Online Appendix: Optimal Income Taxation in Unionized Labor Markets

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In this Appendix, we present some features of the baseline model, we investigate the robustness of our main theoretical results, we provide an elaborate description of the construction of our data set, and we report the sensitivity analyses of our numerical simulations. In particular, we start by linking our measure of union power to the Nash-bargaining weight and derive the first-best outcome in the baseline model. Then, we consider a version of our model where unions respond to marginal tax rates, relax the assumption of efficient rationing (Assumption 2 in the main text), and allow for endogenous occupational choice. In addition, we analyze two alternative bargaining structures: one in which a single, national union bargains with firm-owners over the entire *distribution* of wages, and one in which sectoral unions bargain with firms over wages *and* employment, as in the efficient bargaining model of McDonald and Solow (1981). Then, this Appendix documents the construction of the data set, and it provides some robustness checks of our empirical analysis. The final section presents the results from the sensitivity analysis of our simulations.

1 Derivation of ρ_i from the Right-to-Manage model

In this Appendix, we derive the relationship between our measure of union power ρ_i and the bargaining power in the Nash product that is more commonly used to characterize equilibrium in the RtM-model (see, for instance, Boeri and Van Ours, 2008). In particular, the Nash

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bargaining problem is given by:

$$
\begin{aligned}
\max_{w_i,E_i}\ \Omega_i &= \delta_i \log\left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi)\right) \\
&\quad + (1-\delta_i)\log\left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0} - \sum_{j\neq i} w_j N_j E_j - T_f)\right) \\
\text{s.t. } & w_i = F_i(\cdot), \\
& G_i(w_i - T_i + T_u) - E_i \geq 0, \qquad (1)
\end{aligned}
$$

where $\delta_i \in [0,1]$ is the weight attached to the union's payoff in the Nash product, and $F(\cdot)|_{E_i=0}$ is the firm's output if it does not reach an agreement with the union in sector i, and, hence, none of the workers in sector i find employment. The payoffs are taken in deviation from the payoff associated with the disagreement outcome. It is important to take the voluntary participation constraint in equation (1) explicitly into account, as it will bind for small values of δ_i. If δ_i is close to zero, labor-market equilibrium is characterized by the final two conditions, which jointly determine the competitive equilibrium.

The Lagrangian reads as:

$$
\begin{aligned}
\mathcal{L} &= \delta_i \log\left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi)\right) \\
&\quad + (1-\delta_i)\log\left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0} - \sum_{j\neq i} w_j N_j E_j - T_f)\right) \\
&\quad + \vartheta_i(w_i - F_i(\cdot)) + \mu_i(G(w_i - T_i + T_u) - E_i). \qquad (2)
\end{aligned}
$$

The first-order conditions are given by:

$$
w_i:\ \frac{\delta_i}{(\overline{u_i} - u_u)}\overline{u_i'} - \frac{(1-\delta_i)}{(u_f - u_f^{-i})}u_f' N_i E_i + \vartheta_i + \mu_i G_i' = 0, \qquad (3)
$$

$$
E_i:\ \frac{\delta_i}{E_i(\overline{u_i} - u_u)}(\hat{u}_i - u_u) - \vartheta_i F_{ii} N_i - \mu_i = 0, \qquad (4)
$$

$$
\vartheta_i:\ w_i - F_i = 0, \qquad (5)
$$

$$
\mu_i:\ \mu_i(G_i - E_i) = 0, \qquad (6)
$$

where the bars indicate averages over all employed workers in sector i, $\hat{u}_i$ is the utility of the marginal worker in sector i and $u_f^{-i} \equiv u(F(\cdot)|_{E_i=0} - \sum_{j\neq i} w_j N_j E_j - T_f)$ is the utility of firm-owners if they fail to reach an agreement with the union in sector i. If $\delta_i = 1$, equations (3)–(4) imply that $\mu_i = 0$, and we find the equilibrium of the monopoly-union model. For small values of δ_i, the constraint $G_i = E_i$ becomes binding, and the labor-market equilibrium coincides with the competitive outcome. This can be verified by setting $\delta_i = 0$. Equations (3)–(4) then imply that $\mu_i > 0$. This is the case for all values of $\delta_i \in [0, \delta_i^*]$, where $\delta_i^* \in (0,1)$ solves:

$$\frac{\delta_i^*}{1-\delta_i^*} = \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\overline{u'_i}}. \tag{7}$$

This equation is obtained by setting $G_i = E_i$ and $\mu_i = 0$ in the system of first-order conditions in equations (3)–(6). The reason is that, at exactly this value of δ_i, the constraint $G_i = E_i$ becomes binding. For values of $\delta_i \in [\delta_i^*, 1]$, we thus have $\mu_i = 0$. Combining equations (3)–(4) then leads to:

$$1 - \left(\frac{1-\delta_i}{\delta_i}\right) \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\overline{u'_i}} = \varepsilon_i \frac{(\hat{u}_i - u_u)}{\overline{u'_i} w_i}. \tag{8}$$

If we write the left-hand side of this equation as

$$\rho_i = 1 - \left(\frac{1-\delta_i}{\delta_i}\right) \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\overline{u'_i}}, \tag{9}$$

we arrive at our equilibrium condition (8). Clearly, if $\delta_i = 1$, we have $\rho_i = 1$, so that the MU-model applies. If $\delta_i = \delta_i^*$, from equation (7) it follows that $\rho_i = 0$, and the equilibrium coincides with the competitive outcome. Hence, the relationship between our measure of union power ρ_i and the Nash-bargaining parameter δ_i is:

$$\rho_i = \begin{cases} 0 & \text{if} \quad \delta_i \in [0, \delta_i^*), \\ 1 - \frac{(1-\delta_i)}{\delta_i} \frac{E_i(\overline{u_i} - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\overline{u'_i}} & \text{if} \quad \delta_i \in [\delta_i^*, 1]. \end{cases} \tag{10}$$

For a given tax-benefit system, this equation specifies a direct relationship between δ_i and ρ_i. The mapping clearly depends on endogenous objects such as the tax-benefit system and the threshold δ_i^*. For reasons explained in the main text, we prefer to characterize equilibrium using our measure of union power ρ_i instead of the Nash-bargaining parameter δ_i.[1]

2 First-best allocation

We assume throughout the paper that the government cannot observe participation costs φ. Hence, taxes cannot be conditioned on φ. However, if taxes can be conditioned on participation costs, it is possible to decentralize the first-best allocation as a competitive equilibrium.[2] In this case, the wage in each sector is equated to the marginal productivity of labor, i.e., $w_i = F_i(\cdot)$. Moreover, individuals in sector i with participation costs $\varphi \leq G_i^{-1}(E_i)$ will all be employed. The first-best allocation is characterized by choosing taxes $T_{i,\varphi}$, T_f and employment rates E_i that maximize social welfare subject only to the government budget constraint. The Lagrangian

[1] To the best of our knowledge, the Nash-bargaining parameter δ_i does not have a clear economic interpretation or game-theoretic foundation the current setting. The reason is that the equilibrium is restricted to lie on the labor-demand curve (4), which violates the axiom of Pareto optimality (see also Section 7 in the online Appendix). In addition, as mentioned, for low values of δ_i, the voluntary participation constraint $G_i \geq E_i$ becomes binding.

[2] Because the first-best allocation can be decentralized as a competitive equilibrium, it follows immediately that unions cannot improve on the allocation.

for this maximization problem is given by:

$$\mathcal{L} = \sum_i \psi_i N_i \left[\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(F_i(\cdot) - T_{i,\varphi} - \varphi) \mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_{i,\varphi}) \mathrm{d}G_i(\varphi) \right]$$
$$+ \psi_f u(F(\cdot) - \sum_i F_i(\cdot) N_i E_i - T_f) + \lambda \left[\sum_i N_i \int_{\underline{\varphi}}^{\overline{\varphi}} T_{i,\varphi} \mathrm{d}G_i(\varphi) + T_f - R \right]. \tag{11}$$

The first-order conditions are:

$$T_{i,\varphi}: \quad N_i(\lambda - \psi_i u'(F_i(\cdot) - T_{i,\varphi} - \varphi)) g_i(\varphi) = 0 \quad \text{if} \quad \varphi \leq G_i^{-1}(E_i), \tag{12}$$
$$N_i(\lambda - \psi_i u'(-T_{i,\varphi})) g_i(\varphi) = 0 \quad \text{if} \quad \varphi > G_i^{-1}(E_i), \tag{13}$$
$$T_f: \quad \lambda - \psi_f u'(c_f) = 0, \tag{14}$$
$$E_i: \quad \psi_i N_i (u(F_i(\cdot) - T_{i,\varphi} - G_i^{-1}(E_i)) - u(-T_{i,\varphi}))$$
$$+ N_i \sum_j F_{ji}(\cdot) N_j \left[\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} \psi_j u'(F_j(\cdot) - T_{j,\varphi} - \varphi) \mathrm{d}G_i(\varphi) - \psi_f u'(c_f) \right] = 0, \tag{15}$$
$$\lambda: \quad \sum_i N_i \int_{\underline{\varphi}}^{\overline{\varphi}} T_{i,\varphi} \mathrm{d}G_i(\varphi) + T_f - R = 0. \tag{16}$$

$c_f = F(\cdot) - \sum_i F_i(\cdot) N_i E_i - T_f$ is the consumption of firm-owners. At the first-best allocation, all social welfare weights are equalized: $\psi_f u'(c_f) = \psi_i u'(c_{i,\varphi}) = \lambda$, where $c_{i,\varphi}$ is the consumption of an individual in sector i with participation costs φ. Because all social welfare weights are equalized, the terms in the second line of equation (15) cancel. Equation (15) then implies employment is efficient: $F_i(\cdot) = G_i^{-1}(E_i)$.

3 Union responses to marginal tax rates

This Section derives how our main results are affected if unions respond to marginal tax rates. So far, we have assumed that a union in sector i treats the tax liability T_i for its employed members as given. However, if the government sets a tax *schedule* $T(w_i)$, rather than a tax liability T_i in each sector, unions will anticipate that a higher wage affects the tax liability. Hence, the marginal tax rate will also determine wage demands of the union. To study the implications of union responses to marginal tax rates for optimal income taxation and the desirability of unions, it is convenient to reformulate our model and work with a continuum rather than a discrete set of sectors (or occupations). As before, within each sector, workers are represented by a union that maximizes the expected utility of its members. Sectors are indexed by $i \in \mathcal{I} = [0, 1]$ and ordered in such a way that wages $w(i)$ are increasing in i. $H(i)$ denotes the distribution of workers across sectors with density $h(i)$. Because we work with a continuum, we index sector i by a function argument instead of a subscript. The total measure of workers is normalized to one and the measure of (identical) firm-owners is $1/N$.[3] Within each sector, workers differ in their unobservable participation costs $\varphi \in [\underline{\varphi}, \overline{\varphi}]$, which are distributed

[3] It is slightly more convenient to normalize the measure of workers and not, as in our baseline, the measure of firm-owners to one.

according to a cumulative distribution $G(\varphi)$ that, for simplicity, is assumed to be common across sectors.

To maintain tractability, we assume that workers in each sector produce the final consumption good directly, rather than assuming labor inputs of different types are combined to produce a final consumption good. This guarantees the absence of spillover effects between different sectors. Total output in sector i is thus given by

$$Y(i) = a(i)y(h(i)E(i)). \tag{17}$$

Here, $a(i)$ is an index of productivity, $E(i)$ denotes the employment rate of workers in sector i, and $y(\cdot)$ is a production function that maps total employment $L(i) = h(i)E(i)$ in sector i into units of the final consumption good.

Firms maximize profits by choosing how much labor to hire. The labor-demand curve is, for each i:

$$w(i) = a(i)y'(h(i)E(i)). \tag{18}$$

If $y(\cdot)$ is strictly concave, each union faces a downward-sloping labor-demand curve and firms make profits, which are subject to a non-distortionary profit tax T_f. The government also provides a benefit $-T_u$ to all workers who are not employed (voluntarily or involuntarily). In addition, the government sets a tax *schedule* $T(\cdot)$ on labor income $w(i)$. This is the key difference from our previous set-up, where it was assumed that the government sets the tax *liability* T_i in each sector directly, which unions take as given. The government chooses these instruments to maximize social welfare, subject to its budget constraint, and taking into account how labor-market outcomes are affected by changes in the tax-benefit system.

As in the baseline, we first characterize the equilibrium for any degree of union power $\rho(i) \in [0, 1]$, which is allowed to vary across sectors. Under efficient rationing (Assumption 2 in the main text), workers with participation costs $\varphi \in [\underline{\varphi}, \hat{\varphi}(i)]$ become employed, where $\hat{\varphi}(i) = G^{-1}(E(i))$. By contrast, workers with participation costs $\varphi \in [\hat{\varphi}(i), \overline{\varphi}]$ are not employed (voluntary or involuntary). The union's objective is then given by

$$\Lambda(i) = \int_{\underline{\varphi}}^{\hat{\varphi}(i)} u(c(i) - \varphi)\mathrm{d}G(\varphi) + \int_{\hat{\varphi}(i)}^{\overline{\varphi}} u(c_u)\mathrm{d}G(\varphi), \tag{19}$$

where $c(i) = w(i) - T(w(i))$ is consumption of an employed worker, and $c_u = -T_u$ denotes consumption of an unemployed worker.

If the union representing workers from sector i is a monopoly union (MU), i.e., $\rho(i) = 1$, then it sets the wage $w(i)$ that maximizes the objective (19) subject to $\hat{\varphi}(i) = G^{-1}(E(i))$ and the labor-demand equation (18). The first-order conditions can be combined to find the wage-demand equation:

$$1 - T'(w(i)) = \varepsilon(i)\frac{u(\hat{c}(i)) - u(c_u)}{\overline{u'(c(i))}w(i)}, \tag{20}$$

where $\varepsilon(i) = -y'(L(i))/(L(i)y''(L(i))) > 0$ is the labor-demand elasticity and $\hat{c}(i) = w(i) -$

$T(w(i)) - G^{-1}(E(i))$ is the consumption net of participation costs of the marginally employed worker, i.e., the employed worker with the highest costs of participation. This condition is very similar to equation (6) from the main text, except that the left-hand side is multiplied by the net-of-tax rate $1 - T'(w(i))$. Intuitively, unions only care about demanding a higher wage if it yields a higher after-tax income.

If labor markets are perfectly competitive, i.e., if $\rho(i) = 0$, workers continue to supply labor until the marginally employed worker is indifferent between working and not working: $\hat{c}(i) = c_u$, and, hence, $\hat{\varphi}(i) = w(i) - T(w(i)) + T_u$. In this case, there is no involuntary unemployment. Furthermore, because labor-supply responses are only concentrated on the extensive margin, a local increase in the marginal tax rate at income $w(i)$ that leaves the tax liability unaffected has no impact on labor-market outcomes in sector i.

Following a similar approach as in Section 3.3 of the main text, we can characterize labor-market equilibrium in sector i for any degree of union power $\rho(i) \in [0, 1]$ by combining the labor-demand equation (18) with the following modified wage-demand equation:

$$\rho(i)(1 - T'(w(i))) = \varepsilon(i)\frac{u(\hat{c}(i)) - u(c_u)}{u'(c(i))w(i)}. \tag{21}$$

This condition is analogous to equation (8) from the main text, except that the left-hand side is multiplied by the net-of-tax rate. Clearly, if $\rho(i) = 0$, the competitive equilibrium (CE) prevails, as there is no involuntary unemployment: $u(\hat{c}(i)) - u(c_u) = 0$. By contrast, if $\rho(i) = 1$, equations (21) and (20) coincide and the equilibrium corresponds to the monopoly union (MU) outcome. By varying the degree of union power $\rho(i) \in [0, 1]$, we can obtain any equilibrium from the RtM-model.

There is one key difference between the current formulation and the baseline. In the latter, unions treat the tax liability as given. Consequently, a local increase in the marginal tax rate at income level $w(i)$ that leaves the tax liability unaffected has no impact on labor-market outcomes in sector i. However, in the current setup, unions bargain taking the tax *schedule* $T(\cdot)$ as given. As a result, the equilibrium wage and employment rate in sector i also depend on the marginal tax rate $T'(w(i))$, see equation (21). We demonstrate formally in Appendix A.1 that a local increase in the marginal tax rate $T'(w(i))$ at income level $w(i)$ lowers the equilibrium wage $w(i)$, and, through the labor-demand equation (18), raises the equilibrium employment rate $E(i)$.[4] Intuitively, a higher marginal tax rate lowers the benefits of demanding a higher wage, which induces unions to lower their wage demands, and firms to hire more workers, cf. Hersoug (1984). This effect is referred to in the literature as the wage-moderating effect of a higher marginal tax rate.[5,6]

[4] We also show in Appendix A.1 that, as in the baseline, a higher tax burden or unemployment benefits leads to a higher wage and a reduction in the employment rate.

[5] The wage-moderating effect of a higher marginal tax rate is a robust prediction in models with labor market imperfections. It is derived in the context of unions by Hersoug (1984), but also holds in the context of matching frictions (Pissarides, 1985) and efficiency wages (Pisauro, 1991). See Lehmann et al. (2016) for empirical evidence and Kroft et al. (2020) and Hummel (2021) for a discussion of the implications for optimal income taxation.

[6] Sometimes, this effect is referred to as the wage-moderating effect of 'tax progressivity'. Indeed, if marginal tax rates increase, while average tax rates remain fixed, a higher marginal tax rate also raises the progressivity of the tax system, since a tax system is progressive only if the average tax rate increases in income.

We characterize the optimal tax schedule $T(\cdot)$ using the tax perturbation approach.[7] To do so, we study the welfare effects of a uniform increase in the tax burden $T(w)$ paid by *all* employed workers, a *local* increase in the marginal tax rate $T'(w')$ at some income level w', an increase in the profit tax T_f, and a reduction in the unemployment benefit $-T_u$. If the tax system is optimized, none of these reforms should have an impact on social welfare. This leads to the following Proposition.

Proposition 1. *Suppose Assumptions 1 (independent labor markets), 2 (efficient rationing), and 3 (no income effects at the union level) hold. In addition, suppose the government optimizes a tax schedule $T(\cdot)$ and unions respond to marginal tax rates, cf. equation (21). Then, the optimal tax schedule $T(\cdot)$, unemployment benefits $-T_u$, and profit taxes T_f are determined by:*

$$\omega_u b_u + \int_{\underline{w}}^{\overline{w}} b(w)k(w)\mathrm{d}w = 1, \tag{22}$$

$$b_f = 1, \tag{23}$$

$$\begin{aligned} &\left[\left(\frac{t(w')+\tau(w')}{1-t(w')}\right)\eta_{T'} + (b(w')-1)(1-T'(w'))\kappa_{T'}\right]k(w') \\ &+ \int_{w'}^{\overline{w}} \left[(1-b(w)) - \left(\frac{t(w)+\tau(w)}{1-t(w)}\right)\eta_T + (b(w)-1)(1-T'(w))\kappa_T\right]k(w)\mathrm{d}w = 0, \quad \forall w', \end{aligned} \tag{24}$$

where $b(w)$, $\tau(w)$, $t(w)$ and $\tilde{E}(w)$ denote the social welfare weight, union wedge, participation tax rate, and employment rate at wage w. Moreover, $k(w)$ is the density of the wage distribution, and $\kappa_T \equiv \frac{\partial w}{\partial T}$, $\kappa_{T'} \equiv \frac{\partial w}{\partial T'}$, $\eta_T \equiv -\frac{\partial E}{\partial T}\frac{w(1-t(w))}{\tilde{E}(w)}$, and $\eta_{T'} \equiv \frac{\partial E}{\partial T'}\frac{(1-t(w))w}{\tilde{E}(w)}$ are the elasticities of wages and employment with respect to an increase in the marginal tax rate and total tax burden.

Proof. See Appendix A.2. □

The first two results are the same as in the baseline (see Proposition 1 in the main text), and, hence, their explanation is not repeated here.[8]

The third result is obtained from considering a local increase in the *marginal* tax rate at income level w'. Compared to the baseline, the optimal tax formula (24) is modified in two substantive ways. Both effects are captured on the first line.

