specify $A$, the productivity of physical inputs, as $A \equiv Tf^{1-\alpha}$.

[8] Time subscripts are suppressed where it leads to no confusion

The production function is homogeneous of degree 1 in $K$ and $f$. The reason is that, eventually, we want a steady-state solution in which $u$ and $L$ are fixed and $K$ and $f$ grow at the same rate. Each firm faces a downward-sloping demand function for its variety:

$$x_j = (p_j/P_X)^{-\varepsilon} X \,, \tag{9}$$

where $\varepsilon > 1$ stands for the elasticity of demand. $P_X$ is the aggregate ideal price index and $X$ is aggregate output (the demand curve is derived from demand by consumers and capital assembly firms):

$$P_X \equiv \left( \int_0^1 p_j^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}, \qquad X \equiv \left( \int_0^1 x_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} . \tag{10}$$

We focus on symmetric equilibria, hence prices and levels of production reduce to: $P_X = p_j = p$ and $X = x_j$ for all $j$.

Firms maximize the net present value of profits, which is the discounted stream of instantaneous profit flows $\pi$, at rate $r$: $\Pi_j = \int_0^\infty \pi_j \exp[-\int_0^t r(v)dv]dt$. Instantaneous profits are given by:

$$\pi_j = p_j x_j - rK_j - wL_j \,, \tag{11}$$

where $wL$ is loan payments to workers. Capital $K$ is hired at a rental rate of $r$ from capital-assembly firms. In order to keep producing profitably, the firm devotes some workers to the production of knowledge $f$ (learning):

$$\dot{f}_j = Q(i) f_j (1 - u_j) L_j \,. \tag{12}$$

where a dot denotes a time derivative, $Q(i)$ denotes the productivity of research with the $i^{\text{th}}$ generation GPT. The firm-specific nature of research implies that the productivity of a firm's research today depends on the accumulated stock of firm-specific knowledge capital.

## 3.2 Capital-assembly

Competitive capital-assembly firms assemble the product varieties costlessly into capital. The production function for capital is:

$$I \equiv \left( \int_0^1 i_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{13}$$

$P_I I$ are total purchases of capital goods (where $P_I$ is the ideal price index). Investment demand for each variety is denoted $i_j$. Produced capital adds to the stock of capital (we abstain from depreciation):

$$I = \dot{K} . \tag{14}$$

The stock of capital generates a rent of $rK$ paid by the monopolistic firms. This rent is paid as a dividend to consumers who supply the capital-assembly firms with funds.

## 3.3 Consumption

There is a representative household having a constant saving rate. Hence the consumption function looks like:

$$C = (1-s)pY , \tag{15}$$

where $s$ is the saving rate[9] and $Y$ is real income. Consumption of different varieties is derived from:

$$C \equiv \left( \int_0^1 c_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} . \tag{16}$$

The demand curves from the capital-assembly firms and consumers are identical and can thus be represented by a single downward-sloping demand curve for each variety ($x_j = c_j + i_j$). Goods-market equilibrium gives the resource constraint of the economy:

$$Y = C + I . \tag{17}$$

## 3.4 Firm behaviour

Firms maximize profits subject to the demand function, accumulation constraints on firm specific capital goods and capital. Solving the firm's optimal control problem by setting up the Hamiltonian we can derive the first-order conditions for maximization of profits (see Appendix

[9] We abstract from an intertemporal utility maximizing framework here to reduce the complexity of the model.

A). In symmetric equilibrium we have: $x_j = X$, $p_j = p$, $L_j = L$, $K_j = K$, $f_j = F$,[10] and $u_j = u$. In the remainder we normalize prices: $p \equiv \varepsilon/(\varepsilon - 1)$. The last normalization implies that the 'mark-up' equals the price of the variety.

We can derive the wage and a no-arbitrage condition for the allocation of time of workers in the production of goods and learning:

$$w = (1 - \alpha)TK^{\alpha}F^{1-\alpha}(uL)^{-\alpha} = q_f Q(i)F \,, \tag{18}$$

where $q_f$ is the co-state variable associated with the accumulation constraint on technology. First, the wage equals the marginal value of time of workers in production and second it equals the marginal value of time devoted to research. The latter is to be multiplied by the relative price of technology in terms of final output, as measured by $q_f$.

The dynamics of this economy are described by the evolution of time spend producing $u$, (see Appendix B):

$$\frac{\dot{u}}{u} = Q(i)uL + \left(\frac{1-\alpha}{\alpha}\right)Q(i)L + (sp - 1)TR^{1-\alpha}(uL)^{1-\alpha} \,, \tag{19}$$

where $R$ is the ratio of the two capital stocks ($R \equiv F/K$). The differential equation describing the evolution of $R$ is given by (see Appendix B):

$$\frac{\dot{R}}{R} = Q(i)(1 - u)L - spTR^{1-\alpha}(uL)^{1-\alpha} \,. \tag{20}$$

## 3.5 Steady state

A steady state is an equilibrium in which $\dot{R} = \dot{u} = 0$. It is simple to show that the steady state is characterised by:

$$\frac{\dot{w}}{w} = \frac{\dot{K}}{K} = \frac{\dot{F}}{F} \equiv g \,, \quad \frac{\dot{q}_f}{q_f} = \frac{\dot{p}_X}{p_X} = 0 \,. \tag{21}$$

We can derive the steady-state share of time spend producing ($u^*$):

$$u^* = \frac{\alpha - sp}{\alpha} \,, \tag{22}$$

[10] Capital *F* denotes an an aggregate variable. We distinguish between F and f to emphasize that in this model knowledge capital is firm specific ($F \equiv \int f_j dj$).

where $\alpha > sp$ is assumed in order to arrive at a feasible steady-state allocation ($0<u<1$).[11] Note that the steady-state value of $u$ depends only on the production technology parameters $\alpha$ , the saving rate $s$ and the inverse of the monopoly mark-up, $p$. Moreover, the technology-capital-stock ratio ($R^*$) is:

$$R^* = \left[\frac{Q(i)(1-u^*)}{spTu^*}\right]^{\frac{1}{1-\alpha}} . \tag{23}$$

Appendix B shows that this equilibrium is saddle-point stable under the condition: $u^*<((1-\alpha)/\alpha)^2$.[12] $u$ is the 'jump' variable and $R$ is the 'predetermined' variable.

The steady-state growth rate of the stocks ($K$ and $F$), output ($Y$), and wages will be equal to:

$$g = Q(i)(1 - u^*)L . \tag{24}$$

## 3.6 Transitional dynamics

The transition dynamics are simple. Figure 3.1 gives the phase portrait in ($R$, $u$)-space. The $\dot{R} = 0$ locus gives all combinations ($R$, $u$) where $R$ is not changing. The $\dot{R} = 0$ locus is downward sloping:

$$\left[\frac{du}{dR}\right]_{\dot{R}=0} = -\left(\frac{1-\alpha}{\frac{u}{1-u}+1-\alpha}\right)\frac{u}{R} < 0 . \tag{25}$$

The $\dot{u} = 0$ locus gives, equivalently, all the points where $u$ is not changing. Its slope is:

$$\left[\frac{du}{dR}\right]_{\dot{u}=0} = \left(\frac{(1-\alpha)\Phi\frac{1}{R}}{Q(i)L - \frac{1-\alpha}{u}\Phi}\right) > 0 , \tag{26}$$

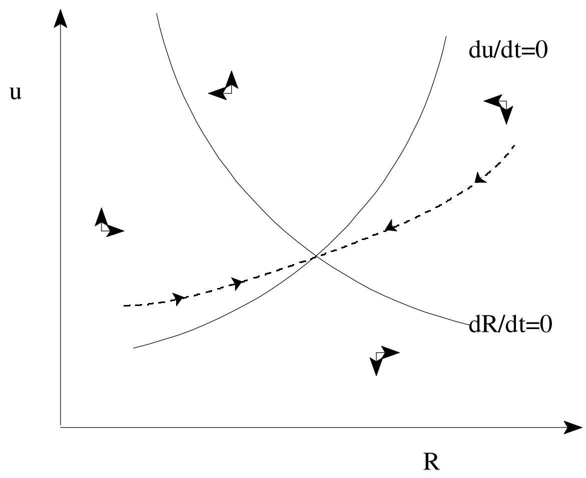
where $\Phi \equiv Q(i)Lu + ((1-\alpha)/(\alpha))Q(i)L$. The denominator is positive as a consequence of the stability condition, i.e. when $u > ((1-\alpha)/\alpha)^2$. The equilibrium can be found where the two phase lines intersect. Starting from an initial ratio of the capital stock and the firm-specific knowledge

[11] This condition is a modified version of the transversality condition known from the Ramsey model that the saving rate must be less than the share of capital or: $\alpha > s$, see e.g. Barro and Sala-i-Martin (1995. p.89).

[12] Alternatively, the stability condition could be rewritten as $(1-u)(1-sp)>(1-\alpha)/\alpha$, which is derived from the transversality condition stating that $f$ cannot grow faster than the interest rate $r$ in the steady state.

stock, a jump in *u* follows to the stable branch, and the economy converges along the stable branch towards the equilibrium.