First, a higher marginal tax rate results in a higher employment rate, since wage demands are reduced: $\eta_{T'} > 0$. The wage-moderation effect of a higher marginal tax rate at w' alleviates labor-market distortions from the explicit tax $t(w')$ on labor participation, and the implicit tax $\tau(w')$ from unions bidding up wages above the market-clearing level. This is captured by the first term on the first line. Intuitively, if unions moderate wage demands in response to a higher marginal tax rate, and employment increases, social welfare increases if employment is distorted downwards, i.e., if $t(w') + \tau(w') > 0$.[9]

[7]The tax-perturbation approach is also employed by, among others, Saez (2001), Golosov et al. (2014), Gerritsen (2016), and Jacquet and Lehmann (2021).

[8]The only differences are that the current extension features a continuum (rather than a discrete number) of types and equation (22) integrates over the income (as opposed to the type) distribution.

[9]A similar term appears in Hummel (2021), who characterizes the optimal tax schedule in a directed search

Second, a higher marginal tax rate reduces the equilibrium wage: $\kappa_{T'} < 0$. As a result, income is redistributed among workers, firm-owners, and the government. In particular, if wages are lowered, firm-owners receive higher profits, workers see their after-tax income reduced, and the government experiences a reduction in tax revenue (provided that $T'(w') > 0$). The reduction in the wage transfers income from workers, whose social welfare weight is $b(w')$, to firm-owners, whose social welfare weight is $b_f = 1$. The reduction in tax payments yields a welfare effect equal to the change in the wage multiplied with $T'(w')(b(w') - 1)$, where $b(w')$ represents the increase in social welfare if the worker pays one unit of income less in tax, while 1 stands for the loss in social welfare if the government receives less tax revenue. The sum of both welfare effects is proportional to $(1 - b(w'))(1 - T'(w'))$, as captured by the second term on the first line. Hence, there is a redistributional gain (loss) due to wage moderation at w' if $b(w') < 1$ ($b(w') > 1$). Note that both the wage and employment effects are proportional to the density $k(w')$ of the wage distribution. The density $k(w')$ is the measure of workers who experience a decrease in the wage or an increase in employment if the marginal tax rate at income w' is increased.

Turning to the second line of equation (24), a higher marginal tax rate not only generates wage-moderation and employment effects at point w' in the income distribution, where the marginal tax is levied, but it also raises tax liabilities for all income levels $w > w'$. This mechanically transfers income from these workers to the government, as captured by the first term $1 - b(w)$. This is the standard mechanical effect of a higher marginal tax rate in all non-linear income tax models, cf. Mirrlees (1971), Diamond (1998), and Saez (2001). Moreover, as in the baseline, a higher tax burden also generates upward pressure on wages and a corresponding reduction in the employment rates for all income levels $w > w'$: $\kappa_T > 0$ and $\eta_T < 0$. The associated reductions in employment are socially costly if participation of these workers is distorted downwards, i.e., if $t(w) + \tau(w) > 0$ for $w > w'$. Moreover, the wage pressure for all income levels $w > w'$ redistributes income from firm-owners to workers and to the government for exactly the same reasons as we discussed above. This generates a loss (gain) in social welfare due to wage pressure at $w > w'$ if $b(w') < 1$ ($b(w') > 1$) for income levels $w > w'$.

To see how equation (24) is linked to the optimal tax formula (18) from the main text, suppose that unions treat the tax liability as given and do not respond to changes in the marginal tax rate. In that case, $\eta_{T'} = \kappa_{T'} = 0$, and both terms on the first line of equation (24) would cancel. Moreover, because equation (24) holds for each w', the term below the integral sign must be equal to zero at each point in the income distribution. By setting $T'(w) = 0$, the result from Proposition 1 coincides with equation (18) from the main text in the special case where there are no spillover effects between different sectors.

Another way to understand how equation (24) and equation (18) from the main text are linked, is to recognize that these optimal tax rules are derived from two, distinct policy experiments. In particular, in the current extension, the increase in the marginal tax rate at w' raises i) the marginal tax rate at w', and ii) the tax liabilities $T(w)$ for all workers with higher wages, i.e., for workers with $w > w'$. By contrast, in the baseline, the optimal tax formula is

model with matching frictions. In that framework, however, there is no wedge due to involuntary unemployment (i.e., no counterpart of the union wedge), because unemployment is constrained efficient.

based on the policy experiment where the tax liability is increased at only one income level. However, if we would, instead, consider increasing the tax liability in our baseline model for all workers with $w > w'$ (like in the current extension), and, in addition, assume independent labor markets (like in the current extension), then the optimal tax formula would become the same as equation (24), but with one important difference: the first line would be zero. Hence, the main modification of the current extension compared to the baseline is to add the terms on the first line of equation (24). That is, the optimal tax formula accounts for wage and employment responses to marginal tax rates.

The wage-moderation effect of a higher marginal tax rate triggers two welfare-relevant effects: it alleviates (exacerbates) labor-market distortions if labor participation is taxed (subsidized) on a net basis, and it gives additional redistributional gains (losses) if $b(w') < 1$. These welfare effects are related. Loosely speaking, the government typically only provides transfers to employed workers that exceed the unemployment benefit, i.e., sets $t(w) < 0$, if these workers have a high social welfare weight, i.e., if $b(w) > 1$. Therefore, we conjecture that, compared to the baseline, wage-moderation effects tend to reduce (raise) optimal marginal tax rates if employment is distorted upwards (downwards) – *ceteris paribus*. However, we are not sure whether the ceteris paribus condition holds, since the optimal marginal tax schedule is dependent on all social welfare weights, the entire income distribution, and participation distortions at all income levels. Only a more elaborate quantitative analysis can shed light on the implications of wage moderation for optimal taxes, but this is beyond the scope of the current paper.

Next, we ask if unions are desirable for income redistribution if the government optimizes the non-linear tax schedule and unions respond to changes in marginal tax rates. To that end, we study the welfare effect of increasing union power at income level w. This leads to the following result.

Proposition 2. *Suppose Assumptions 1 (independent labor markets) and 2 (efficient rationing) hold. In addition, suppose that the tax-benefit system is optimized as in Proposition 1. Then, an increase in union power for workers whose wage is w raises social welfare if and only if:*

$$(b(w) - 1)(1 - T'(w)) - (t(w) + \tau(w))\tilde{\varepsilon}(w) > 0, \tag{25}$$

where $\tilde{\varepsilon}(w)$ is the labor-demand elasticity at wage w.

Proof. See Appendix A.3. □

To understand this result, consider a local increase in union power for workers who are employed at wage w. An increase in union power at income level w boosts wage demands and reduces employment at w. The increase in the equilibrium wage then transfers income from firm-owners, whose social welfare weight is $b_f = 1$, to workers, whose social welfare weight is $b(w)$. As in the baseline, the welfare effect is proportional to $b(w) - 1$. Moreover, a higher equilibrium wage also transfers income from workers to the government if $T'(w) > 0$. This explains why the first term is multiplied by the net-of-tax rate $1 - T'(w)$. Turning to the second term, the increase in the wage due to higher union power also results in a lower employment rate. By how much depends on the labor-demand elasticity $\tilde{\varepsilon}(w)$. A lower employment rate, in turn, affects social welfare through the explicit tax $t(w)$ and the implicit tax $\tau(w)$ on labor participation. These effects are captured by the second term. Equation (25) states that an increase in union power results in a welfare gain i) if participation is distorted upwards, and/or ii) if the wage increase is associated with a positive redistributional gain, which requires $b(w) > 1$.

The main difference compared to the baseline (see Proposition 2 in the main text) is that whether an increase in union power raises social welfare depends on *both* social welfare weights, $b(w)$, and net taxes on labor participation, $t(w)+\tau(w)$. Importantly, as mentioned before, these are not independent. The government typically only provides transfers to employed workers that exceed the unemployment benefit, i.e., sets $t(w) < 0$, if these workers have a high social welfare weight, i.e., if $b(w) > 1$. Therefore, we view our adjusted desirability condition as only slightly weaker. Moreover, we can derive a sufficiency condition for the desirability of unions: an increase in union power unambiguously raises social welfare if participation is subsidized on a net basis ($t(w) + \tau(w) < 0$) *and* the social welfare weights of the workers represented by the union is above-average ($b(w) > 1$). Conversely, a sufficient condition for unions not to be desirable is that workers pay positive participation taxes ($t(w) > 0$) and have a below-average social welfare weight ($b(w) < 1$).[10] Given that we empirically find that participation taxes are never negative, the desirability condition implies that a necessary condition for unions to be desirable is that the social welfare weight of the workers that are represented by the union is above average, i.e., $b(w) > 1$. Hence, Proposition 2 from the main text largely carries over to the current setting.

In the baseline without spillover effects, participation taxes and social welfare weights are tightly linked. From equation (22) in the main text, labor participation for workers with wage w is subsidized on a net basis, i.e., $t(w) + \tau(w) < 0$, if and only if these workers have an above-average social welfare weight, i.e., $b(w) > 1$. This explains why $b(w) > 1$ is both necessary and sufficient for an increase in union power to be welfare-improving in the baseline, see also Proposition 2 in the main text. Intuitively, both participation distortions and distributional effects are proportional to $1 - b(w)$. Therefore, only knowledge of social welfare weights is required to judge whether an increase in union power raises social welfare. If unions respond to marginal tax rates, however, such a tight link between social welfare weights and net taxes on participation no longer exists, since participation taxes at each income level are determined by the complete optimal non-linear tax schedule, which, in turn, depends on all social welfare weights, the income distribution, and participation distortions at all income levels. Consequently, judging whether an increase in union power raises social welfare generally requires knowledge of both participation taxes and social welfare weights.

4 Inefficient rationing

We have deliberately biased our findings in favor of unions by assuming that unemployment rationing is efficient: the burden of involuntary unemployment is borne by the workers with the highest participation costs. However, there are neither theoretical nor empirical reasons to expect that labor rationing is always efficient, see Gerritsen (2017) and Gerritsen and Jacobs

[10] This sufficiency condition only requires that participation taxes are positive, since implicit taxes from unions are always weakly positive (i.e., $\tau(w) \geq 0$). Hence, a positive participation tax is sufficient to guarantee downward distortions on participation.

(2020). In this Section, we analyze how the optimal tax formulas should be modified, and under which conditions unions are desirable, if the assumption of efficient rationing is relaxed. For analytical convenience, we assume that labor markets are independent and there are no income effects at the union level.

We follow Gerritsen (2017) and Gerritsen and Jacobs (2020) by defining the rationing schedule as a continuously differentiable function

$$e_i(E_i, \varphi_i^*, \varphi), \quad e_{iE_i}(\cdot), -e_{i\varphi_i^*}(\cdot) > 0, \tag{26}$$

which specifies the probability $e_i \in [0, 1]$ that workers with participation costs $\varphi \in [\underline{\varphi}, \varphi_i^*]$, find employment in sector i for a given sectoral employment rate E_i and a given participation threshold φ_i^*. The probability $e_i(\cdot)$ of finding a job in sector i increases in employment E_i and decreases if labor participation rises, i.e., if φ_i^* is higher.[11] For all values of employment E_i and the participation cut-off φ_i^*, the following relationship must hold:

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) = E_i. \tag{27}$$

Hence, integrating over all employment probabilities of the workers in sector i (who differ in terms of their participation costs) yields sectoral employment.

Under independent labor markets and no income effects, we can describe the equilibrium using reduced-form equations $w_i = w_i(\rho_i, T_i - T_u)$ and $E_i(\rho_i, T_i - T_u)$, which pin down the equilibrium wage and employment rate in sector i as a function of union power ρ_i and the participation tax $T_i - T_u$. The following Proposition characterizes the optimal tax formulas if labor rationing is inefficient.

Proposition 3. *If Assumptions 1 (independent labor markets), 3 (no income effects at the union level) are satisfied, and labor rationing is described by the rationing schedule (26), then optimal unemployment benefits* $-T_u$*, optimal profit taxes* T_f*, and optimal participation taxes* $T_i - T_u$ *are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{28}$$

$$b_f = 1, \tag{29}$$

$$\left(\frac{t_i + \hat{\tau}_i}{1 - t_i}\right)\eta_{ii} - \left(\frac{\varrho_i}{1 - t_i}\right)\gamma_i = (1 - b_i) + (b_i - b_f)\kappa_{ii}, \tag{30}$$

where the union wedge is redefined as

$$\hat{\tau}_i \equiv \psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(E_i, \varphi_i^*, \varphi)\left(\frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda w_i}\right)\mathrm{d}G_i(\varphi), \tag{31}$$

[11] An example of a rationing schedule that satisfies these criteria is a uniform rationing scheme. All participating workers then face the same probability of finding a job, i.e., $e_i(E_i, \varphi_i^*, \varphi) = E_i/G_i(\varphi_i^*)$ for all values of $\varphi \in [\underline{\varphi}, \varphi_i^*]$.

and ϱ_i denotes the rationing wedge, which is defined as

$$\varrho_i \equiv \frac{\psi_i e_i(E_i,\varphi_i^*,\varphi_i^*)}{E_i/G_i(\varphi_i^*)} \int_{\underline{\varphi}}^{\varphi_i^*} \frac{e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)}{\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)\mathrm{d}G_i(\varphi)} \left(\frac{u(w_i-T_i-\varphi)-u(-T_u)}{\lambda w_i}\right) \mathrm{d}G_i(\varphi) \quad (32)$$

and $\gamma_i \equiv -\frac{\partial G_i(\varphi_i^*)}{\partial(T_i-T_u)}\frac{\varphi_i^*}{G_i(\varphi_i^*)}$ captures the participation response.

Proof. See Appendix B.1. □

The expressions for the optimal unemployment benefit and profit tax are identical to those stated in Proposition 1 in the main text and their explanation is not repeated here. The expression for the optimal participation tax in equation (30) equates the marginal distortionary costs of a higher participation tax (left-hand side) to the marginal distributional gains of a higher participation tax (right-hand side). The expression for the optimal participation tax is modified in two ways compared to the one with efficient rationing. First, with a general rationing scheme, the union wedge $\hat{\tau}_i$ no longer measures the monetized utility loss of a *marginal* worker losing her job, but the expected utility loss of *all rationed workers*, i.e., the workers who lose their job if the wage is marginally increased. Second, in addition to the union wedge $\hat{\tau}_i$, there is a distortion associated with the inefficiency of the rationing scheme, which is captured by the rationing wedge ϱ_i.

To understand the rationing wedge ϱ_i, consider a decrease in the participation tax $T_i - T_u$. Moreover, suppose the reduction in the participation tax is combined with an increase in union power ρ_i so that the equilibrium wage (and hence, the equilibrium employment rate) remains unaffected. More people want to participate if the participation tax is lowered. A fraction $e_i(E_i,\varphi_i^*,\varphi_i^*)$ of the workers who are at the participation margin (i.e., those who are indifferent between employment and unemployment) will succeed in finding a job. However, if employment remains constant, other workers become unemployed. Since these workers are not indifferent between work and unemployment, a welfare loss occurs. The latter is captured by the term ϱ_i, which measures the marginal welfare costs associated with an inefficient allocation of jobs over those who are willing to work. These costs are weighted by the participation response γ_i.

According to equation (30), the higher is ϱ_i, i.e., the more inefficient is the rationing scheme, the *higher* should be the optimal participation tax. The intuition is similar to Gerritsen (2017): by setting a higher participation tax, the workers who care least about finding a job opt out of the labor market. This, in turn, increases the employment prospects of the workers who experience a larger surplus from finding a job. Consequently, the government replaces involuntary unemployment by voluntary unemployment, which reduces the inefficiency of labor-market rationing.

The next Proposition gives the condition under which an increase in union power raises social welfare if rationing is no longer efficient.

Proposition 4. *If labor rationing is described by the rationing schedule (26), and taxes and transfers are set according to Proposition 3, then an increase in union power ρ_i in sector i raises social welfare if and only if*

$$b_i > 1 + \left(\frac{\varrho_i}{1-t_i}\right)\gamma_i. \quad (33)$$

Proof. See Appendix B.2. □

To understand whether it is optimal to increase union power, consider again a policy reform starting from a situation where taxes are optimally set. We marginally raise union power ρ_i in sector i, while simultaneously reducing the participation tax $T_i - T_u$ in sector i such that the wage w_i, and hence employment E_i, is kept constant. The reduction in the participation tax is financed by an increase in the profit tax T_f to ensure that the government budget remains balanced.[12] If the tax system is optimized, the tax reform has no impact on social welfare. Therefore, any impact on social welfare must come from the increase in union power. The reform transfers income from firm-owners to workers in sector i. As before, the associated welfare effect is proportional to $b_i - 1$. By construction, there are no welfare effects associated with changes in equilibrium wages and employment. However, the increase in net earnings raises participation of workers in sector i. If some of the (previously voluntarily) unemployed workers find a job, a welfare loss occurs because – with constant employment – some participants who experience a surplus from working will not be able to find a job. For a given social welfare weight, the more inefficient is the rationing scheme, or the higher is the participation response (i.e., the higher ϱ_i or γ_i), the higher should be the social welfare weight of workers b_i for unions in sector i to be desirable. The welfare costs of inefficient rationing could be so large that they completely off-set the potential welfare gains of unions. Consequently, if rationing is inefficient, increasing union power in a sector where $b_i > 1$ does not necessarily raise social welfare.

5 Occupational choice

So far we have abstracted from an intensive margin of labor supply: each individual can only work a fixed number of hours in one particular sector. The main reason for doing so is that an intensive margin raises a number of very complicated issues that we cannot yet address. For example, which party (i.e., unions or individuals) decides on the number of hours worked? Does the incidence of unemployment fall on the intensive (hours) or extensive (participation) margin? How do unions aggregate worker preferences if they can switch between sectors? In this Section, we do not attempt to answers these difficult questions. Instead, we will demonstrate that our main insights carry over to a setting where workers can switch between occupations. This is what Saez (2002) refers to as the 'intensive margin' in discrete labor-supply models.

To model occupational choice, we assume that each of the N workers draws a vector $\boldsymbol{\varphi} \equiv (\varphi_0, \varphi_1, \cdots, \varphi_I) \in \Phi$ of participation costs according to some cumulative distribution function $G(\boldsymbol{\varphi})$. The i-th element of vector $\boldsymbol{\varphi}$ indicates how costly it is for an individual to work in sector i. Based on their participation costs, individuals choose in which sector (or: occupation) to look for a job. Without labor unions, this choice simply boils down to finding the occupation j where the net payoff from working $w_j - T_j - \varphi_j$ is maximized, provided the latter exceeds the payoff from not working $-T_u$. With labor unions, however, this problem is more complicated, because individuals may not be able to find a job if wages are set above the market-clearing level. An additional difficulty is that it is no longer clear how the union objective should be

[12] The reduction in the participation tax can also be financed by a uniform increase in the tax on all (employed and non-employed) workers. This does not matter for the outcomes.

specified if individuals can switch between sectors. To overcome these issues, we adopt a similar approach as with inefficient rationing (see Section 4). In particular, we assume that there exist reduced-form equations $p_i(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u)$ that are differentiable functions of all participation taxes, which specify a probability $p_i \in [0, 1]$ that an individual becomes employed if she looks for a job in sector i. If the individual is unsuccessful, she cannot move to another sector but instead becomes unemployed. Each individual then solves:

$$\max_{j \in \{0,1,...,I\}} \quad u(-T_u) + p_j(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u)(u(w_j - T_j - \varphi_j) - u(-T_u)), \tag{34}$$

where occupation 0 refers to non-employment, with $w_0 = \varphi_0 = 0$, $T_0 = T_u$, and $p_0 = 1$.

As before, we assume that there are no income effects at the union level and we denote by $w_i(T_1 - T_u, ..., T_I - T_u)$ and $E_i(T_1 - T_u, ..., T_I - T_u)$ the equilibrium wage and *total* employment (as opposed to the employment rate) in sector i as a function of the participation taxes. Furthermore, let Φ_i denote the set of all individuals who look for a job in sector i (including non-employment):

$$\Phi_i \equiv \{\boldsymbol{\varphi} \in \Phi | \text{ arg max}_j \; p_j(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u)(u(w_j - T_j - \varphi_j) - u(-T_u)) = i\}. \tag{35}$$

In equilibrium, the following relationship holds for all i and for all participation taxes:

$$N \int_{\Phi_i} p_i(\boldsymbol{\varphi}, T_1 - T_u, ..., T_I - T_u) \mathrm{d}G(\varphi) = E_i(T_1 - T_u, ..., T_I - T_u). \tag{36}$$

We make the following assumption regarding the functions $p_i(\cdot)$, which ensures that rationing is efficient.