**Figure 3.1 Phase diagram**

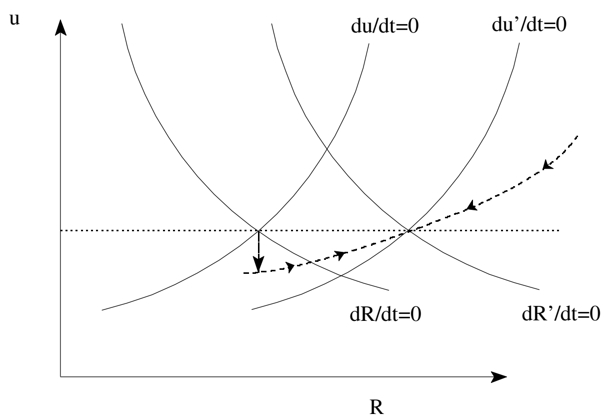



## 3.7 A technology revolution...

A GPT as the computer revolution affects the marginal productivity of research as it opens new opportunities for knowledge-creating activities throughout the economy. This paper ignores the details of a GPT and simply mimics the computer revolution by an increase in *Q*.[13] Accordingly $Q(i)$ increases to $Q(i + 1)$. Hence, the productivity of learning or time spent doing research increases. Time devoted to learning and assimilating the new technology becomes more valuable. Graphically, this can be represented by a rightward shift of both the $\dot{u} = 0$ and the $\dot{R} = 0$ loci, see Figure 3.2.

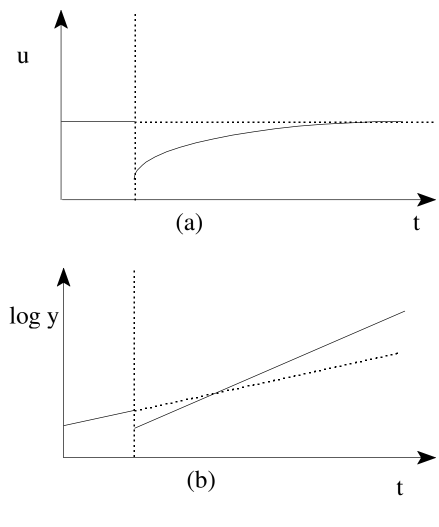
**Figure 3.2 Adjustment to the new steady state**

[13] See for example Helpman and Trajtenberg (1998).




On impact $u$ falls to equate the marginal productivity of workers in learning and producing. Therefore, more time will be spend learning and less time will be spend producing in order to accumulate $F$. Output falls, exactly because $u$ falls, see Figure 3.3a. Over time production recovers as the stock of knowledge increases. Hence, workers reduce time learning because a higher stock of knowledge increases the value of an hour producing, see also Figure 3.3b. The increase in $Q$ leads to a permanently higher growth rate in the long run. The reason is that the productivity of an hour spent learning has increased permanently.[14]

**Figure 3.3 Effects of an increase in Q**




[14] This feature of the model is due to our simple way of introducing the GPT. Nahuis (1998) provides an alternative way of introducing a GPT that, however, comes at the cost of losing analytical tractability.

## 3.8

... and the reason why we don't see itUsing the propositions above it is simple to derive the expression for the difference between the measured and ideal TFP for the specific equations used in the model. Correcting normally measure TFP for input use or learning activities is done by calculating:

$$\hat{A} = \hat{D} - \frac{(1 - u)wL}{Y}\hat{u} \ . \tag{27}$$

Correcting also for non-marketable knowledge-capital production implies a perfect measure for productivity:

$$\hat{A}^* \equiv \hat{Y}^B - \frac{rK}{Y^B}\hat{K} - \frac{wL}{Y^B}\hat{L} = \frac{AF(K,uL)}{Y^B}\hat{A} + \frac{(1-u)wL}{Y^B}\hat{w} \ . \tag{28}$$

This measure simply says that true productivity growth is weighted productivity growth in the two sectors. Substituting for $\hat{w}=\hat{q}_f+\hat{Q}+\hat{F}$ and use that $\hat{q}_f = \ - \alpha\left(\hat{u} + \hat{R}\right) - \hat{Q}$.

$$\hat{D} - \hat{A}^* = \frac{(1-u)\,w}{Y^B}\hat{u} + \frac{(1-u)\,w}{Y^B}\left(\alpha\left(\hat{u} + \hat{R}\right)\right) \ . \tag{29}$$