Assumption 1. (Efficient rationing with occupational choice) *$p_i = 0$ on the boundary of the set Φ_i for all sectors i.*

Assumption 1 extends our notion of efficient rationing to this environment by assuming that if there is involuntary unemployment, individuals who are indifferent between choosing sector i and another sector (possibly non-employment) do not find a job. This form of rationing is efficient in the sense that individuals with the lowest surplus from working in a particular sector (compared to their second-best alternative) do not find a job if wages are set above the market-clearing level. This notion of efficient rationing is similar to Lee and Saez (2012).

The following Proposition characterizes the optimal tax system with an intensive, occupational-choice margin.

Proposition 5. *If Assumptions 3 (no income effects at the union level) and 1 (efficient rationing with occupational choice) are satisfied, and individuals optimally choose their occupation according to equation (34), then the optimal unemployment benefit $-T_u$, profit taxes T_f, and participation taxes $T_i - T_u$ are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{37}$$

$$b_f = 1, \tag{38}$$

$$\sum_j \omega_j \left(\frac{t_j+\tau_j^o}{1-t_j}\right)\eta_{ji} = \omega_i(1-b_i) + \sum_j \omega_j (b_j - b_f)\kappa_{ji}, \quad \forall i, \tag{39}$$

where the union wedge with endogenous occupational choice is

$$\tau_j^o \equiv \psi_j N \int_{\Phi_j} \frac{\partial p_j/\partial(T_i-T_u)}{\partial E_j/\partial(T_i-T_u)} \left(\frac{u(w_j - T_j - \varphi_j) - u(-T_u)}{\lambda w_j}\right) \mathrm{d}G(\varphi).$$

Proof. See Appendix C.1. □

The optimal tax formulas are almost identical to the ones in the model without an occupational choice and the interpretation is similar. There are a few, subtle differences between equation (39) and the expression for the optimal participation tax without an occupational choice. First, the union wedge no longer captures the utility loss of the marginal worker, but instead captures the average utility loss of all workers who lose their job if employment in sector j is marginally reduced.[13] This term is similar to the union wedge $\hat{\tau}_j$ with inefficient rationing.

A second difference is that the employment and wage responses η_{ji} and κ_{ji} not only capture 'demand interactions' (through complementarities in production), but also 'supply interactions' (through occupational choice). To illustrate this, suppose that the participation tax in sector i is increased. *Ceteris paribus* this leads to a higher wage and a lower employment rate in sector i. Without an occupational choice, employment and wages in other sectors go down if labor types are complementary factors in production. With an occupational choice, a higher participation tax in sector i might lead some individuals to switch to sector $j \neq i$. This puts further downward pressure on wages in other sectors, but mitigates (and possibly overturns) the negative impact on employment in other sectors. An occupational choice thus affects the magnitude, and possibly the sign, of wage and employment responses. However, *given* these responses, i.e., given η_{ji} and κ_{ji}, the optimal tax formulas are the same as we had before.

Our second main result on the desirability of unions also generalizes to an environment with an occupational choice.

Proposition 6. *If Assumptions 3 (no income effects at the union level) and 1 (efficient rationing with occupational choice) are satisfied, individuals optimally choose their occupation according to equation (34), and taxes and transfers are set according to Proposition 5, then an increase in union power ρ_i in sector i raises social welfare if and only if $b_i > 1$.*

Proof. See Appendix C.2. □

The key to understanding why the desirability condition from Proposition 2 from the main text also holds in the current setting with occupational choice is that labor rationing is efficient. To see this, consider again a marginal increase in union power in sector i: $\mathrm{d}\rho_i > 0$. This reform puts upward pressure on the wage in sector i, which can be off-set by lowering the income tax in sector i: $\mathrm{d}T_i < 0$. The reduction in the income tax, in turn, can be financed

[13] To see why τ_j^o captures an average welfare loss, differentiate equation (36) for $i = j$ with respect to $T_i - T_u$

$$N \int_{\Phi_j} \frac{\partial p_j(\varphi, T_1 - T_u, ..., T_I - T_u)}{\partial(T_i - T_u)} \mathrm{d}G(\varphi) = \frac{\partial E_j(T_1 - T_u, ..., T_I - T_u)}{\partial(T_i - T_u)}.$$

by raising the profit tax: $\mathrm{d}T_f > 0$. As before, the tax reform has no impact on social welfare if the tax system is optimized. Furthermore, as in the model without an occupational choice, this combined reform transfers resources from firm-owners (whose social welfare weight equals one) to workers in sector i (whose social welfare weight equals b_i). However, unlike before, the higher net income of workers in sector i could attract workers from other sectors (possibly non-employment) to look for a job in sector i. These individuals experience the smallest surplus from working in sector i compared to their second-best alternative. Under our assumption of efficient rationing, they will not find a job. Anticipating this, workers on the boundary of Φ_i will not switch between sectors following an increase in union power ρ_i. The impact on social welfare is therefore the same as without an occupational-choice margin, which explains why the desirability condition is unaffected.[14]

6 Bargaining over the wage distribution

In our baseline model, bargaining takes place at the sectoral level and wages vary only across (and not within) sectors. Each sectoral union faces a trade-off between employment and wages, but does not care about the overall *distribution* of wages. There is, however, ample empirical evidence that a higher degree of unionization is associated with lower wage inequality.[15] How do our results for optimal taxes and the desirability of unions change if unions care about the entire distribution of wages?

To answer this question, we now analyze a single union which bargains with firm-owners over *all* wages. To maintain tractability, we assume efficient rationing and we assume away income effects at the union level. The union has a utilitarian objective: it maximizes the sum of all workers' expected utilities. As in the RtM-model, wages are determined through bargaining between the national union and firms, while firms (unilaterally) determine employment. Since the utility function $u(\cdot)$ is concave, the union has an incentive to compress the wage distribution. Doing so is only possible if labor markets are interdependent, since in that case marginal productivity (and hence, the wage) for any group of workers depends on employment in other sectors. If labor markets would be independent, a national union would simply set the same wages in each sector as a sectoral union would, and our previous results apply.

We explicitly solve the Nash-bargaining problem to characterize labor-market equilibrium, where the national union's bargaining power is denoted by $\delta \in [0, 1]$. Since there is only one union, we can no longer use a sector-specific measure of union power ρ_i to analyze the union's desirability. However, under Nash-bargaining, equilibrium wages and employment also depend on profit taxes, which is not the case if we use ρ_i to parameterize union power. To maintain comparability with our previous findings, we therefore assume that firm-owners are risk neutral. This ensures that equilibrium wages and employment can be written only in terms of participation taxes, like before. In Appendix D.1, we set up the bargaining problem, characterize labor-market equilibrium, and extensively discuss its properties. Here, we only highlight the most important features.

[14]This result is similar to Lee and Saez (2012) who find that a minimum wage is desirable if and only if $b_i > 1$.

[15]See, for instance, Freeman (1980, 1993), Lemieux (1993, 1998), Machin (1997), Card (2001), DiNardo and Lemieux (1997), Card et al. (2004), Visser and Checchi (2011), and Western and Rosenfeld (2011).

First, if the union has no bargaining power at all ($\delta = 0$), the labor-market equilibrium coincides with the competitive outcome. Second, if union power δ is sufficiently high, there is at least one group of workers whose wage is raised above the market-clearing level. This follows from the assumptions that, first, the union has an incentive to compress the wage distribution and, second, labor rationing is efficient. Hence, starting from the competitive labor-market outcome, a marginal increase in the bargained wage in the sector with the lowest wage compresses the wage distribution, but entails negligible welfare losses due to involuntary unemployment. Third, it may not be in the union's best interest to raise *all* wages above the market-clearing level. This is because an increase in the wage for high-skilled workers depresses the wages for low-skilled workers. A national union may therefore refrain from demanding an above market-clearing wage for high-skilled workers.

The next proposition shows how taxes should be optimized if there is a single union, which bargains with firm-owners over the entire distribution of wages. To abstain from conflicting union and government objectives, we assume that both the government and the union maximize a utilitarian objective.

Proposition 7. *If Assumptions 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, labor markets are interdependent, and a single union bargains over all wages* w_i *in all sectors* i*, then the expressions for the optimal unemployment benefits* $-T_u$*, optimal profit taxes* T_f*, and optimal participation taxes* $T_i - T_u$ *are the same as in Proposition 1 from the main text.*

Proof. In the absence of income effects, the reduced-form wage and employment equations can be written as $w_i = w_i(T_1 - T_u, ..., T_I - T_u)$ and $E_i = E_i(T_1 - T_u, ..., T_I - T_u)$. Since the optimal tax formulas from Proposition 1 in the main text are derived for any relationship between tax instruments and labor-market outcomes, they remain the same. □

The reason why Proposition one generalizes to a national union bargaining over the entire wage distribution is that the optimal tax rules are expressed in terms of sufficient statistics and equilibrium wages and employment only depend on participation taxes in both cases.[16]

How is the desirability condition for unions modified if the union negotiates the wages for all workers? Once more, we can answer this question by analyzing the welfare effects of a (marginal) increase in union power δ combined with a tax reform that leaves wages and employment in all sectors unaffected. If the tax system is optimized, the tax reform has no impact on social welfare. Any effect on social welfare must then necessarily come from the increase in union power. To analyze the effects of such a reform, we need to keep track of the sectors where the wage is set above the market-clearing level. Denote by $k(\delta) \equiv \{i : G(w_i - (T_i - T_u)) > E_i\}$ the set of sectors where the wage is raised above the market-clearing level. This set $k(\cdot)$ depends – among other things – on union power $\delta \in [0, 1]$. If the union has no power ($\delta = 0$), no wage is raised above the market-clearing level, and consequently $k(\cdot)$ is empty. On the other hand, $k(\delta)$ contains at least one element if $\delta = 1$, since a utilitarian monopoly union always has an incentive to increase the wage for the workers in the sector with the lowest wage. We assume

[16] The optimal tax levels are not necessarily the same because the elasticities and wedges generally differ between the different bargaining structures.

that the set of sectors where wages are above market-clearing levels $k(\delta)$ does not change in response to a marginal increase in union power.[17]

The rise in union power puts upward pressure on the wages of workers $i \in k(\delta)$ for whom the wage already exceeds the market-clearing level (the 'direct' effect). Through spillovers in production, the wages for workers in other sectors $j \notin k(\delta)$ will be affected as well (the 'indirect' effect). Now, consider a tax reform that leaves all wages and employment levels unaffected. Such a tax reform *only* requires an adjustment in the income taxes T_i for those workers whose wage exceeds the market-clearing level, i.e., for sectors $i \in k(\delta)$. Intuitively, if the adjustment in the tax system offsets the 'direct' effects, there will also be no 'indirect' effects. As before, the marginal changes in the participation taxes can be financed by a marginal increase in the profit tax such that the government budget remains balanced. The tax reform that leaves equilibrium wages and employment constant is characterized by the solution to the following system of equations:

$$\forall i \in k(\delta): \quad \sum_{j \in k(\delta)} \frac{\partial w_i(T_1 - T_u, ..., T_I - T_u, \delta)}{\partial T_j} \mathrm{d}T_j^* + \frac{\partial w_i(T_1 - T_u, ..., T_I - T_u, \delta)}{\partial \delta} \mathrm{d}\delta = 0. \quad (40)$$

Here, the functions $w_i = w_i(T_1 - T_u, ..., T_I - T_u, \delta)$ are the reduced-form equations that solve the bargaining problem (see Appendix D.1 for details). The next Proposition derives the desirability condition for the national union.

Proposition 8. *If Assumptions 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, there is a national utilitarian union bargaining with firm-owners over all wages, and the tax-benefit system is optimized according to Proposition 7, then an increase in union power δ increases social welfare if and only if*

$$\sum_{i \in k(\delta)} \omega_i (b_i - 1)(-\mathrm{d}T_i^*) > 0, \quad (41)$$

where the changes in income taxes $\mathrm{d}T_i^*$ follow from equation (40) and $k(\delta) \equiv \{i : G(w_i - (T_i - T_u)) > E_i\}$.

Proof. See Appendix D.3 □

Proposition 8 is an intuitive counterpart of Proposition 2 from the main text: an increase in union power raises social welfare if and only if doing so allows the government to increase the incomes of workers with an above-average social welfare weight. By the same logic as before, the joint increase in union power and the tax reform leaves all labor-market outcomes unaffected, while raising the *net* incomes for the low-skilled. Therefore, increasing union power raises social welfare if and only if the weighted average social welfare weight of workers whose wage is above the market-clearing level exceeds the average social welfare weight of all (employed and unemployed) workers. The weight depends on the share ω_i of workers in sector i and on the change in the income taxes $-\mathrm{d}T_i^*$ in the policy reform.

[17] Assuming $k(\delta)$ does not change following a marginal change in δ is without loss of generality, since there is a discrete number of sectors.

Since the desirability condition remains unaltered, the union's desire to compress the wage distribution does not provide an *additional* reason why a welfarist government would like to raise union power. As was the case with a restriction on profit taxes, the government can achieve the same wage compression as the labor union through the tax-transfer system, without creating involuntary unemployment. Hence, unions cannot redistribute income via wage compression any better than the government can.

7 Efficient bargaining

Up to this point, we have assumed that bargaining takes place in a right-to-manage setting. This bargaining structure generally leads to outcomes that are not Pareto efficient, because firm-owners – who take wages as given – do not take into account the impact of their hiring decisions on the union's objective (McDonald and Solow, 1981). This inefficiency can be overcome if unions and firm-owners bargain over both wages *and* employment. This Section explores whether our results generalize to a setting with efficient bargaining (EB), as in McDonald and Solow (1981). For analytical convenience we do impose the assumptions of independent labor markets, efficient rationing, and no income effects at the union level.

We would like to emphasize from the outset that we consider the EB-model less appealing for two main reasons. First, the assumption that firms and unions can write contracts on both wages *and* employment is problematic with national or sectoral unions, since individual firm-owners then need to commit to employment levels that are not profit-maximizing (Boeri and Van Ours, 2008). Oswald (1993) argues that firms unilaterally set employment, even if bargaining takes place at the firm level. Second, employment is higher in the EB-model compared to the competitive outcome, since part of firm profits are converted into jobs. This property of the EB-model is difficult to defend empirically. Therefore, we maintain the RtM-model as our baseline.

The key feature of the EB-model is that any potential equilibrium (w_i, E_i) in sector i lies on the *contract curve*, which is the line where the union's indifference curve and the firm's iso-profit curve are tangent:

$$\frac{u(w_i - T_i - \hat{\varphi}_i) - u(-T_u)}{E_i \overline{u'(w_i - T_i - \varphi)}} = \frac{w_i - F_i(\cdot)}{E_i}. \tag{42}$$

Intuitively, if the equilibrium wage and employment level are on the contract curve, then it is impossible to raise either union i's utility while keeping firm profits constant, or vice versa.

The contract curve defines a set of potential labor-market equilibria (w_i, E_i) in sector i. Which contract is negotiated depends on the power of union i relative to that of the firm. We model union i's power as its ability to bargain for a wage that exceeds the marginal product of labor. In particular, let υ_i denote the power of union i. We select the equilibrium in labor market i using the following rent-sharing rule:

$$w_i = (1 - \upsilon_i) F_i(\cdot) + \upsilon_i \phi_i(E_i), \tag{43}$$

where $\phi_i(E_i) \equiv \frac{\hat{\phi}_i(N_i E_i)}{N_i E_i}$ is the average productivity of a worker in sector i and $\hat{\phi}_i$ is the contribution of sector i to total output:[18]

$$\hat{\phi}_i(N_iE_i) \equiv F(K, N_1E_1, \cdots, N_iE_i, \cdots, N_IE_I) - F(K, N_1E_1, \cdots, 0, \cdots, N_IE_I). \tag{44}$$

If unions have zero bargaining power, i.e., $\upsilon_i = 0$, the outcome in the EB-model coincides with the competitive equilibrium: $w_i = F_i(\cdot)$. Efficiency then requires $\hat{\varphi}_i = w_i - (T_i - T_u) = \varphi_i^*$. If, on the other hand, union i has full bargaining power, i.e., $\upsilon_i = 1$, it can offer a contract which leaves no surplus to firm-owners. In the latter case, the wage equals average labor productivity and the firm makes zero profits from hiring workers in sector i: $w_iN_iE_i = \hat{\phi}_i(\cdot)$. We refer to this outcome as the full expropriation (FE) outcome.

The characterization of labor-market equilibrium is graphically illustrated in Figure 1. As in

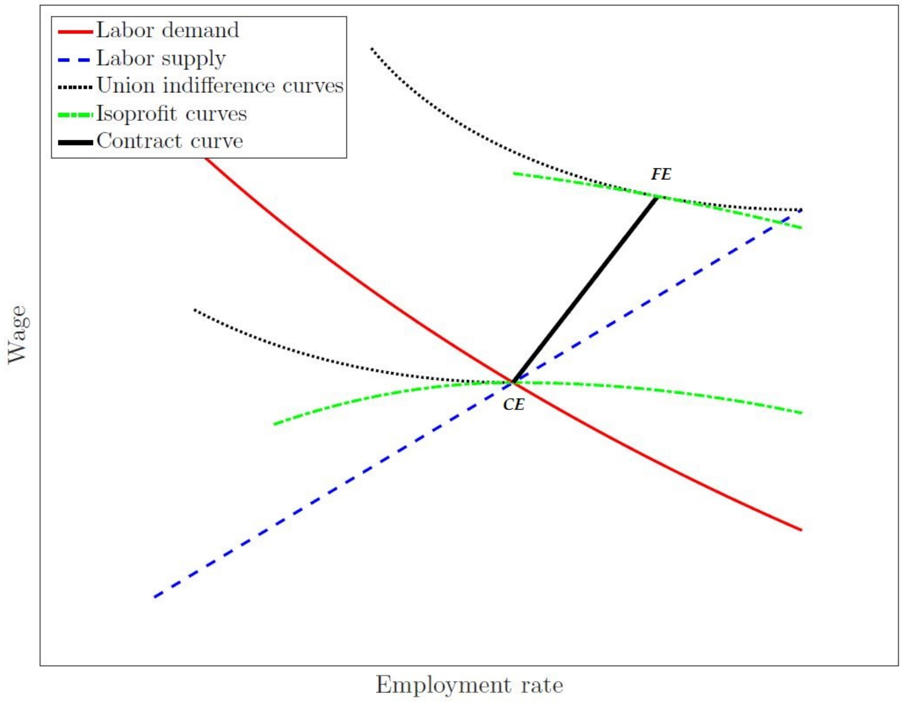

Figure 1: Labor-market equilibria in the efficient bargaining model

the RtM-model, the equilibrium coincides with the competitive outcome if the union has zero bargaining power. If union power increases, the equilibrium moves along the contract curve towards the FE-equilibrium, where the union has full bargaining power. Which equilibrium is selected depends on union power υ_i.

Figure 1 provides three important insights. First, as in the RtM-model, there is involuntary unemployment if union power υ_i is positive. Without involuntary unemployment, unions are marginally indifferent to changes in employment, since labor rationing is efficient. Hence, unions are always willing to bargain for a slightly higher wage and accept some unemployment. Second, in contrast to the RtM-model, there is also a labor-demand distortion: the wage exceeds the marginal product of labor if $\upsilon_i > 0$, see equation (43). Consequently, the labor-market equilibrium is no longer on the labor-demand curve. Intuitively, if the wage equals the marginal

[18] It should be noted that ϕ_i is different from the one used in Section 8.1 of the main text, where it denotes the wage share of sector i in aggregate labor income.

product of labor, firms are indifferent to changes in employment, whereas unions are generally not. Hence, it is possible to negotiate a labor contract with a lower wage and higher employment, which benefits both parties. As a result, efficient bargaining results in implicit subsidies on labor demand. Third, and in stark contrast to the RtM-model, an increase in union power will not only result in a higher wage, but also in *higher* employment. As illustrated in Figure 1, the contract curve is upward sloping. The higher is union power, the larger is the share of the bargaining surplus that accrues to union members. Due to the concavity of the utility function $u(\cdot)$, this surplus is translated partly into higher wages, and partly into higher employment.