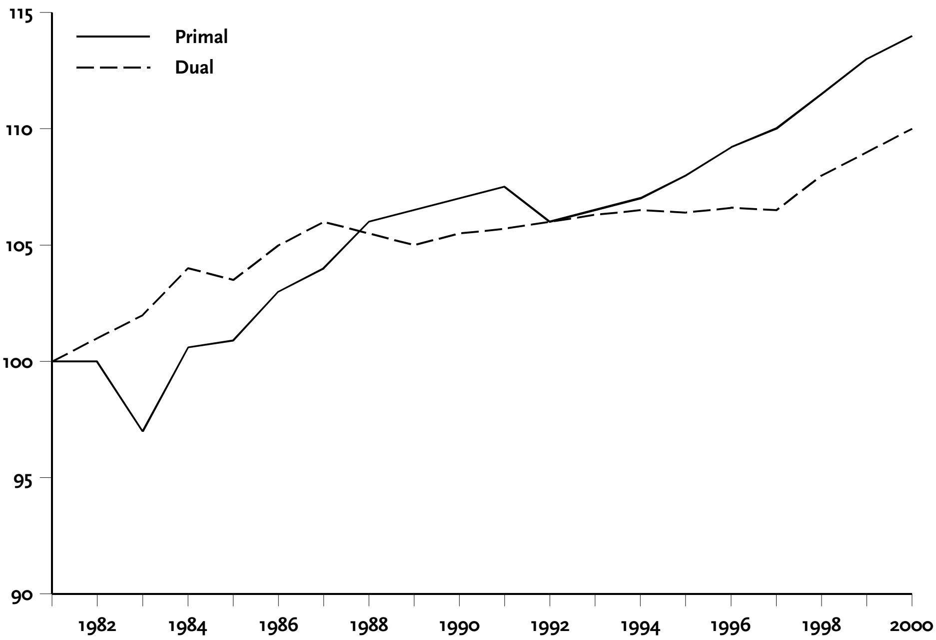
Hence $\hat{D}$ underestimates true productivity growth of broad output ($\hat{A}^*$) for two reasons outside the steady state (remember $\hat{u} < 0$ at the arrival of a GPT). First, the first term on the rhs is the *input* exaggeration; increased learning time or research time leads to an underestimation. The second term indicates productivity growth in the research sector; this again leads to an underestimation of productivity growth. A third factor would appear if growth in the non-measured sector differs from that in the measured sector (we constructed the example model such that both growth rates are equal).[15] It is clear that the Solow residual might record a slowdown while the economy actually is booming. Recall that the true state of the economy is measured by the dual. The next section explores whether there is a difference between actual measures for the dual and the primal.

# 4 Empirical relevance

[15] Different growth rates in the knowledge production function and the final outputs production is not at adds with a steady state (see Barra and Sala-i-Martin, 1995 for an exposition).

The figure below depicts two measures for productivity growth, one based on prices and one based on quantities. It is clear that the dual exceeds the primal measurement until the late 1980s-early 1990s.

**Figure 4.1 US productivity calculated by the primal and the dual**




Source: Marquis and Trehan (2002), figure 3b.

We interpret this as a confirmation of our hypothesis that the productivity effects of the ICT revolution were already present, though not measured in the primal, when Solow stated his paradox (1987).

Before we can claim this, we need to sort out whether it is indeed the 'learning' mechanism that is responsible for the differences between the dual and the primal.[16] The way we proceed is to test the following hypothesis: the degree of underestimation of the primal growth account compared to the ideal TFP measure is higher if learning is more important and knowledge accumulation is thus higher (read: if the ideal or true TFP measure is higher). Thus we estimate $(\hat{\boldsymbol{A}}^* - \hat{\boldsymbol{D}}) = \beta \hat{\boldsymbol{A}}^*$.[17] If there are no imperfections whatsoever, $\beta$ should be zero (the primal and dual would be identical). Our hypothesis is that $\beta$ is (positively) different from zero. We use estimates of primal and dual TFP measures for 19 Singaporese industries for 1975 to 1994 to test for this (Bloch and Tang, 1998). The table shows the regression result testing this hypothesis. There is indeed a significant relation between the degree of learning (dual growth)

[16] Roeger (1995) shows that changes in mark-up rates can also explain the difference between the two measures.

[17] We could analogously estimate $\hat{\boldsymbol{D}} = \beta \hat{\boldsymbol{A}}^*$ and test whether $\beta$ is different from one. Moreover, adding a constant to the regression does also yield an estimate for $\beta$, significantly different from zero.

and the degree of mis-measurement. Given the relatively high R-square of about 0.3 the mechanism is important indeed.[18]

**Table 4.1 Regression results. Dependent variable (Dual TFP - Primal TFP)**

| | (I) |
|---|---|
| Dual TFP | 0.865 |
| | (3.71) |
| Constant | No |
| $R^2$-adj. | 0.33 |
| # observations | 19 |

T-statistics are reported between parentheses

# 5 Conclusion

This paper argued that the arrival of the computer revolution caused a slowdown in measured productivity growth. This is due to erroneous measurement of inputs and erroneous measurement of outputs. We showed that the appropriate way to account for growth in a knowledge-based economy is to apply the dual account.