In the absence of income effects at the union level, and assuming independent labor markets, the contract curve (42) and the rent-sharing rule (43) jointly determine the equilibrium wage w_i and employment E_i in sector i solely as a function of the participation taxes $T_i - T_u$. If the participation tax increases, fewer workers want to participate. In terms of Figure 1, the labor-supply schedule shifts upward. As a result, the equilibrium wage (employment rate) will be higher (lower) following the increase in the participation tax. Therefore, the comparative statics are qualitatively the same as in the RtM-model. We replicate Lemma 1 from the main text for the EB-model in Appendix E.1. The following Proposition characterizes optimal taxes.

Proposition 9. *If Assumptions 1 (independent labor markets), 2 (efficient rationing), and 3 (no income effects at the union level) are satisfied, and the efficient-bargaining equilibrium in labor market i is determined by the contract curve (42) and the rent-sharing rule (43), then optimal unemployment benefits $-T_u$, profit taxes T_f, and participation tax rates t_i are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{45}$$

$$b_f = 1, \tag{46}$$

$$\left(\frac{t_i + \tau_i - m_i}{1 - t_i}\right)\eta_{ii} = (1 - b_i) + (b_i - 1)\kappa_{ii}, \tag{47}$$

where $m_i \equiv \frac{w_i - F_i}{w_i} = \upsilon_i \left(\frac{\phi_i - F_i}{w_i}\right)$ is the implicit subsidy on labor demand. The wage and employment elasticities with respect to the participation tax rate t_i are given by:

$$\kappa_{ii} = \frac{u'_u w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right)}{\frac{\hat{u}'_i E_i}{G'_i(\hat{\varphi}_i)} + u'_u w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right) + (\hat{u}_i - u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i} - 1 + \frac{(\hat{u}'_i - \overline{u'_i})}{\overline{u'_i}}\right)} > 0, \tag{48}$$

$$\eta_{ii} = \frac{-u'_u w_i(1-t_i)}{\frac{\hat{u}'_i E_i}{G'_i(\hat{\varphi}_i)} + u'_u w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right) + (\hat{u}_i - u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i} - 1 + \frac{(\hat{u}'_i - \overline{u'_i})}{\overline{u'_i}}\right)} > 0. \tag{49}$$

Proof. See Appendix E.2. □

The optimality conditions in the EB-model are very similar to their counterparts in the RtM-model. Except from differences in the definitions of the elasticities, the main difference is the implicit subsidy on labor demand m_i in the expression for the optimal participation tax rate t_i in equation (47). Since the equilibrium wage exceeds the marginal product of labor, a decrease in employment in sector i positively affects the firm's profits, which the government can tax without generating distortions. The higher is the implicit subsidy on labor demand m_i, the higher should optimal participation tax rates be set – *ceteris paribus*.

The optimal participation tax aims to redistribute income and to counter the implicit taxes on labor participation τ_i and the implicit subsidies on labor demand m_i. The equilibrium is neither on the labor-supply nor on the labor-demand curve if the union has some bargaining power. On the one hand, employment is too low, because unions generate involuntary unemployment (as captured by the union wedge τ_i), which calls for lower participation tax rates. On the other hand, employment is too high, because unions generate implicit subsidies on labor demand (as captured by m_i), which calls for higher participation tax rates. Hence, it is no longer unambiguously true that participation taxes should optimally be lower in unionized labor markets. This result contrasts with our finding from the RtM-model.

How is the desirability condition for unions affected if we assume efficient bargaining? The next Proposition answers this question.

Proposition 10. *If Assumption 2 (efficient rationing) is satisfied, the equilibrium in labor market i is determined by the contract curve (42) and the rent-sharing rule (43), and taxes and transfers are set according to Proposition 9, then increasing union power υ_i in sector i raises social welfare if and only if $b_i > 1$.*

Proof. See Appendix E.3. □

According to Proposition 10, the condition under which an increase union power in sector i is desirable is the same as in the RtM-model. Therefore, the question whether unions are desirable or not does not depend on the bargaining structure. This might seem surprising, given that – unlike in the RtM-model – employment increases in union power in the EB-model. However, also *unemployment* increases in union power, since the contract curve is steeper than the labor-supply curve. Intuitively, the union trades off employment and wages, which is not the case at the individual level. Only the effect on unemployment is critical to assess the desirability of unions. Stronger unions still generate more involuntary unemployment. Hence, an increase in union power is desirable only if there is too much employment as a result of net subsidies on participation. Therefore, the intuition for the desirability of unions in the RtM-model carries over to the EB-model: unions are only useful only if net participation subsidies lead to overemployment.

8 Data

8.1 Union data

For data on union density by sector, we draw on the "Jelle Visser database", which is officially referred to as the Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts (Visser, 2019). This database forms the basis of the OECD Bargaining and Trade Union Data. We use database version 6.1 from 2019.[19] The ICTWSS-V6.1 is an unbalanced panel data set spanning 55 countries over the time-period 1960-2018. It contains 234 union-related variables of which we use union density at the sectoral level (variables 202-220 in the database). The union density is defined as total, net union membership as a proportion of all wage and salary earners in employment. Net union membership is defined as the total number of union members minus union members that are outside the active, dependent and employed labor force (i.e., retired workers, independent workers, students, and unemployed workers).

We focus our analysis on the latest year in our database for which we have the most comprehensive coverage of union density data. The union database contains many missing observations, because union densities are not measured every year, not for every country, and not for every sector. To obtain a more complete data set, we pool the observations on union membership for each country-sector over a 10-year time window. This procedure rests on the assumption that union membership rates are only slow-changing over time.[20] Doing so gives us a coverage of union densities at the sectoral level of approximately 75%.

8.2 Wage data

We use data on gross earnings per worker in local currency units at the sectoral level for the (latest) year where we have observations. To do so we exploit three data sources.

First, for most countries in our sample (Austria, Canada, Germany, Denmark, Spain, Finland, France, United Kingdom, Hungary, Ireland, Italy, Latvia, Netherlands, Norway, Slovakia, Sweden, and United States) we draw on the STAN (Structural Analysis) industry database, which is collected by the Organisation for Economic Co-operation and Development (OECD, 2022b). The STAN database is a panel data set containing information on output, value added, and its underlying components, as well as labor input, investment, and capital stocks at the sectoral level. The STAN database covers sectoral data on all OECD countries at the International Standard Industrial Classification of All Economic Activities, version 4 (ISIC4), at the 2-digit level from 1970-2021. From this database, we extract the wage (WAGE), employment (EMPN), full-time equivalent employment (FTEN) variables. Wage refers to gross wages and salaries for employees, *excluding* employer contributions, for example for social insurance and pensions. The total wage bill is the corresponding item in each country's National Accounts. Moreover, by focusing on the wage bill minus employer contributions, this wage measure most closely corresponds to the gross earnings variable in the OECD tax-benefit calculator, which is used to compute participation tax rates. Employment refers to the total number of persons engaged in domestic production, including the self-employed. Full-time equivalent employment is employment in persons corrected for hours worked. The wage per full-time equivalent worker is calculated as the total sectoral wage bill divided by the total number of full-time equivalent workers.

Second, we rely on the Statistics on Wages Database of the International Labor Organization (ILO, 2022e) for Switzerland, Japan, (South) Korea, New Zealand, and Turkey, since the OECD

[19]The most recent version (6.2) dates to 2021. The latter, however, no longer contains sectoral data on union densities.

[20]We confirm that union densities are slow-changing by inspecting sectoral union densities over time for countries that have more comprehensive data coverage over time.

STAN database does not contain sectoral wage data for these countries. Furthermore, the STAN wage data cover fewer sectors than the ILO data for Australia. Therefore, we also use the ILO wage data for Australia. The ILO database contains mean monthly gross earnings of employees measured in local currency units at the ISIC4 1-digit level. This unbalanced panel data set covers 187 countries and spans the time-period 1969-2021. Gross earnings are defined as monthly gross remuneration in cash and in-kind paid to employees, as a rule at regular intervals, for time worked or work done together with remuneration for time not worked, such as annual vacation, other type of paid leave or holidays. Monthly earnings data are converted to yearly earnings by multiplication with 12.

To merge the earnings data with the union data, we chose the year of the wage data that matched with the latest year for which we had the most comprehensive coverage of union data. This was possible for all countries, except for Switzerland. Here, we substituted wage data for 2016, since sectoral wage data were not available in the ILO data for 2015. Table 1 reports the coverage of our data.

Third, we calculate wages per full-time equivalent worker using data from the STAN database and the OECD. The STAN data contain information on full-time equivalent employment for the following 7 countries: Austria, Spain, France, Italy, Netherlands, Norway, and the United States. For the 16 remaining countries, only data on total employment are available (Australia, Canada, Germany, Denmark, Finland, Hungary, Ireland, Japan, (South) Korea, Latvia, New Zealand, Slovakia, Sweden, Switzerland, Turkey, and United Kingdom). Therefore, we calculate full-time equivalent employment ourselves by means of a country-sector specific part-time factor, which is defined as the ratio of average weekly hours worked relative to the statutory length of the working week in that country. We divide the wage per worker by the part-time factor to obtain the wage per full-time equivalent worker. Data on weekly hours worked come from the ILO (2022d). Data on the statutory working week are taken from the Employment Outlook of the OECD (2021). The statutory length of the working week is taken to be standard working week. Due to data availability, for the following countries we used negotiated hours rather than statutory hours: Denmark, Germany, and Switzerland. No data on the standard working week were available for Ireland and the UK. For these countries, we impute the working week at 40 hours.

8.3 Merging union and wage data

To merge the sectoral union densities from the ICTWSS-database and the sectoral wage data from the STAN and ILO-databases, we make a concordance between the sectoral classification of each database, since the sectoral division in each data set is based on a different industry classification. Table 2 shows the sectoral mapping between all datasets. The baseline sectoral classification is the one from the ICTWSS (union) data.

We exactly map the sectoral wage data from the STAN database onto the sectoral classification of the union data by aggregating and disaggregating a number of sectors in the STAN database for each country-year observation, see Table 2. In particular, we construct the Manufacturing sector to exclude the Metal sector, which is taken as a separate sector. In addition, we merge the Transport and communication sectors. Further, we create the aggregate sector

Table 1: Mapping of years in ICTWSS, STAN and ILO data

Country	ICTWSS	STAN	ILO
1. Australia	2016	2016	
2. Austria	2016	2016	
3. Canada	2017	2017	
4. Germany	2016	2016	
5. Denmark	2016	2016	
6. Spain	2016	2016	
7. Finland	2016	2016	
8. France	2016	2016	
9. Hungary	2015	2015	
10. Ireland	2016	2016	
11. Italy	2014	2014	
12. Japan	2014	2014	
13. Korea	2013		2013
14. Latvia	2016		2016
15. Netherlands	2016	2016	
16. Norway	2017	2017	
17. New Zealand	2017		2017
18. Slovakia	2016	2016	
19. Sweden	2017	2017	
20. Switzerland	2015		2016
21. Turkey	2016		2016
22. United Kingdom	2018	2018	
23. United States	2018	2018	

'Industry' by aggregating the underlying sectors. For the countries for which we rely on the ILO wage data, which are only available at the 1-digit ISIC level, we map the sectoral division in the ILO data directly onto the ICTWSS data. We could not do this for the Metal sector and the (aggregate) sector Commercial services. Due to mismatches between the sector definitions in the union data and the ILO data we drop the ILO-sectors E. Water supply; sewerage, waste management and remediation activities, J. Information and communication, and N. Administrative and support service activities. This merge of data leaves us with (potentially) 19 different sectors, of which 3 are aggregated sectors.

Our final sample contains 23 countries: Australia, Austria, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Hungary, Ireland, Italy, Japan, Latvia, (South) Korea, Netherlands, Norway, New Zealand, Slovakia, Sweden, Turkey, and United States. This sample is based on the joint availability of sectoral union density data, sectoral wage data from the OECD and the ILO, and the tax-benefit calculator of the OECD for these countries. Our sample ultimately consists of data during the time period 2014-2018. For the sector Metal we have only one observation for the union density in Australia, but no corresponding wage data. Therefore, we are left with 18 sectors. Given that the coverage of sectoral union densities and wage data is incomplete, our final sample contains 294 observations spread out over 23 countries and 18 sectors.

Table 2: Sector concordance between union and earnings data

Merged data	ICTWSS	STAN	ILO
Agriculture	agr	D01T03	A. Agriculture; forestry and fishing
Industry*	ind	D05T44	
Services*	serv	D45T99	
Mining	mining	D05T09	B. Mining and quarrying
Manufacturing	manuf	D10T33 - D24T25	C. Manufacturing
Metal	metal	D24T25	
Utilities	util	D35T39	D. Electricity; gas, steam and air conditioning supply
Construction	constr	D41T43	F. Construction
Trade	trade	D45T47	G. Wholesale and retail trade; repair of motor vehicles and motorcycles
Transport and communication	transport	D49T53 + D58T63	H. Transportation and storage
Hotels and restaurants	hotels	D55T56	I. Accommodation and food service activities
Finance	finance	D64T66	K. Financial and insurance activities
Real estate and business services	business	D68T82	L. Real estate activities
Commercial services*	commercial	D45T82	
Social services	socialserv	D87T88	
Public administration	publadmin	D84	O. Public administration and defence; compulsory social security
Education	educ	D85	P. Education
Health care	health	D86	Q. Human health and social work activities
Other services	otherserv	D90T99	S. Other service activities

* Denotes an aggregate sector

8.4 Tax data

We employ the OECD tax-benefit web calculator to manually compute participation tax rates for all 294 country-sector observations in our data set (OECD, 2022a). This tax-benefit calculator computes the gross-net income trajectory for pre-specified income levels and demographics of households. The tax-benefit calculator is available for many years and we pick the year for which we used the union data, see also Table 1.

To determine participation taxes, we first calculate the sum of taxes paid minus transfers received at the household level if the primary earner is full-time employed at the sectoral wage. Subsequently, we calculate the sum of taxes paid minus transfers received at the household level when the primary earner is unemployed and entitled to social assistance-benefits (in the baseline) or unemployment benefits (in the robustness check).[21] In line with our theoretical definition, the participation tax rate is then defined as the difference between taxes paid minus transfers received when the primary earner is employed and unemployed, expressed as a fraction of gross earnings of the primary earner. The total net tax burden in work is the sum of the income tax and social-security contributions minus family benefits, and in-work tax credits.[22] The total tax burden for households where the primary earner is out of work is based on the same tax items except that we account for social-assistance benefits (in the baseline) or unemployment benefits (in the robustness check).

[21]Because our theoretical model is static, it is not obvious if the empirical counterpart of income in non-employment includes only social assistance or also unemployment benefits, which only have a limited duration. Therefore, we decided to calculate the participation tax rate at each country-sector observation twice.

[22]We set the housing benefits (e.g., rent assistance) to zero, since we do not want to distinguish between renters and home-owners.

We focus on a two-earner couple with two dependent children (the default setting in the OECD tax-benefit web calculator). The earnings of the primary earner are taken to be the sector-specific yearly full-time equivalent wage. Regarding the secondary earner, we assume positive assortative mating, such that there is a perfect correlation between earnings of primary and secondary earners. We then calculate the secondary earner's income by multiplying the primary earner's income with a country-specific ratio that measures the earnings differential between primary and secondary earners. In particular, the ratio is calculated as the product of average monthly earnings of females multiplied by total female employment divided by the product of average monthly earnings of males multiplied by total male employment, using data from ILO (2022a,b,c). In our data set, this fraction is always between 0 and 1, see Table 3. It captures differences in labor participation, unemployment rates, working hours and hourly wages between females and males (e.g., due to labor-market discrimination). For all other choices, we use the default settings of the tax-benefit calculator.

Table 3: Employment and earnings males and females

Country	Year	Employment male	Employment female	Earnings male	Earnings female	Ratio
Australia	2016	5189	4483	3958	2651	0.58
Austria	2016	1922	1719	3836	2447	0.57
Canada	2017	7916	7245	3605	2767	0.70
Denmark	2016	1213	1074	4709	3882	0.73
Finland	2016	1101	1020	3647	2923	0.74
France	2016	12393	11736	3786	3110	0.78
Germany	2016	19289	16985	5353	4371	0.72
Hungary	2015	2106	1788	1913	1586	0.70
Ireland	2016	983	841	3950	3374	0.73
Italy	2014	12032	8848	3228	2662	0.61
Japan	2014	29362	22026	3019	2180	0.54
Korea	2013	13107	9000	3633	2347	0.44
Latvia	2016	392	408	1707	1445	0.88
Netherlands	2016	3758	3209	3631	2281	0.54
New Zealand	2017	1083	978	3518	2474	0.63
Norway	2017	1173	1060	4564	3975	0.79
Slovakia	2016	1263	1048	1987	1550	0.65
Spain	2016	9465	7897	2889	2312	0.67
Sweden	2017	2249	2062	3764	3343	0.81
Switzerland	2015	2043	1758	5821	4805	0.71
Turkey	2016	15678	6749	1296	1203	0.40
United Kingdom	2018	14447	12912	3726	2459	0.59
United States	2018	67672	59203	4618	3521	0.67

8.5 Robustness check: unemployment benefits

In the robustness exercise, we assume that households collect unemployment benefits when the primary earner becomes unemployed. Table 4 shows the descriptive statistics of participation tax rates based on unemployment benefits by country, while Table 5 shows the descriptive statistics by sector.

Table 4: Participation tax rates based on unemployment benefits by country

Country	No. sectors	Participation tax rate Mean	Std. dev.	Min.	Max.
Sample	294	67.75	15.87	30.19	151.4
Australia	13	47.05	4.54	39.89	54.48
Austria	5	77.54	12.57	71.47	100.0
Canada	18	69.01	8.14	55.52	81.89
Denmark	14	75.46	5.68	66.27	86.95
Finland	12	79.24	1.23	76.19	81.53
France	15	74.00	6.57	51.74	78.81
Germany	7	84.09	5.65	71.91	89.84
Hungary	18	66.32	6.32	52.79	78.14
Ireland	16	57.79	25.66	46.58	151.4
Italy	9	80.89	4.59	70.59	86.44
Japan	11	79.75	4.75	73.17	87.34
Korea	12	56.40	11.84	38.05	74.32
Latvia	10	88.43	2.24	82.16	89.79
Netherlands	17	73.47	3.48	65.08	78.19
New Zealand	13	35.60	3.04	30.19	39.99
Norway	13	75.51	2.87	66.33	77.05
Slovakia	10	76.25	3.56	69.34	79.75
Spain	17	76.40	15.95	57.43	131.8
Sweden	15	67.74	6.21	59.71	78.63
Switzerland	7	84.50	0.39	84.12	85.21
Turkey	6	64.46	4.79	60.79	73.00
United Kingdom	18	50.02	11.55	35.80	67.00
United States	18	57.55	12.41	47.76	80.78

Not surprisingly, participation tax rates are substantially higher once we take unemployment benefits into account: on average 68% based on unemployment benefits, compared to an average of 37% in the baseline based on social-assistance benefits, see also Figure 2.

Participation tax rates based on unemployment benefits also feature quite some cross-country heterogeneity, and generate a different country ranking than based on social assistance, because unemployment benefit systems differ a lot across countries. The countries with the highest participation tax rates based on unemployment benefits are: Latvia (88%), Switzerland (84%), and Germany (84%). The countries with the lowest participation tax rates are: New Zealand (36%), Australia (47%), and United Kingdom (48%).

The participation tax rates based on unemployment benefits show a bit more variation across sectors, but are generally in the order of 60-70%, with Agriculture again being an outlier, see Figure 2.

Figure 3 gives the scatter plot of participation tax rates against union densities. This scatter plot shows the same pattern as in the main text. A simple regression of participation tax rates on union density returns a positive coefficient of 0.076 (s.e. 0.041), which is significant at the 10-percent level.

Table 5: Participation tax rates based on unemployment benefits by sector

Sector	No. countries	Participation tax rate Mean	Std. dev.	Min.	Max.
Sample	294	67.75	15.87	30.19	151.4
Agriculture	19	78.84	26.38	36.12	151.4
Commercial	17	71.54	12.70	46.58	89.21
Construction	21	69.67	13.44	37.31	89.45
Education	17	64.30	16.09	34.39	88.56
Finance	17	58.48	15.71	30.19	85.21
Health care	10	65.45	13.83	45.98	89.34
Hotels and restaurants	15	69.87	13.32	35.26	81.89
Industry	15	71.00	13.05	47.43	89.34
Manufacturing	22	66.10	14.94	36.40	88.53
Mining	8	56.59	15.12	35.80	81.93
Other services	15	70.84	12.72	39.99	86.90
Public administration	19	66.58	16.81	33.07	89.79
Real estate and business services	14	63.81	13.34	35.75	78.90
Services	16	71.43	12.57	47.53	89.27
Social services	23	73.14	13.01	38.77	89.84
Trade	17	66.84	13.26	39.14	84.12
Transport and communication	18	66.00	15.17	36.11	87.34
Utilities	11	55.28	17.00	30.36	79.75

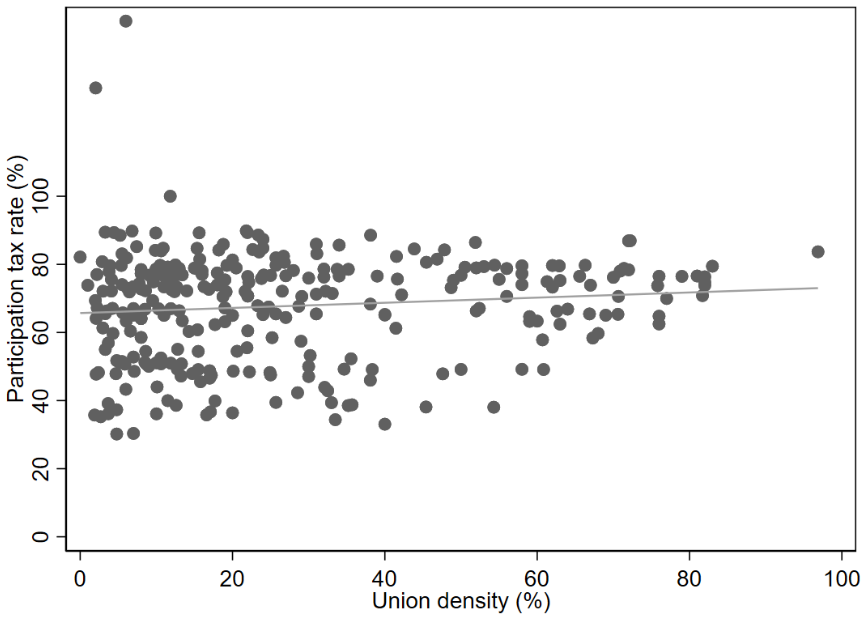

Figure 3: Participation tax rates and union densities

Finally, Table 6 presents the regression results of a fixed-effects regression of participation tax rates on union densities. This regression strengthens the baseline results.

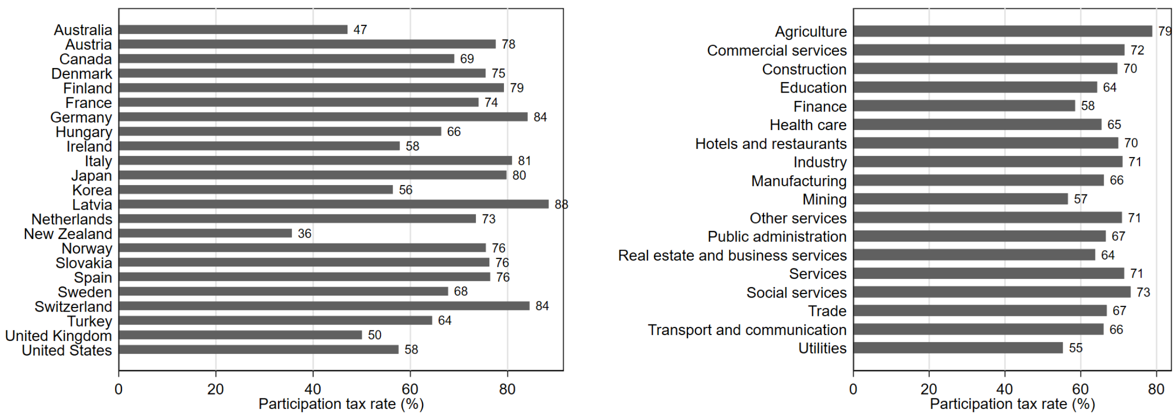

Figure 2: Average participation tax rates across countries and sectors based on unemployment benefits

Table 6: Fixed-effects regressions of participation tax rates on union densities

Variable	Coefficient	Standard error	t-value
Union density	-0.172	0.042	-4.11
Constant	59.49	2.28	26.0
R^2	0.67	R^2 adj.	0.64

Country-fixed effects included, United States is the reference country

8.6 Robustness check: micro data for the US

The analysis conducted so far is based on sector data. The main reason for doing so is that micro data on union membership are scarce and generally not available for the countries that we include in our analysis. However, for the United States, the Current Population Survey (CPS) contains information on union membership for a subsample of individuals. Therefore, our final robustness check investigates the relationship between union membership and participation tax rates based on micro data for the United States.

For our analysis, we use the March 2018 supplement of the CPS (2018), which is the same year for which the sector data are available, see Table 1. We define the wage variable as income from wage and salary payments. For a subsample of individuals, approximately 6.7% of the total sample, the CPS contains information on union membership. From this subsample, we focus on individuals who work full-time (defined as working at least 35 hours per week and 45 weeks per year) and have an hourly wage that exceeds half the federal minimum wage of $7.25 per hour. This leaves us with a sample of 9,052 observations.

Because for each individual union membership takes a value of either zero or one, we group individuals by their annual labor income. The lowest income bin contains all individuals with earnings below $20,000. The next bins proceed in steps of $10,000 and the final bin groups all individuals with earnings above $150,000.[23] Within each income bin, we calculate the union density as the fraction of individuals who are member of a union. As it turns out, union

[23]The minimum number of observations within an income group is 85 (for earnings between $140,000 and $150,000) and the maximum number of observations is 1,457 (for earnings between $30,000 and $40,000).

density shows less dispersion between income bins than what is found based on sector data. Specifically, for different income groups union density varies between 5% (for individuals with earnings below \$20,000 and above \$150,000) and 18% (for individuals with earnings between \$70,000 and \$80,000), with an average of approximately 11%. The corresponding figures based on sector data for the United States are 2% (Agriculture) and 30% (Education), again with an average of approximately 11%, see Table 4 from the main text. To calculate participation tax rates, we apply the OECD tax benefit calculator to the average annual wage within each income bin in exactly the same way as before, see Section 8.4.

The left panel of Figure 4 shows the relationship between the union density and the participation tax rates based on the micro data from the CPS. For comparability, the right panel shows the relationship based on sector data (which is also shown in Figure 9 from the main text). For

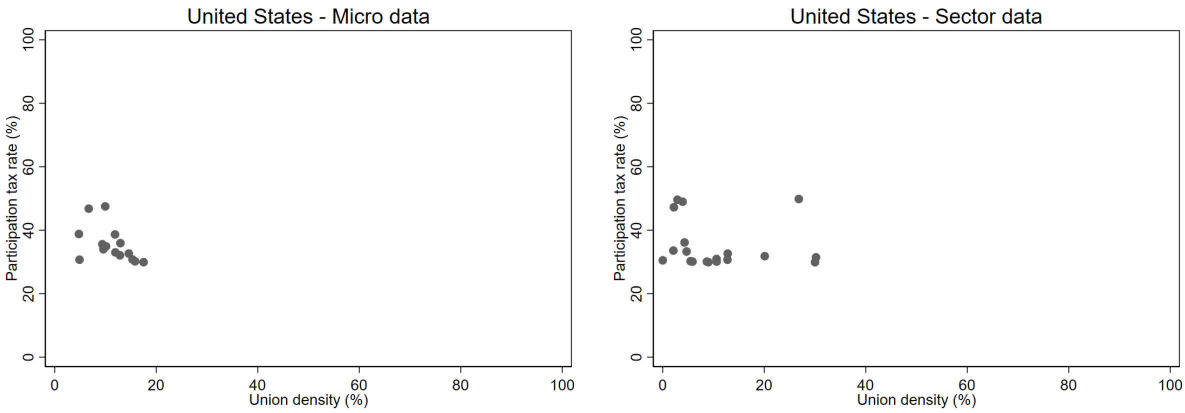

Figure 4: Union densities and participation tax rates based on micro and sector data

the United States, the sector data reveal a weak negative correlation between participation tax rates and union densities that is far from significant. The CPS data show a stronger negative association, which is (weakly) significant at the 10-% level despite the fact that there is less variation in union density. This confirms the result from our analysis based on sector data for a much larger sample of countries, see Table 1 from the main text.

Finally, Figure 5 plots union densities against wages scaled by the national average based on micro (left panel) and sector (right panel) data. Figure 5 from the main text shows the pattern for the entire sample. While the latter reveals a positive association between union density and wages for the entire sample of countries, such a relationship is not visible for the United States. The correlation between union density and wages, based on both micro and sector data, is far from significant. Hence, we do not find evidence that unions are actually strongest among low-income workers. Combined with the observation that participation tax rates are positive at all income levels, this corroborates our finding based on sector data that unions are not a socially desirable complement to the redistributive tax system if taxes are optimally set.

9 Simulations: sensitivity analysis

This section analyzes how the results from our simulations are affected if some of the key parameters of our model are changed. For each of the robustness checks, we change one of the

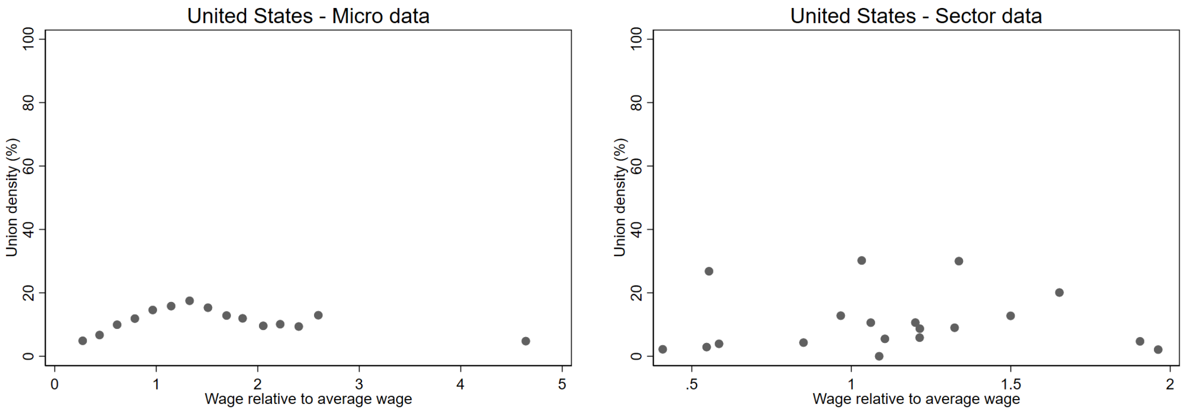

Figure 5: Union densities and wages based on micro and sector data

parameters and recalibrate the model to match the same empirical targets as in the baseline.[24]

9.1 Labor-demand elasticity

We first examine how our results are affected if we consider different values for the labor-demand elasticity. This elasticity ultimately determines the trade-off between wages and employment for the union. Figures 6 and 7 (8 and 9) show the optimal participation tax rates and social welfare weights if the average labor-demand elasticity is doubled to $\bar{\varepsilon} = 1.4$ or cut in half to $\bar{\varepsilon} = 0.35$. The implied elasticity of substitution in the production then equals $\sigma = 1.354$ and $\sigma = 0.355$, respectively. The average participation tax rate at the optimal tax system with unions is comparable to the baseline scenario as it ranges from 58.0% to 59.0%, depending on the elasticity of labor demand.

We confirm our finding that optimal participation tax rates are significantly lower in unionized labor markets: optimal participation tax rates in competitive labor markets are on average between 7.4 and 7.5 percentage points higher, depending on the elasticity of labor demand. Furthermore, we find that if the tax system is optimized, an increase in union power does not improve social welfare in both cases: the social welfare weight for all employed workers remains below the average of one.

It might be surprising that the impact of unions on the average participation tax rate is so similar for different labor-demand elasticities. The explanation for this finding is that the degree of union power ρ is recalibrated to make sure that the unemployment rate in the calibrated economy corresponds to the actual unemployment rate of 6.9%. A higher labor-demand elasticity raises the costs of demanding higher wages, and, hence, requires higher union power to match the unemployment rate observed in the data. Hence, the impact of a larger labor-demand elasticity on the union wedge is countered by larger union power.

[24]When we compare the optimal tax system with and without unions, we do *not* recalibrate the model, but instead conduct a comparative statics exercise by setting $\rho = 0$.

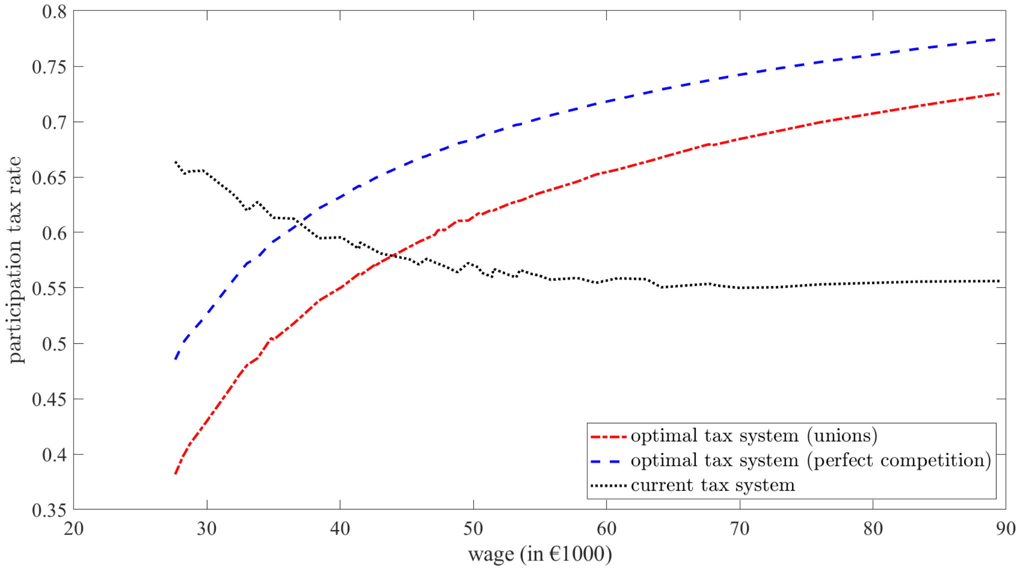

Figure 6: Optimal participation tax rates (high labor-demand elasticity)

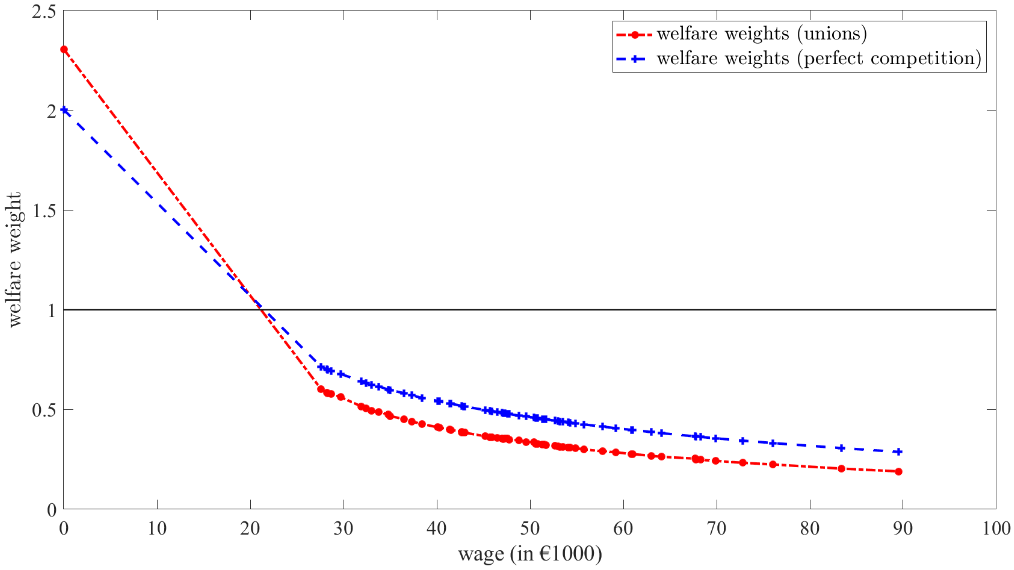

Figure 7: Social welfare weights (high labor-demand elasticity)

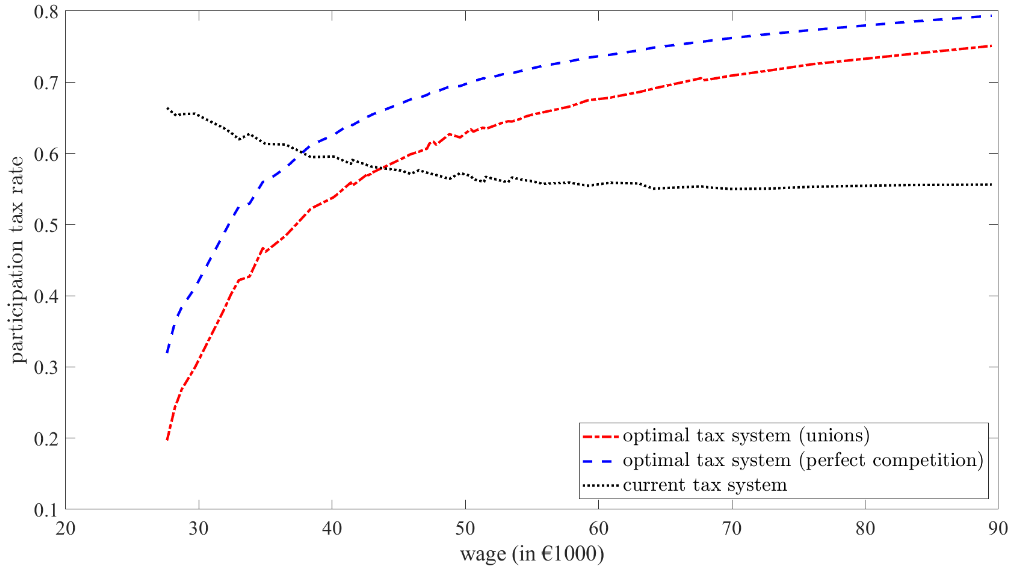

Figure 8: Optimal participation tax rates (low labor-demand elasticity)

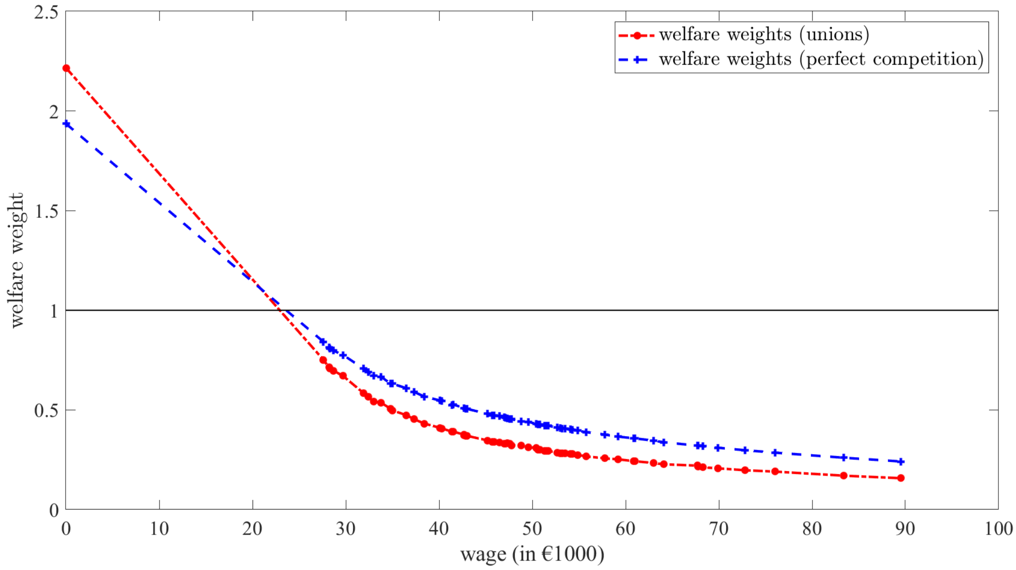

Figure 9: Social welfare weights (low labor-demand elasticity)

9.2 Union power

In this robustness check, we investigate how our results are affected if the degree of union power increases, see Figures 10 and 11. To that end, we calibrate the degree of union power at $\rho = 0.330$ to match an involuntary unemployment rate of 13.8%, which is twice as high as the rate of 6.9% in the baseline year 2015. Not surprisingly, the impact of unions on the optimal participation tax rates is larger. On average, the optimal participation tax rate with unions is approximately 10.5 percentage points below the optimal participation tax rate with perfectly competitive labor markets (compared to 7.4 percentage points in the baseline). Furthermore, we confirm our baseline finding that an increase in union power does not raise social welfare:

all social welfare weights for employed workers are below the average of one if the tax system is optimized.