We constructed a simple endogenous growth model in which workers can allocate their time to production or to learning and R&D to better cope with the new technology. Upon arrival of the new technology workers allocate more time to learning. With respect to productivity growth two conclusions can be drawn. First, output indeed falls on the impact of a computer revolution, as workers reallocate their time from producing to learning, you might argue that the computer revolution *caused* a decline in productivity. This is no actual decline in productivity; less inputs produce less output. To correctly assess productivity, inputs should be corrected for by adjusting for actual time spend *producing* (instead of the common empirical practice to adjust only for hours *worked*). A second perspective, however, is that the computer revolution caused a growth boom. Realising that workers, in the first place, start to learn more *because* learning opportunities are ample makes clear that a productivity measure of broad output -- measuring both knowledge and physical production -- must have gone up!

We showed that our mechanism is relevant empirically and that the difference between primal and dual growth matches the computer revolution.

[18] We assume that both the dual and the primal are measured accurately; at least measured accurately subject to usual measurement error. In the hypothetical case that the dual measure is white noise only we would find a positive relation too.

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## Appendices

# A. Accounting for growth in a knowledge based economy

### A.1 Standard growth accounting

Standard growth accounting ignores the accumulation of firm-specific knowledge capital and hence does not take into account investment time for firm-specific knowledge capital. So start with a standard production function:

$$Y = DG(K,L) \ , \tag{A.1}$$

where $K$, and $L$ respectively measure physical capital and labour. The growth of output is associated with growth in production factors and technology. By differentiating with respect to time, dividing by $Y$ and rewriting this yields:

$$\hat{D} = \hat{Y} - \chi_K \hat{K} - \chi_L \hat{L} \ , \tag{A.2}$$

where $\chi_i \equiv (D(\partial G/\partial i)i)/Y, \ i = K,L$. In practice the $\chi$'s are approximated by the factor income share, which is appropriate if the social marginal product equals the factor prices. The model presented in the main text does not have any externalities related to factor inputs. Therefore, the approximation is indeed appropriate.

### A.2 Growth accounting in a knowledge-based in-house learning economy (proof of prop. 3)

Recall the income-expenditure identity:

$$Y^B = rK + wL = Y + q_f \Delta f \ , \tag{A.3}$$

saying that income should equal broad production. Where $Y = A\,G(K, (1 - u)\,L)$ and that $\Delta f = H(f_t, (1 - u_t)L_t Q)$.

Taking the total differential of the income-production equality yields, after rearranging:

$$\begin{aligned} \dot{Y}^B = \ & wL(\hat{L} + \hat{w}_L) + rK(\hat{r} + \hat{K}) = \\ & G_F \dot{F} + G_K \dot{K} + G_L u \dot{L} + G_u L \dot{u} + H(.)\dot{q}_f + q_f H_F \dot{F} + q_F H_Q \dot{Q} + q_f(1 - u) H_L \dot{L} - q_F L H_U \dot{u} \end{aligned} \tag{A.4}$$

where a subscript denote a partial derivative, a dot denotes a difference and a hat denotes a growth rate. Rearranging and assuming that $\hat{K} = \hat{L} = 0$ gives:

$$\hat{Y}^B = \frac{wL}{Y^B}\hat{w} + \frac{rK}{Y^B}\hat{r} = \frac{G_F F}{Y^B}\hat{F} + \frac{G_u uL - H_u q_F uL}{Y^B}\hat{u} + \frac{H(.)q_f}{Y^B}\hat{q}_f + \frac{q_f H_F F}{Y^B}\hat{F} + \frac{q_F H_Q Q}{Y^B}\hat{Q} \,. \tag{A.5}$$

The second term on the right hand side is zero if there is an efficient allocation of research or learning time (this we assume). If we, furthermore, assume that learning function *H* is homogeneous of degree 1 in *F*, the two weights for $\hat{F}$ sum to unity. Thus we obtain:

$$\hat{Y}^B = \frac{wL}{Y^B}\hat{w} + \frac{rK}{Y^B}\hat{r} = \hat{F} + \frac{Y^B - Y}{Y^B}\hat{q}_f + \frac{q_F H_Q Q}{Y^B}\hat{Q} \,. \tag{A.6}$$

If we assume that learning function *H* is homogeneous of degree 1 in *Q*,we obtain:

$$\hat{Y}^B = \frac{wL}{Y^B}\hat{w} + \frac{rK}{Y^B}\hat{r} = \hat{F} + \frac{Y^B - Y}{Y^B}\left(\hat{q}_f + \hat{Q}\right) . \tag{A.7}$$