Figures 12 and 13 plot the optimal participation tax rates and social welfare weights if the degree of union power is calibrated at $\rho = 0.125$, to match an unemployment rate of 3.45%, which is half the actual unemployment rate in the year 2015. This could capture, for instance, that only a fraction of involuntary unemployment is driven by unions demanding above market-clearing wages. We again find that an increase in union power reduces social welfare. Furthermore, unions lead to lower optimal participation tax rates compared to the competitive benchmark, but the difference is less pronounced (4.7 percentage points versus 7.4 percentage points in the baseline).

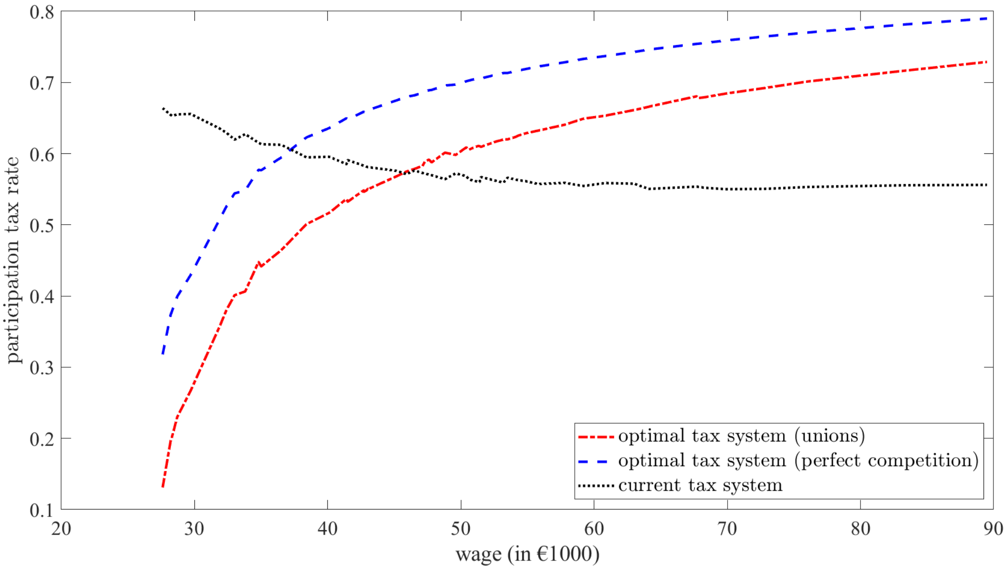

Figure 10: Optimal participation tax rates (strong unions)

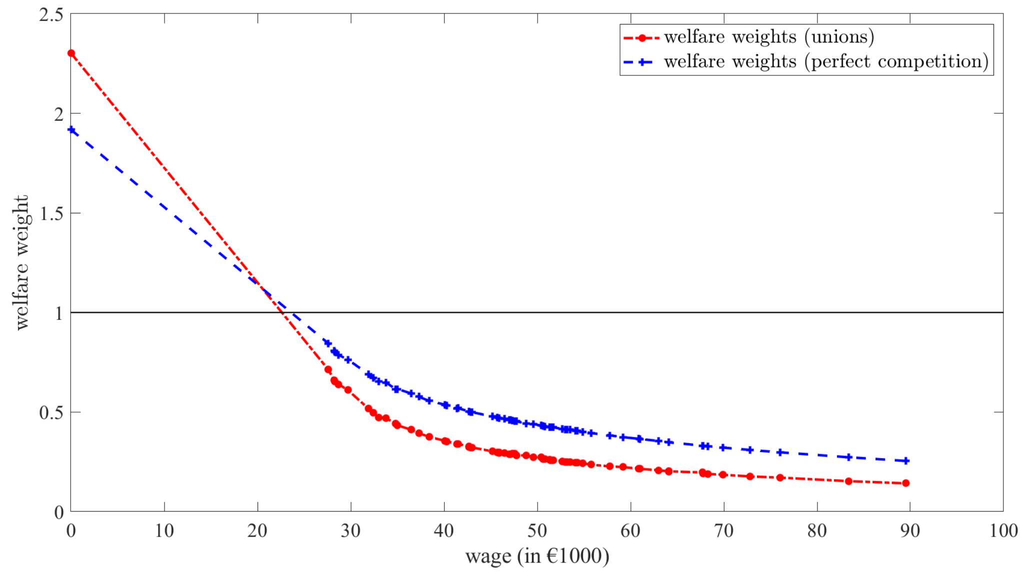

Figure 11: Social welfare weights (strong unions)

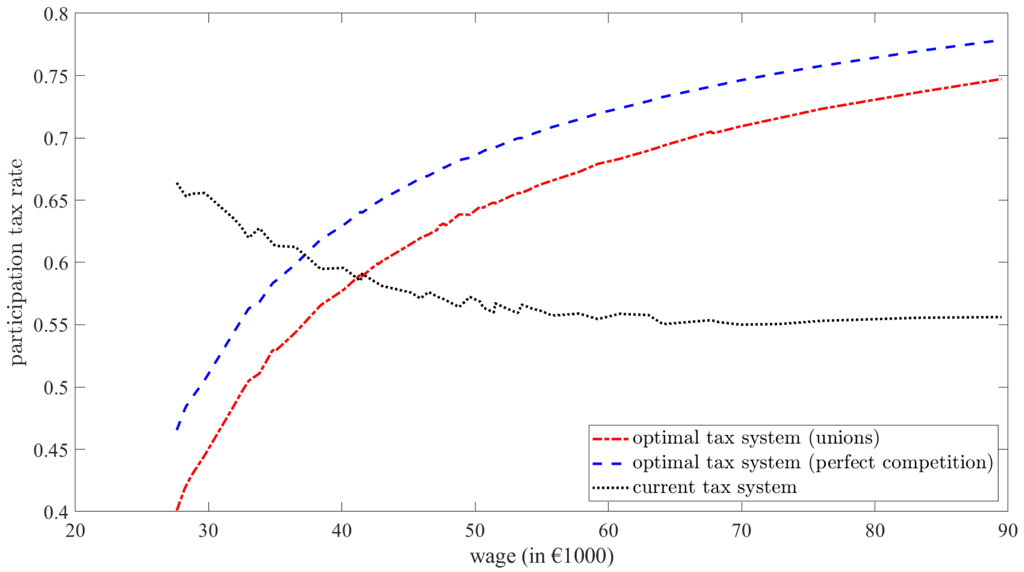

Figure 12: Optimal participation tax rates (weak unions)

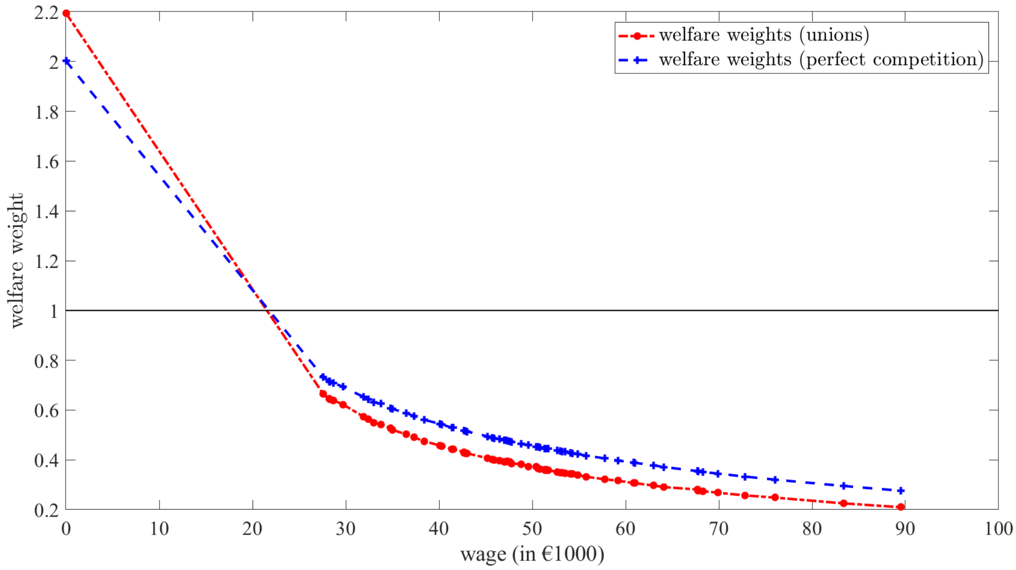

Figure 13: Social welfare weights (weak unions)

9.3 Participation elasticity

Next, we increase the average participation elasticity in the calibrated economy from its value of $\bar{\pi} = 0.25$ in the baseline to a value of $\bar{\pi} = 0.50$. Figures 14 and 15 plot the optimal participation tax rates and the social welfare weights at the optimal tax system with and without unions. In line with the theoretical findings from Diamond (1980), optimal participation tax rates are lower than before, as can be seen by comparing Figures 7 and 14. The difference is most pronounced for low- and middle-income groups. The reason is that the participation elasticity is declining in income, cf. equation (34). Targeting a higher average participation elasticity, in turn, leads to larger increases in the participation elasticity at lower levels of income. Consequently, compared to the baseline, optimal participation tax rates are lowered especially for these workers.

Interestingly, the impact of unions on the optimal tax system is less pronounced if the participation elasticity is increased. Optimal participation tax rates with unions are on average only 3.6 percentage points below the optimal participation tax rates with competitive labor markets (compared to a difference of 7.4 percentage points in the baseline). Intuitively, if the participation elasticity is large, an increase in the wage above the market-clearing level leads to a sharp increase in involuntary unemployment. Consequently, the degree of union power that is required to match the unemployment rate of 6.9% in the data is lower than in the baseline. The reduction in union power, in turn, lowers the union wedge. As a result, the impact of unions on the optimal tax-benefit system is smaller. Lastly, we again find that the social welfare weights for all employed workers are below the average of one, which according to Proposition 2 implies that an increase in union power lowers social welfare.

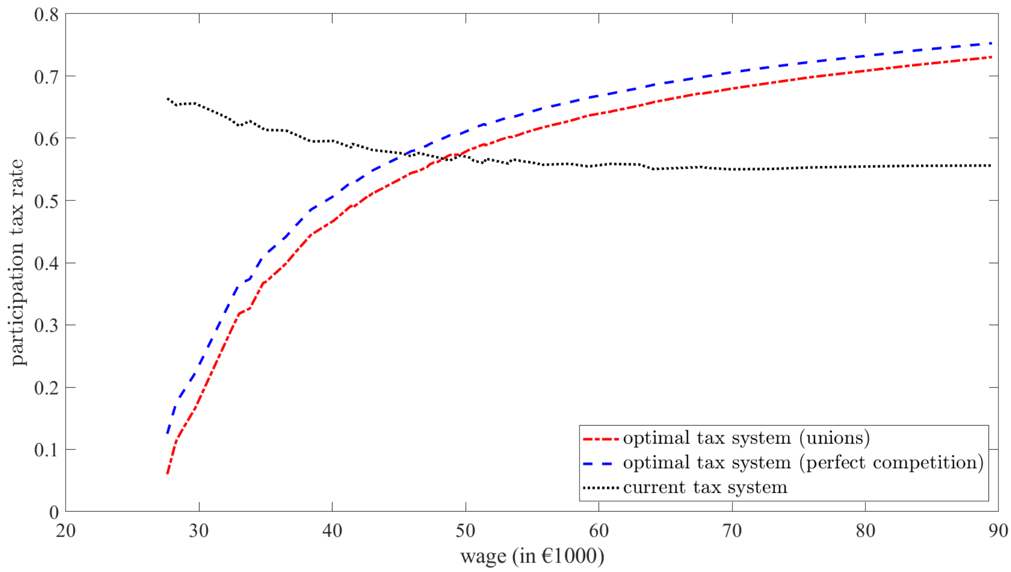

Figure 14: Optimal participation tax rates (high participation elasticity)

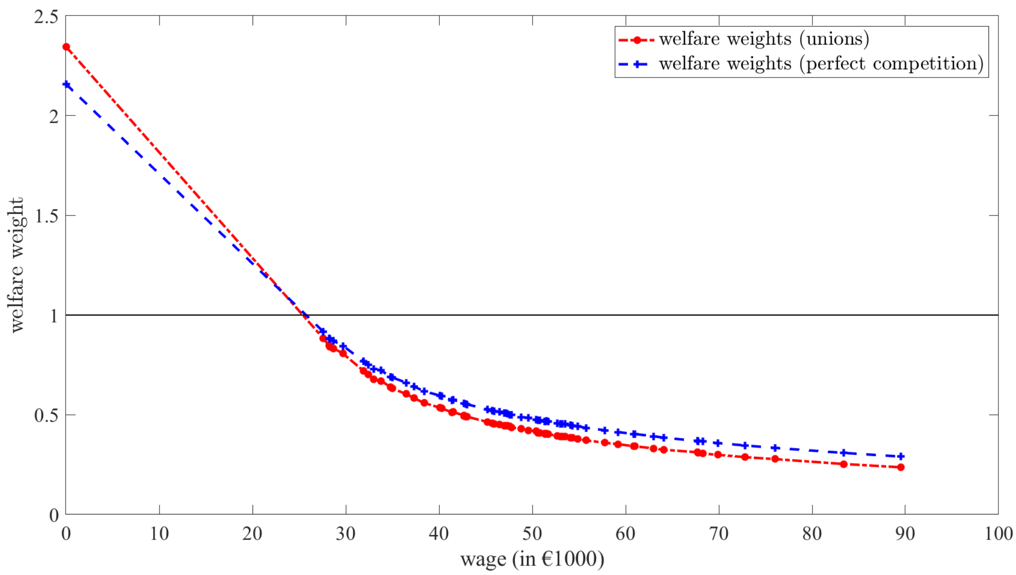

Figure 15: Social welfare weights (high participation elasticity)

9.4 Inequality aversion

We significantly decrease inequality aversion by reducing the coefficient of absolute risk aversion to $\theta = 0.016$. At this value of θ, the coefficient of *relative* risk-aversion is 0.50 for an individual with zero participation costs who earns the average wage. The optimal participation tax rate with unions equals approximately 27.8% on average, which is less than half the current rate of 58.3%. With a lower degree of inequality aversion, the government redistributes less income towards the unemployed and more towards low-income workers. In particular, the optimal participation tax rate at the bottom of the income distribution is now *negative*, as can be seen from Figure 16. Hence, low-income workers receive a subsidy of approximately €8,228, which exceeds the non-employment benefit of €5,323 at the optimal tax system with unions (which is much lower than the value of €12,560 in the baseline).

The finding that participation is optimally subsidized for low-skilled workers has an important implication: the social welfare weight of low-skilled workers exceeds the average of one, see Figure 17. Therefore, according to Proposition 2, an increase in union power for low-skilled workers *raises* social welfare – which does not occur in the baseline. This is true for individuals whose current earnings are below €28,300, where participation is subsidized at the optimal tax system.[25] Hence, an increase in union power alleviates these upward distortions in labor participation.

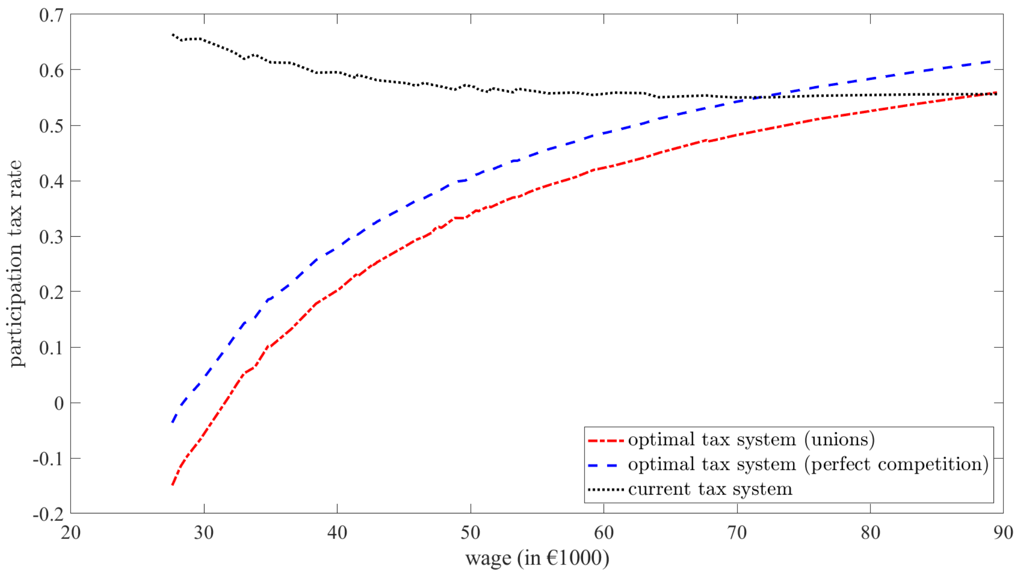

Figure 16: Optimal participation tax rates (low inequality aversion)

[25]At the *current* tax system, however, participation taxes for these workers are positive. This explains why in the analysis from Section 7 in the main text we do not find that an increase in union power for these workers raises social welfare.