## B. Solution of the Model

### B.1. Firms

The current-value Hamiltonian of the optimal control problem for firm *j* reads as:

$$\mathcal{H}_j = p_j x_j - wL_j - rK_j + q_{fj} Q(i) f_j (1-u_j) L_j \,. \tag{B.1}$$

The first order conditions for an optimum are:

$$\frac{\partial \mathcal{H}_j}{\partial K_j} = r \Leftrightarrow p_j \left(\frac{\varepsilon - 1}{\varepsilon}\right) \alpha T K_j^{\alpha-1} f_j^{1-\alpha} (u_j L_j)^{1-\alpha} = r, \tag{B.2}$$

$$\frac{\partial \mathcal{H}_j}{\partial u_j} = 0 \Leftrightarrow p_j \left(\frac{\varepsilon - 1}{\varepsilon}\right) (1-\alpha) T K_j^{\alpha} f_j^{1-\alpha} (u_j L_j)^{1-\alpha} u_j^{-1} - q_{fj} Q(i) f_j L_j = 0. \tag{B.3}$$

$$\frac{\partial \mathcal{H}_j}{\partial L_j} = p_j\left(\frac{\varepsilon - 1}{\varepsilon}\right)(1-\alpha)TK_j^{\alpha} f_j^{1-\alpha}(u_j L_j)^{1-\alpha} L_j^{-1} - w - q_{fj} Q(i) f_j (1-u_j) = 0 \ , \tag{B.4}$$

$$\begin{aligned}\frac{\partial \mathcal{H}_j}{\partial f_j} = rq_{fj} - \dot{q}_{fj} \Leftrightarrow p_j\left(\frac{\varepsilon - 1}{\varepsilon}\right)(1-\alpha)TK_j^{\alpha} f_j^{-\alpha}(u_j L_j)^{1-\alpha} + \\ q_{fj} Q(i)(1-u_j) L_j = rq_{fj} - \dot{q}_{fj} \ .\end{aligned} \tag{B.5}$$

In addition we impose the transversality condition:

$$\lim_{t \to \infty} f_j \exp\left[-\int_0^t r(v)\, dv\right] = 0 \ . \tag{B.6}$$

The second and third FOC's give the no-arbitrage condition for the allocation of time of workers over production of goods and learning:

$$\begin{aligned} w &= p_j\left(\frac{\varepsilon - 1}{\varepsilon}\right)(1-\alpha)TK_j^{\alpha} f_j^{1-\alpha}(u_j L_j)^{-\alpha} \\ &= q_{fj} Q(i) f_j \ . \end{aligned} \tag{B.7}$$

**B.2. Equilibrium**

First, the differential equation for $R = F/K$ can be obtained using the economy's resource constraint:

$$\frac{\dot{R}}{R} = Q(i)(1-u)L - spTR^{1-\alpha}(uL)^{1-\alpha} \ , \tag{B.8}$$

The derivation of the differential equation describing $u$ requires two additional steps. First, the no-arbitrage condition (B.3) can be differentiated with respect to time to arrive at:

$$\alpha\frac{\dot{R}}{R} + \alpha\frac{\dot{u}}{u} = -\frac{\dot{q}_f}{q_f} \ . \tag{A.9}$$

Second, we can substitute the first term in FOC (B.5) out by rewriting the no-arbitrage condition:

$$\frac{1}{q_f}(1-\alpha)TR^{-\alpha}(uL)^{1-\alpha} = Q(i)uL \ . \tag{B.10}$$

Using the last three results we obtain the differential equation in $u$:

$$\frac{\dot{u}}{u} = Q(i)uL + \left(\frac{1-\alpha}{\alpha}\right)Q(i)L + (sp-1)TR^{1-\alpha}(uL)^{1-\alpha} \ . \tag{B.11}$$

### B.3 Stability

The stability of the equilibrium can be checked by evaluating the determinant of Jacobian matrix $J$ at the equilibrium $E$:

$$J = \begin{bmatrix} \dfrac{\partial \dot{R}}{\partial R} & \dfrac{\partial \dot{R}}{\partial u} \\ \dfrac{\partial \dot{u}}{\partial R} & \dfrac{\partial \dot{u}}{\partial u} \end{bmatrix} . \tag{B.12}$$

The four partial derivatives of $J$ at $E$ are:

$$\left[\frac{\partial \dot{R}}{\partial R}\right]_E = -(1-\alpha)Q(i)(1-u^*)L < 0 \ , \tag{B.13}$$

$$\left[\frac{\partial \dot{R}}{\partial u}\right]_E = -Q(i)LR^*\left(1+(1-\alpha)\frac{1-u^*}{u^*}\right) < 0 \ , \tag{B.16}$$

$$\left[\frac{\partial \dot{u}}{\partial u}\right]_E = 2Q(i)Lu^* + \left(\frac{1-\alpha}{\alpha}\right)Q(i)L - \alpha\Phi >< 0 \ , \tag{B.15}$$

$$\left[\frac{\partial \dot{u}}{\partial R}\right]_E = -(1-sp)\left(\frac{1-\alpha}{\alpha}\right)\frac{r^*u^*}{R} < 0 \ . \tag{B.14}$$

where $\Phi \equiv (1 - sp)T(R^*)^{(1-\alpha)}(u^*L)^{(1-\alpha)}$. From the $\dot{u} = 0$ locus s follows that at *E* we have:

$$\Phi = Q(i)Lu^* + \frac{1-\alpha}{\alpha}Q(i)L \; . \tag{B.17}$$

The equilibrium is saddle-point stable if: $\partial\dot{u}/\partial u > 0$; then the determinant of the Jacobian is negative. This will be the case if:

$$Q(i)Lu^* - \frac{1-\alpha}{u^*}\Phi > 0 \; . \tag{B.18}$$

substitution of $\Phi$ gives:

$$u^* > \left(\frac{1-\alpha}{\alpha}\right)^2 \; . \tag{B.19}$$

## C. Accounting in the example model (not for publication)

### C.1 Accounting

The following assumptions with respect to the accounting procedure are necessary to obtain consistency. We assume the following: (1) Households own the shares of firms producing differentiated goods. (3) The manager of the firm decides on investment in firm-specific capital where after (4) firms pay profits ($\pi_i = x_i p_i - rK_i - wL_i$) to households. (5) Firm *i* pays rental rate *r* to the capital assembly firms who pay this as dividends to the household. (6) Households decide on saving over total income (wage income, dividend and profits).

Firm $i$

| | |
|---|---|
| $r\,K_i$ | $p_i\,C_i$ |
| $u\,w\,L_i$ | $p_i\,I_i$ |
| $(1\text{-}u)w\,L_i$ | |
| $\pi_i$ | |
| $S_f\,(\equiv q_f\,dF/dt\,)$ | $(1\text{-}u)w\,L_i$ |

Households

| | |
|---|---|
| $p\,C$ | $D$ |
| $S$ | $w\,L$ |
| | $\pi$ |

K-firms

| | |
|---|---|
| $D$ | $r\,K$ |
| $P_I\,I$ | $S$ |
| $dK/dt$ | $P_I\,I$ |

Note that due to these assumptions total investment (below the dashed lines) is equal to savings of households and savings generated internally in the firm. The assumptions with respect to accounting (and hence the model) could, without altering the main results, be changed such that total investment equals total household saving. It would unnecessarily complicate the analysis of the dynamics of the model.

**C.2 Growth accounting with endogenous growth and firm-specific capital**

The accounting procedure presented in Appendix A.2 shows that alongside physical output, Y (Y=C+I), knowledge capital is produced. Define the sum of these as broad output: $Y^B$. Obviously total production equals total income, see also Appendix C.1 (note that profits are ignored in the remainder), therefore:

$$Y^B = wL + rK\ . \tag{C.1}$$

Differentiating the definition for income-production equality yields, after rearranging:

$$\dot{Y}^B = wL(\hat{L} + \hat{w}_L) + rK(\hat{r} + \hat{K}) \ . \tag{C.2}$$

Noting that total production includes knowledge production $Y^B = Y + q_f \dot{F}$, differentiating this equation with respect to time yields:

$$\dot{Y}^B = \dot{Y} + \dot{q}_f \dot{F} + q_f \ddot{F} \ . \tag{C.3}$$

Differentiating the accounting identity, $q_f \dot{F} = (1-u)wL$ , and substituting this into equation (B.5) and substituting the result into (B.4) and dividing by *Y* yields:

$$\begin{aligned} &\hat{Y} + \frac{1}{Y}((1-u)(L\dot{w} + w\dot{L}) - \dot{u}wL) = \\ &\frac{w_L L}{Y}(\hat{L} + \hat{w}) + \frac{rK}{Y}(\hat{r} + \hat{K}) \ . \end{aligned} \tag{C.4}$$

Rewriting yields:

$$\begin{aligned} &\hat{Y} + (1-u)\frac{wL}{Y}\left(\hat{w} + \hat{L} - \frac{u}{1-u}\hat{u}\right) = \\ &\frac{wL}{Y}(\hat{L} + \hat{w}) + \frac{rK}{Y}(\hat{r} + \hat{K}) \ . \end{aligned} \tag{C.5}$$

Now introduce the assumption that the $\chi$'s equal the income shares and substitute (A.2) into the previous expression and rewrite to get:

$$\frac{wL}{Y}\hat{w} + \frac{rK}{Y}\hat{r} = \hat{D} + (1-u)\frac{wL}{Y}\left(\hat{w} + \hat{L} - \frac{u}{1-u}\hat{u}\right) \ , \tag{C.6}$$

The expression on the rhs is closely related to the perfect measure for productivity growth.