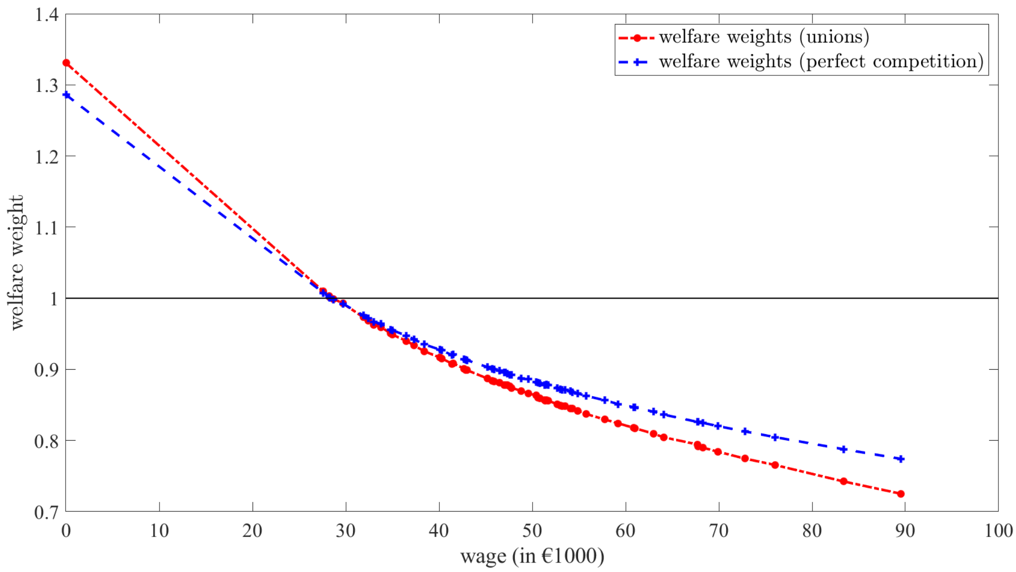

Figure 17: Social welfare weights (low inequality aversion)

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A Union responses to marginal tax rates

A.1 Comparative statics

This Appendix studies how a (local) increase in the unemployment benefit $-T_u$, the tax burden $T(w(i))$, the marginal tax rate $T'(w(i))$ at income level $w(i)$, and union power $\rho(i)$ affects the equilibrium wage $w(i)$ and employment rate $E(i)$ of workers in sector i. To do so, we, first, use $\hat{\varphi}(i) = G^{-1}(E(i))$ and the labor-demand equation (18) to substitute for $w(i)$ in the wage-demand equation (21). Second, we introduce tax reform parameters ν and ξ and define[26]

$$\begin{aligned}
&\Upsilon(E(i), T_u, \nu, \xi, \rho(i)) \equiv \rho(i)(1 - T'(a(i)y'(h(i)E(i))) - \xi) \\
&\times \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(a(i)y'(h(i)E(i)) - T(a(i)y'(h(i)E(i))) - \nu - \varphi)\mathrm{d}G(\varphi) \times a(i)h(i)y''(h(i)E(i)) \\
&+ \Big[u(a(i)y'(h(i)E(i)) - T(a(i)y'^{-1}(E(i))) - u(-T_u)\Big] = 0.
\end{aligned} \tag{50}$$

This equation pins down the equilibrium employment rate $E(i)$ for workers in sector i as a function of the unemployment benefit $-T_u$, union power $\rho(i)$, and the reform parameters ν and ξ. The parameter ν can be used to study how a local increase in the tax burden $T(w(i))$ affects the equilibrium employment rate. The parameter ξ can be used to study the effect of locally increasing the marginal tax rate $T'(w(i))$ at income level $w(i) = a(i)y'(h(i)E(i))$. The impact on $E(i)$ follows from applying the implicit function theorem, and evaluating the resulting expressions at $\nu = \xi = 0$. With a slight abuse of notation, we denote by $E_T(i) = \partial E(i)/\partial T(w(i))$ the impact of a higher tax burden on the employment rate. The latter is given by

$$\begin{aligned}
E_T(i) = \frac{\partial E(i)}{\partial T(w(i))} = -\frac{\Upsilon_\nu(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} = \frac{u'^{-1}(E(i)))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} \\
+ \left[\frac{\rho(i)(1 - T'(w(i)))a(i)h(i)y''(h(i)E(i)) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u''(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))}\right] < 0.
\end{aligned} \tag{51}$$

The sign follows from the concavity of $y(\cdot)$ and $u(\cdot)$, and concavity of the union objective implies that $\Upsilon_E < 0$.

The impact of a local increase in the marginal tax rate on the equilibrium employment rate

[26] See, for example, Jacquet et al. (2013) and Jacobs et al. (2017), among many others.

is:

$$E_{T'}(i) = \frac{\partial E(i)}{\partial T'(w(i))} = -\frac{\Upsilon_\xi(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} \tag{52}$$
$$= \left[\frac{\rho(i)a(i)h(i)y''(h(i)E(i)) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))}\right] \geq 0,$$

with a strict inequality if $\rho(i) > 0$. Again, the sign follows from the assumptions on the utility and production function and concavity of the union objective.

The effect of lowering the unemployment benefit (i.e., increasing T_u) is

$$E_{T_u}(i) = \frac{\partial E(i)}{\partial T_u} = -\frac{\Upsilon_{T_u}(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} = \frac{-u'(-T_u)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} > 0. \tag{53}$$

Lastly, the impact of a local increase in union power $\rho(i)$ is:

$$E_\rho(i) = \frac{\partial E(i)}{\partial \rho(i)} = -\frac{\Upsilon_\rho(E(i), T_u, 0, 0, \rho(i))}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} \tag{54}$$
$$= \frac{-(1 - T'(w(i)))a(i)h(i)y''(h(i)E(i)) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\Upsilon_E(E(i), T_u, 0, 0, \rho(i))} < 0.$$

From equations (52) and (54) follows that the impact of union power and the marginal tax rate are closely related: $E_{T'}(i) = -\frac{\rho(i)}{1-T'(w(i))}E_\rho(i)$. Intuitively, by making wage increases more attractive, a lower marginal tax rate has a similar impact as an increase in union power: both lead to an increase in the wage and a reduction in the employment rate.

To summarize, a (local) increase in the tax burden $T(w(i))$ (captured by $\mathrm{d}\nu > 0$), unemployment benefit $-T_u$, or union power $\rho(i)$ has a negative impact on the employment rate, whereas a local increase in the marginal tax rate $T'(w(i))$ (captured by $\mathrm{d}\xi > 0$) positively affects the employment rate $E(i)$. The impact on the equilibrium wage in sector i then follows directly from the labor-demand equation (18). Because the latter is downward-sloping, a higher tax burden, union power or unemployment benefit positively affects the wage $w(i)$ of workers in sector i, whereas a higher marginal tax rate leads to a lower equilibrium wage $w(i)$.

A.2 Optimal taxation

In the current framework with a continuum of types, the government's objective is given by

$$\mathcal{W} = \int_0^1 \psi(i)\left[\int_{\underline{\varphi}}^{G^{-1}(E(i))} u(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi) + \int_{G^{-1}(E(i))}^{\overline{\varphi}} u(-T_u)\mathrm{d}G(\varphi)\right]h(i)\mathrm{d}i$$
$$+ N^{-1}\psi_f u\left(N\left(\int_0^1 a(i)y(h(i)E(i))\mathrm{d}i - \int_0^1 w(i)E(i)h(i)\mathrm{d}i - T_f\right)\right). \tag{55}$$

Recall that the measure of workers is normalized to one and the measure of firm-owners is $1/N$. The government's budget constraint, in turn, is

$$\int_0^1 \Big[E(i)T(w(i)) + (1-E(i))T_u + T_f - R\Big]h(i)\mathrm{d}i = 0. \tag{56}$$

For each i, the equilibrium wage and employment rate are pinned down by

$$w(i) = a(i)y'(h(i)E(i)), \tag{57}$$

$$\rho(i)(1-T'(w(i))) \times \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i)-T(w(i))-\varphi)\mathrm{d}G(\varphi) \times a(i)h(i)y''(h(i)E(i))$$
$$+ \Big[u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)\Big] = 0. \tag{58}$$

Equation (57) is the labor-demand equation and equation (58) the modified wage-demand equation. The government's problem is to find the tax schedule $T(\cdot)$ that maximizes social welfare (55) subject to the budget constraint (56), taking into account the impact on equilibrium wages and employment rates in each sector i as determined by the labor-market equilibrium conditions (57)–(58).

The Lagrangian of the government's problem is given by

$$\mathcal{L} = \int_0^1 \psi(i)\Big[\int_{\underline{\varphi}}^{G^{-1}(E(i))} u(w(i)-T(w(i))-\varphi)\mathrm{d}G(\varphi) + \int_{G^{-1}(E(i))}^{\overline{\varphi}} u(-T_u)\mathrm{d}G(\varphi)\Big]h(i)\mathrm{d}i$$
$$+ N^{-1}\psi_f u\Big(N\Big(\int_0^1 a(i)y(h(i)E(i))\mathrm{d}i - \int_0^1 w(i)E(i)h(i)\mathrm{d}i - T_f\Big)\Big)$$
$$+ \lambda\int_0^1 \Big[E(i)T(w(i)) + (1-E(i))T_u + T_f - R\Big]h(i)\mathrm{d}i, \tag{59}$$

where λ is the multiplier on the government budget constraint.

To solve this problem, we proceed in a similar way as Jacquet and Lehmann (2021). Specifically, we start by replacing the tax schedule $T(w)$ by a perturbed tax schedule $T(w)+m\tilde{R}(w)$. Under the perturbed tax schedule, the equilibrium wage w and employment rate E in sector i are pinned down by

$$w = a(i)y'(h(i)E), \tag{60}$$

$$\rho(i)(1-T'(w)-m\tilde{R}'(w)) \times \int_{\underline{\varphi}}^{G^{-1}(E)} u'(w-T(w)-m\tilde{R}(w)-\varphi)\mathrm{d}G(\varphi) \times a(i)h(i)y''(h(i)E)$$
$$+ \Big[u(w-T(w(i))-m\tilde{R}(w)-G^{-1}(E)) - u(-T_u)\Big] = 0, \tag{61}$$

which are the counterparts of equations (57)–(58) under the perturbed tax schedule. Denote by $w^R(i,m)$ and $E^R(i,m)$ the equilibrium wage and employment rate in sector i under the perturbed tax schedule, so that $w^R(i,0) = w(i)$ and $E^R(i,0) = E(i)$.

Assuming that the tax function is twice differentiable, and equations (60)–(61) pin down a unique solution, we can apply the implicit function theorem to derive[27]

$$\frac{\partial E^R(i,0)}{\partial m} = E_T(i)\tilde{R}(w(i)) + E_{T'}(i)\tilde{R}'(w(i)), \tag{62}$$
$$\frac{\partial w^R(i,0)}{\partial m} = w_T(i)\tilde{R}(w(i)) + w_{T'}(i)\tilde{R}'(w(i)), \tag{63}$$

where $E_T(i) < 0$ and $E_{T'}(i) > 0$ are as defined in equations (51)–(52). Furthermore, from the labor-demand equation (18), $w_T(i)$ and $w_{T'}(i)$ satisfy

$$w_T(i) = E_T(i)a(i)y''(h(i)E(i))h(i) > 0, \quad w_{T'}(i) = E_{T'}(i)a(i)y''(h(i)E(i))h(i) < 0, \tag{64}$$

which capture the impact of a local increase in the tax burden and marginal tax rate on the equilibrium wage, respectively.[28]

The government's Lagrangian under the perturbed tax schedule is

$$\begin{aligned}\tilde{\mathcal{L}}(m) = &\int_0^1 \psi(i)\Big[\int_{\underline{\varphi}}^{G^{-1}(E^R(i,m))} u(w^R(i,m) - T(w^R(i,m)) - m\tilde{R}(w^R(i,m)) - \varphi)\mathrm{d}G(\varphi) \\ &+ \int_{G^{-1}(E^R(i,m))}^{\overline{\varphi}} u(-T_u)\mathrm{d}G(\varphi)\Big]h(i)\mathrm{d}i \\ &+ N^{-1}\psi_f u\Big(N\Big(\int_0^1 a(i)y(h(i)E^R(i,m))\mathrm{d}i - \int_0^1 w^R(i,m)E^R(i,m)h(i)\mathrm{d}i - T_f\Big)\Big) \\ &+ \lambda\int_0^1 \Big[E^R(i,m)(T(w^R(i,m)) + m\tilde{R}(w^R(i,m))) + (1-E^R(i,m))T_u + T_f - R\Big]h(i)\mathrm{d}i.\end{aligned} \tag{65}$$

This is the Lagrangian (59) if the tax schedule is $T(w) + m\tilde{R}(w)$ and equilibrium wages and employment rates are denoted by $w^R(i,m)$ and $E^R(i,m)$.

The welfare impact of perturbing the tax schedule $T(w)$ in the direction $\tilde{R}(w)$, evaluated at $m = 0$, is given by

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}(0)}{\partial m} = &\int_0^1 \Big[-\psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)\tilde{R}(w(i)) \\ &+ \psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)(1 - T'(w(i)))\frac{\partial w^R(i,0)}{\partial m} \\ &+ \psi(i)\Big[u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)\Big]\frac{\partial E^R(i,0)}{\partial m}\Big]h(i)di \\ &+ \psi_f u'\Big(N\Big(\int_0^1 a(i)y(h(i)E(i))\mathrm{d}i - \int_0^1 w(i)E(i)h(i)\mathrm{d}i - T_f\Big)\Big) \\ &\times\Big[\int_0^1 a(i)y'(h(i)E(i))\frac{\partial E^R(i,0)}{\partial m}h(i)\mathrm{d}i - \int_0^1 w(i)\frac{\partial E^R(i,0)}{\partial m}h(i)\mathrm{d}i - \int_0^1 \frac{\partial w^R(i,0)}{\partial m}E(i)h(i)\mathrm{d}i\Big] \\ &+ \lambda\Big[\int_0^1 \Big(E(i)\tilde{R}(w(i)) + E(i)T'(w(i))\frac{\partial w^R(i,0)}{\partial m} + (T(w(i)) - T_u)\frac{\partial E^R(i,0)}{\partial m}\Big)h(i)\mathrm{d}i\Big].\end{aligned} \tag{66}$$

[27]See Jacquet and Lehmann (2021) for further details.

[28]Note that these effects capture the behavioral responses along the actual (and not a linearized) tax schedule, so they account for the non-linearity of the tax schedule. See Jacquet and Lehmann (2021) for further details

The first three lines capture the welfare effect associated with changes in the utility of workers, the next two lines capture the welfare effect on firm-owners, and the final line captures the impact of the tax reform on the government budget.

The right-hand side of this equation can be simplified in a number of steps. First, note that profit maximization implies that the first two terms on the fifth line cancel (this is an application of the envelope theorem). Second, define the social welfare weight of firm-owners and of workers who are employed in sector i as

$$b_f = \frac{\psi_f u'(c_f)}{\lambda}, \qquad \hat{b}(i) = \frac{\psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)}{\lambda E(i)}, \tag{67}$$

where c_f denotes the consumption of firm-owners. Third, combine the terms with $\tilde{R}(w(i))$, $\frac{\partial w^R(i,0)}{\partial m}$ and $\frac{\partial E^R(i,0)}{\partial m}$. Rearranging gives

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \int_0^1 \Bigg[(1-\hat{b}(i))E(i)\tilde{R}(w(i)) + \big[\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\big]E(i)\frac{\partial w^R(i,0)}{\partial m}$$
$$+ \left(\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u)\right)\frac{\partial E^R(i,0)}{\partial m}\Bigg]h(i)\mathrm{d}i. \tag{68}$$

Using equations (62)–(63) and collecting terms, we get

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \int_0^1 \Bigg[\left(1-\hat{b}(i) + \Big(\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\Big)w_T(i)\right)E(i)\tilde{R}(w(i)) \tag{69}$$
$$+ \left(\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u)\right)E_T(i)\tilde{R}(w(i))$$
$$+ \Big(\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\Big)E(i)w_{T'}(i)\tilde{R}'(w(i))$$
$$+ \left(\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i)) - T_u)\right)E_{T'}(i)\tilde{R}'(w(i))\Bigg]h(i)\mathrm{d}i.$$

To proceed, let $K(w)$ denote the wage distribution with associated density $k(w)$, defined over the support $[\underline{w}, \overline{w}]$, where $\underline{w} = w(0)$ is the lowest wage and $\overline{w} = w(1)$ the highest wage. As some workers are not employed, the wage distribution has a mass point at zero: $\omega_u = \int_0^1 (1 - E(i))h(i)\mathrm{d}i$. Monotonicity of wages, in turn, implies

$$K(w(i)) = \omega_u + \int_0^i E(j)h(j)\mathrm{d}j, \quad \leftrightarrow \quad k(w(i))w'(i) = E(i)h(i). \tag{70}$$

The fraction of workers with a wage below $w(i)$ contains the employed workers whose type is below i and *all* unemployed workers (irrespective of their type). Next, we change the index of all variables from i to w. Moreover, we denote by $b(w)$ the social welfare weight of workers who are paid w, so that $b(w(i)) = \hat{b}(i)$. Further, we define by $\tilde{E}(w)$ the employment rate among workers whose wage if employed equals w, so that $\tilde{E}(w(i)) = E(i)$. Then, by substituting all these definitions into equation (69), we arrive at

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \int_{\underline{w}}^{\overline{w}} \Bigg[\bigg(1 - b(w) + \Big(b(w)(1 - T'(w)) - b_f + T'(w)\Big) w_T\bigg) \tilde{R}(w) \tag{71}$$

$$+ \left(\tau(w)w + t(w)w\right)\frac{E_T}{\tilde{E}(w)}\tilde{R}(w) + \Big(b(w)(1 - T'(w)) - b_f + T'(w)\Big) w_{T'}\tilde{R}'(w)$$

$$+ \left(\tau(w)w + t(w)w\right)\frac{E_{T'}}{\tilde{E}(w)}\tilde{R}'(w)\Bigg] k(w)\mathrm{d}w,$$

where, to avoid further notation, we ignored the function arguments on the partial effects $w_{T'}$, w_T, E_T and $E_{T'}$. Moreover, the union wedge and participation tax rate are defined as:

$$\tau(w(i)) \equiv \psi(i)\frac{u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)}{\lambda w(i)}, \quad t(w(i)) \equiv \frac{T(w(i)) - T_u}{w(i)}. \tag{72}$$

By integrating by parts the terms featuring $\tilde{R}(w)$, equation (71) can be rewritten as

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \tag{73}$$

$$\int_{\underline{w}}^{\overline{w}} \left[\Big(b(w)(1 - T'(w)) - b_f + T'(w)\Big) w_{T'} + \left(\tau(w)w + t(w)w\right)\frac{E_{T'}}{\tilde{E}(w)}\right] k(w)\tilde{R}'(w)\mathrm{d}w$$

$$+ \int_{\underline{w}}^{\overline{w}} \left(\int_{w}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1 - T'(z)) - b_f + T'(z)\Big) w_T + \left(\tau(z)z + t(z)z\right)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z\right) \tilde{R}'(w)\mathrm{d}w$$

$$- \int_{\overline{w}}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1 - T'(z)) - b_f + T'(z)\Big) w_T + \left(\tau(z)z + t(z)z\right)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z \times \tilde{R}(\overline{w})$$

$$+ \int_{\underline{w}}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1 - T'(z)) - b_f + T'(z)\Big) w_T + \left(\tau(z)z + t(z)z\right)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z \times \tilde{R}(\underline{w}).$$

Because the upper and lower bound coincide, the term on the fourth line is equal to zero. Combining the first and second lines gives

$$\frac{\partial \tilde{\mathcal{L}}(0)}{\partial m}\frac{1}{\lambda} = \tag{74}$$

$$\int_{\underline{w}}^{\overline{w}} \Bigg[\bigg(\Big(b(w)(1 - T'(w)) - b_f + T'(w)\Big) w_{T'} + \left(\tau(w)w + t(w)w\right)\frac{E_{T'}}{\tilde{E}(w)}\bigg) k(w)$$

$$+ \left(\int_{w}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1 - T'(z)) - b_f + T'(z)\Big) w_T + \left(\tau(z)z + t(z)z\right)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z\right)\Bigg] \tilde{R}'(w)\mathrm{d}w$$

$$+ \int_{\underline{w}}^{\overline{w}} \left[1 - b(z) + \Big(b(z)(1 - T'(z)) - b_f + T'(z)\Big) w_T + \left(\tau(z)z + t(z)z\right)\frac{E_T}{\tilde{E}(z)}\right] k(z)\mathrm{d}z \times \tilde{R}(\underline{w}).$$

Below we use this formula to derive a number of properties of the optimal tax schedule.

Before doing so, however, we first consider the welfare impact of raising the profit tax T_f. If the profit tax is optimized, increasing it should have no impact on social welfare. This requires

$$\frac{\partial \mathcal{L}}{\partial T_f} = -\psi_f u'(c_f) + \lambda = 0, \quad \leftrightarrow \quad b_f = \frac{\psi_f u'(c_f)}{\lambda} = 1, \tag{75}$$

which follows from differentiating the Lagrangian (59) with respect to T_f and setting the resulting expression equal to zero. Equation (75) coincides with the second result from Proposition 1.