The perfect measure for productivity growth is derived directly from the macroeconomic production function used in the model:

$$Y^B = AF(K,uL) + q_f Q(i) F(1-u)L \ . \tag{C.7}$$

In growth rates:

$$\hat{Y}^B = \frac{AF(K,uL)}{Y^B}\hat{A} + \psi_K\hat{K} + \psi_{uL}(\hat{u}+\hat{L}) + \frac{q_f Q(i)\,F(1-u)L}{Y^B}\left(\hat{q}_f + \hat{Q}(i) + \hat{F} + \hat{L} - \frac{u}{1-u}\hat{u}\right) , \tag{C.8}$$

where $\psi_i \equiv AF_i i/Y^B,\ i=K,uL$. Use $w=q_f Q(i) F$ (see (B.7)) to get $\hat{w}=\hat{q}_f+\hat{Q}+\hat{F}$ and substitute this in the previous expression. Bringing all factors relating to changes in factor inputs to the lhs yields an expression for productivity growth.

$$\hat{Y}^B - \psi_K\hat{K} - \psi_{uL}\left(1+\frac{1-u}{u}\hat{L}\right) = \frac{AF(K,uL)}{Y^B}\hat{A} + \left(\psi_{uL} - \frac{uwL}{Y^B}\right)\hat{u} + \psi_{uL}\frac{w}{AF_{uL}}\frac{u}{1-u}\hat{w} , \tag{C.9}$$

Now substitute for the marginal products, the factor prices. This gives us the following expression for the perfect measure for productivity growth of broad output:

$$\hat{A}^* \equiv \hat{Y}^B - \frac{rK}{Y^B}\hat{K} - \frac{wL}{Y^B}\hat{L} = \frac{AF(K,uL)}{Y^B}\hat{A} + \frac{(1-u)wL}{Y^B}\hat{w} . \tag{C.10}$$

This expression for true productivity growth ($\hat{A}^*$) has an obvious interpretation: productivity growth of broad output is the weighted sum of the increase in the technology of physical production and the increase in productivity of knowledge production. The latter is in turn equal to the change in the productivity of knowledge (as $\hat{w}=\hat{q}_f+\hat{Q}+\hat{f}$): that is the increase in organisational or firm-specific capital, $\hat{f}$, the productivity effect of the GPT, plus the change in the shadow price, which is the value of knowledge capital in consumption goods. The weights are the factor-income shares in broad output.

Our final task is to relate $\hat{A}^*$ to our observable $\hat{D}$. Use , $Y=AF(K,uL)$ to arrive at:

$$\hat{D} = \hat{A} + \chi_u\hat{u} . \tag{C.11}$$

This expression prevails in the main text as the knowledge-investment-*input*-corrected growth account. Multiplying (B.12) by $Y^B/Y$ and substituting (C.10) for $\hat{A}$ yields:

$$\frac{Y^B}{Y}\left(\hat{Y}^B - \frac{rK}{Y^B}\hat{K} - \frac{wL}{Y^B}\hat{L}\right) = \hat{D} + \frac{(1-u)wL}{Y^B}\hat{w} + \chi_u\hat{u} . \tag{C.12}$$

Substituting in (B.12) for $\chi$, yields on the rhs exactly the expression derived by the dual growth account (rhs of (C.8)).

$$\begin{aligned} \frac{w_L L}{Y}\hat{w}_L + \frac{rK}{Y}\hat{r} &= \hat{D} + (1-u)\frac{wL}{Y}\left(\hat{w} + \hat{L} - \frac{u}{1-u}\hat{u}\right) \\ &= \frac{Y^B}{Y}\left(\hat{Y}^B - \frac{rK}{Y^B}\hat{K} - \frac{wL}{Y^B}\hat{L}\right) \\ &= \frac{Y^B}{Y}\hat{A}'. \end{aligned} \tag{C.13}$$

Hence the dual growth account overestimates the growth rate, as it is the ideal measure times $Y^B/Y$, and $Y^B > Y$. It is obvious that the dual growth account overestimates the growth rate as the weights on the lhs of equation (C.10) are too high as their sum exceeds one. However, as mentioned above, in practice the factor income shares that sum to unity are used, and hence the dual approach tends to the perfect measure.