Next, we consider a reduction in the unemployment benefit $-T_u$. Unlike the profit tax, equilibrium wages and employment rates are affected by a change in the unemployment benefit, see Appendix A.1. If the unemployment benefit is optimized, a small change leaves social welfare unaffected. Therefore,

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial T_u} &= \int_0^1 \left[-\psi(i)\int_{G^{-1}(E(i))}^{\overline{\varphi}} u'(-T_u)\mathrm{d}G(\varphi) + \lambda(1-E(i))\right] h(i)\mathrm{d}i, \qquad (76)\\ &+ \int_0^1 \left[\psi(i)\int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i)-T(w(i))-\varphi)\mathrm{d}G(\varphi)(1-T'(w(i)))\right.\\ &\left. - E(i)\psi_f u'(c_f) + \lambda E(i)T'(w(i))\right] w_{T_u}(i)h(i)\mathrm{d}i\\ &+ \int_0^1 \left[\psi(i)\left[u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)\right] + \lambda(T(w(i))-T_u)\right] E_{T_u}(i)h(i)\mathrm{d}i = 0.\end{aligned}$$

which is obtained from differentiating equation (59) with respect to T_u, taking into account the impact of a higher T_u on equilibrium wages and employment rates. Here, $E_{T_u}(i)$ is as defined in equation (53) and $w_{T_u}(i) = E_{T_u}(i) \times a(i)y''(h(i)E(i))h(i) < 0$.

To proceed, divide equation (76) by λ and impose the definition for $\hat{b}(i)$, b_f, and the define the social welfare weight of the unemployed as

$$b_u = \frac{\int_0^1 \psi(i)(1-E(i))u'(-T_u)h(i)\mathrm{d}i}{\lambda\int_0^1 (1-E(i))h(i)\mathrm{d}i}. \qquad (77)$$

Substitution of equation (77) into equation (76) gives:

$$\begin{aligned}&\int_0^1 (1-b_u)(1-E(i))h(i)\mathrm{d}i + \int_0^1 \left[\hat{b}(i)(1-T'(w(i))) - b_f + T'(w(i))\right] w_{T_u}(i)E(i)h(i)\mathrm{d}i \qquad (78)\\ &+ \int_0^1 \left[\psi(i)\frac{u(w(i)-T(w(i))-G^{-1}(E(i))) - u(-T_u)}{\lambda} + (T(w(i))-T_u)\right] E_{T_u}(i)h(i)\mathrm{d}i = 0.\end{aligned}$$

Next, note that the first term equals $(1-b_u)\omega_u$. For the second and third term, apply the same change in indexation of variables from i to w as before, and substitute the definitions of the social welfare weight $b(w)$, the union wedge $\tau(w)$, the employment rate $\tilde{E}(w)$ at wage w, the participation tax rate $t(w)$, and the optimal profit tax $b_f = 1$ to find:

$$\begin{aligned}&\omega_u(1-b_u) + \int_{\underline{w}}^{\overline{w}} (b(w)-1)(1-T'(w))w_{T_u}k(w)\mathrm{d}w \qquad (79)\\ &+ \int_{\underline{w}}^{\overline{w}} \left[\tau(w)w + t(w)w\right]\frac{E_{T_u}}{\tilde{E}(w)}k(w)\mathrm{d}w = 0,\end{aligned}$$

where again we dropped the function arguments of the behavioral responses w_{T_u} and E_{T_u} to avoid additional notation.

To derive the first result from Proposition 1, consider equation (74). If the tax function $T(\cdot)$ is optimized, it must be that the welfare impact of perturbing the tax function in *any* direction

$\tilde{R}(w)$ equals zero. Therefore, the term on the last line must be equal to zero. Imposing $b_f = 1$ then yields

$$\int_{\underline{w}}^{\overline{w}} \left[1 - b(w) + (b(w) - 1)(1 - T'(w))w_T + \big(\tau(w)w + t(w)w\big)\frac{E_T}{\tilde{E}(w)}\right]k(w)\mathrm{d}w = 0. \tag{80}$$

Suppose that there are no income effects at the union level, cf. Assumption 3. In that case, for each worker type, $w_T = -w_{T_u}$ and $E_T = -E_{T_u}$. Combining equations (79) and (80) then gives:

$$\omega_u(1 - b_u) = \int_{\underline{w}}^{\overline{w}} (b(w) - 1)k(w)\mathrm{d}w. \tag{81}$$

Rearranging leads to the first result from Proposition 1.

To arrive at the final result from Proposition 1, consider again equation (74) and substitute $b_f = 1$. As mentioned before, if the tax function $T(\cdot)$ is optimized, the welfare impact of perturbing it in *any* direction $\tilde{R}(w)$ must be equal to zero. By the fundamental lemma of the calculus of variations, it follows that the term below the integral sign that is multiplied by $\tilde{R}'(w)$ equals zero:

$$\begin{aligned}&\left[(b(w) - 1)(1 - T'(w))w_{T'} + (t(w) + \tau(w))\frac{wE_{T'}}{\tilde{E}(w)}\right]k(w)\\&+ \int_{w}^{\overline{w}} \left[(1 - b(z)) + (b(z) - 1)(1 - T'(z))w_T + (t(z) + \tau(z))\frac{zE_T}{\tilde{E}(z)}\right]k(z)\mathrm{d}z = 0,\end{aligned} \tag{82}$$

which must hold for each $w \in [\underline{w}, \overline{w}]$. Evaluate this result at $w = w'$ and changing the index of integration from z to w, equation (82) coincides with equation (24) from Proposition 1 after making the substitutions for the wage and employment elasticities with respect to an increase in the marginal tax rate and tax burden.

To obtain an intuitive derivation of this result in the spirit of Saez (2001), consider Figure 18. The black, dotted (red, solid) line shows the tax schedule $T(w)$ before (after) the tax reform. The reform increases the marginal tax rate in the small interval $[w', w' + \Delta]$ by an amount equal do $\mathrm{d}T'$. As a result, the tax burden for individuals with earnings above $w' + \Delta$ increases by an amount $\Delta\mathrm{d}T'$. Such a reform has three welfare-relevant effects. First, the tax burden *mechanically* increases for employed individuals with earnings above $w'+\Delta$. As a result, income is transferred from these workers to the government. This mechanical effect is captured by the first term below the integral sign on the second line of equation (82). Second, for individuals with earnings above $w' + \Delta$, a higher tax burden raises the equilibrium wage and lowers the equilibrium employment rate, cf. equation (51). The welfare effects of a higher tax liability on wages and employment are captured by the second and third term (proportional to w_T and E_T, respectively) below the integral sign on the second line of equation (82). The wage effect reflects the distributional impact of a higher wage if the tax burden increases; a higher wage redistributes income from firms to workers and the government. The term associated with the employment effect reflects the impact of higher tax burdens on participation distortions. Third, the increase in the marginal tax rate by an amount $\mathrm{d}T'$ in the interval $[w', w' + \Delta]$ generates lower equilibrium wages and higher equilibrium employment rates for individuals within this

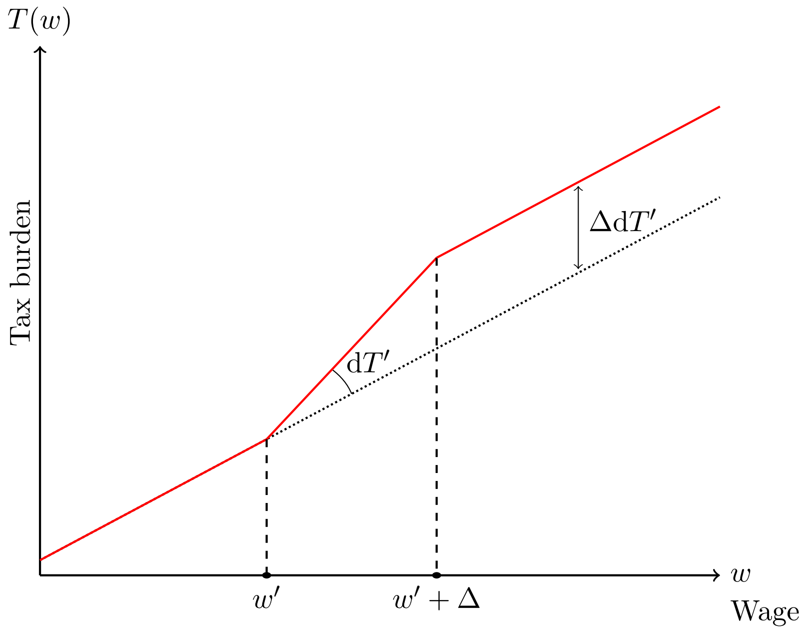

Figure 18: Tax perturbation approach

interval, which are captured by the terms on the first line of equation (82). In particular, a reduction in the negotiated wage redistributes income from workers and the government to firm-owners for individuals within this interval. Moreover, a higher marginal tax rate leads to higher equilibrium employment rates for individuals within this interval, in line with the wage-moderating effect of a higher marginal tax rate, cf. equation (52). Both effects are proportional to the density $k(w)$ of the income distribution, which determines for how many individuals the marginal tax rate is increased. Equating to zero the sum of the welfare-relevant effects leads to equation (82).

A.3 Desirability of unions

To study whether unions are desirable, suppose there is an increase in union power $\rho(i)$ for workers who are employed in sector i at wage $w(i)$. The increase in union power generates a (local) increase in the wage $w(i)$ and a (local) reduction in the employment rate $E(i)$, cf. equation (54). To determine the welfare impact of a local increase in union power, differentiate the Lagrangian (59) with respect to $\rho(i)$. With a slight abuse of notation, this gives

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \rho(i)} &= h(i)\Bigg(w_\rho \Bigg[\psi(i) \int_{\underline{\varphi}}^{G^{-1}(E(i))} u'(w(i) - T(w(i)) - \varphi)\mathrm{d}G(\varphi)(1 - T'(w(i))) - \psi_f u'(c_f)E(i) \\ &+ \lambda E(i)T'(w(i)) \Bigg] + E_\rho \Bigg[\psi(i)(u(w(i) - T(w(i)) - G^{-1}(E(i))) - u(-T_u)) + \lambda(T(w(i)) - T_u) \Bigg] \Bigg) \\ &= \lambda E(i)h(i) \Bigg[w_\rho(b(w(i)) - 1)(1 - T'(w(i))) + \frac{E_\rho}{\tilde{E}(w(i))} \Big(\tau(w(i))w(i) + t(w(i))w(i) \Big) \Bigg], \end{aligned} \tag{83}$$

where we used the property $b_f = 1$ and substituted the definitions for the social welfare weight $b(w)$, the union wedge $\tau(w)$, and the participation tax rate $t(w)$. Furthermore, $w_\rho > 0$ and

$E_\rho < 0$ capture the impact of a local increase in union power on the equilibrium wage and employment rate.

A local increase in union power at wage w positively affects social welfare if and only if equation (83) is positive. Because $\lambda E(i)h(i) > 0$, the latter requires

$$w_\rho(b(w)-1)(1-T'(w)) + \frac{E_\rho}{\tilde{E}(w)}\Big(\tau(w)w + t(w)w\Big) > 0. \tag{84}$$

To proceed, note that we can write $E_\rho = E_w w_\rho$, where E_w is the slope of the labor-demand equation (18). The latter can be obtained from implicitly differentiating $w = ay'(hE)$. Next, define the labor-demand elasticity as $\tilde{\varepsilon}(w) = -E_w w/\tilde{E}(w) > 0$ so that $\tilde{\varepsilon}(w(i)) = \varepsilon(i)$. Equation (84) then becomes:

$$w_\rho(b(w)-1)(1-T'(w)) - w_\rho\tilde{\varepsilon}(w)(t(w)+\tau(w)) > 0. \tag{85}$$

Dividing by $w_\rho > 0$ leads to equation (25) of Proposition 2.

B Inefficient rationing

B.1 Optimal taxation

To prove Proposition 3, we start by characterizing some properties of the general rationing schedule, which satisfies, for all values of E_i and φ_i^*

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) = E_i. \tag{86}$$

Differentiate equation (86) with respect to E_i and φ_i^* to obtain:

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) = 1, \tag{87}$$

$$\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)\mathrm{d}G_i(\varphi) + e_i(E_i, \varphi_i^*, \varphi_i^*)G_i'(\varphi_i^*) = 0. \tag{88}$$

As stated in the main text, rather than deriving labor-market equilibrium explicitly for a general rationing scheme, we instead assume that income effects at the union level are absent and labor markets are independent. In this case, the equilibrium wage and employment rate in sector i depend only on union power ρ_i and the participation tax: $E_i = E_i(\rho_i, T_i - T_u)$ and $w_i = w_i(\rho_i, T_i - T_u)$. To derive the social welfare function, first use equation (86) to write

$$(1-E_i)u(-T_u) = u(-T_u) - \int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)u(-T_u)dG_i(\varphi). \tag{89}$$

Consequently, the Lagrangian for maximizing social welfare is:

$$\mathcal{L} = \sum_i \psi_i N_i \left(u(-T_u) + \int_{\underline{\varphi}}^{\varphi_i^*} e_i(E_i, \varphi_i^*, \varphi)(u(w_i - (T_i - T_u) - T_u - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi) \right)$$
$$+ \psi_f u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i(T_u + E_i(T_i - T_u)) + T_f - R \right). \quad (90)$$

The first-order conditions for T_u, T_f, and $T_i - T_u$ are given by:

$$T_u: \quad -\sum_i \psi_i N_i (E_i \overline{u_i'} + (1 - E_i) u_u') + \lambda \sum_i N_i = 0, \quad (91)$$

$$T_f: \quad -\psi_f u_f' + \lambda = 0, \quad (92)$$

$$T_i - T_u: \quad -N_i E_i (\psi_i \overline{u_i'} - \lambda) + N_i E_i \left[\psi_i \overline{u_i'} - \psi_f u_f' \right] \frac{\partial w_i}{\partial (T_i - T_u)}$$
$$+ N_i E_i \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(u_i(\varphi) - u_u)\mathrm{d}G_i(\varphi) + \lambda(T_i - T_u) \right] \frac{\partial E_i}{\partial (T_i - T_u)}$$
$$+ N_i \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(u_i(\varphi) - u_u)\mathrm{d}G_i(\varphi) \right] \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} = 0. \quad (93)$$

Here, we used the assumption that labor markets are independent. The expected utility of the employed workers in sector i is given by:

$$\overline{u_i'} \equiv \int_{\underline{\varphi}}^{\varphi_i^*} \frac{e_i(E_i, \varphi_i^*, \varphi)}{E_i} u'(w_i - T_i - \varphi)\mathrm{d}G_i(\varphi), \quad (94)$$

and $u_i(\varphi) \equiv u(w_i - T_i - \varphi)$ is the utility of the worker with participation costs $\varphi \in [\underline{\varphi}, \varphi_i^*]$ who is employed in sector i.

Equations (91) and (92) lead to the first two results in Proposition 3. Next, divide equation (93) by $N_i E_i \lambda$. Define the expected utility loss of labor rationing in sector i for those workers who lose their job if the employment rate E_i is marginally reduced as:

$$\hat{\tau}_i \equiv \psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{iE_i}(E_i, \varphi_i^*, \varphi) \left(\frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda w_i} \right) \mathrm{d}G_i(\varphi). \quad (95)$$

Substitute equation (95) into equation (93) and use the definition of the elasticities η_{ii} and κ_{ii} to find

$$\left(\frac{t_i + \hat{\tau}_i}{1 - t_i} \right) \eta_{ii} = (1 - b_i) + (b_i - b_f)\kappa_{ii}$$
$$+ \frac{\partial \varphi_i^*}{\partial (T_i - T_u)} \frac{1}{E_i} \left[\psi_i \int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi) \frac{(u_i(\varphi) - u_u)}{\lambda} \mathrm{d}G_i(\varphi) \right]. \quad (96)$$

Next, use equation (88) to rewrite the last part of equation (96) as:

$$\begin{aligned}&\frac{\partial\varphi_i^*}{\partial(T_i-T_u)}\frac{1}{E_i}\left[\psi_i\int_{\underline{\varphi}}^{\varphi_i^*}e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)\frac{(u_i(\varphi)-u_u)}{\lambda}\mathrm{d}G_i(\varphi)\right]\\ =&-\frac{\partial\varphi_i^*}{\partial(T_i-T_u)}\frac{G_i'(\varphi_i^*)}{G_i(\varphi_i^*)}\frac{\varphi_i^*}{1-t_i}\frac{e_i(E_i,\varphi_i^*,\varphi_i^*)}{E_i/G_i(\varphi_i^*)}\\ &\times\left[\psi_i\int_{\underline{\varphi}}^{\varphi_i^*}\frac{e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)}{\int_{\underline{\varphi}}^{\varphi_i^*}e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)\mathrm{d}G_i(\varphi)}\left(\frac{u_i(\varphi)-u_u}{\lambda w_i}\right)\mathrm{d}G_i(\varphi)\right].\end{aligned}\tag{97}$$

As a final step, define the rationing wedge as

$$\varrho_i\equiv\frac{\psi_i e_i(E_i,\varphi_i^*,\varphi_i^*)}{E_i/G_i(\varphi_i^*)}\int_{\underline{\varphi}}^{\varphi_i^*}\frac{e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)}{\int_{\underline{\varphi}}^{\varphi_i^*}e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)\mathrm{d}G_i(\varphi)}\left(\frac{u(w_i-T_i-\varphi)-u(-T_u)}{\lambda w_i}\right)\mathrm{d}G_i(\varphi)\tag{98}$$

and the participation response by

$$\gamma_i\equiv-\frac{\partial G_i(\varphi_i^*)}{\partial(T_i-T_u)}\frac{\varphi_i^*}{G_i(\varphi_i^*)},\tag{99}$$

where the threshold depends on the participation tax through $\varphi_i^*=w_i(\rho_i,T_i-T_u)-(T_i-T_u)$. After substituting these definitions in equation (96), we arrive at:

$$\left(\frac{t_i+\hat{\tau}_i}{1-t_i}\right)\eta_{ii}-\left(\frac{\varrho_i}{1-t_i}\right)\gamma_i=(1-b_i)+(b_i-b_f)\kappa_{ii}.\tag{100}$$

B.2 Desirability of unions

To study the welfare effects of the reform described in Section 4, one can differentiate the Lagrangian in equation (90) with respect to T_i and T_f under the assumptions that the reform is budget neutral, and leaves wages and employment in sector i (i.e., w_i and E_i) unaffected. The welfare effect is then:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{W}}{\lambda}&=N_iE_i(1-b_i)\mathrm{d}T_i+(1-b_f)\mathrm{d}T_f\\ &+N_iE_i\left[\psi_i\frac{1}{E_i}\int_{\underline{\varphi}}^{\varphi_i^*}e_{i\varphi_i^*}(E_i,\varphi_i^*,\varphi)\left(\frac{u_i(\varphi)-u_u}{\lambda}\right)\mathrm{d}G_i(\varphi)\right]\frac{\partial\varphi_i^*}{\partial(T_i-T_u)}\mathrm{d}T_i.\end{aligned}\tag{101}$$

The first term reflects the (direct) change in workers' utility in sector i following the change in the participation tax, whereas the second term reflects the change in firm-owners' utility induced by a change in the profit tax. The third term reflects the utility loss due to a change in labor participation: if T_i is lowered, more workers want to participate. If some of these workers find a job and employment remains constant, then it must be that some other workers lose their jobs and thus experience a utility loss, since rationing is not fully efficient.

Under the balanced-budget assumption, we have $N_iE_i\mathrm{d}T_i+\mathrm{d}T_f=0$. In addition, if the government can levy a non-distortionary profit tax, then $b_f=1$. Substituting these results in equation (101), the change in social welfare can be written as:

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = N_i E_i \left(1 - b_i + \left[\psi_i \frac{1}{E_i}\int_{\underline{\varphi}}^{\varphi_i^*} e_{i\varphi_i^*}(E_i, \varphi_i^*, \varphi)\left(\frac{u_i(\varphi) - u_u}{\lambda}\right)\mathrm{d}G_i(\varphi)\right]\frac{\partial \varphi_i^*}{\partial(T_i - T_u)}\right)\mathrm{d}T_i. \quad (102)$$

Given that T_i is lowered in the policy experiment (i.e., $\mathrm{d}T_i < 0$), the welfare effect is positive provided that the term in between brackets is negative. Using the definitions for ϱ_i and γ_i from equations (98) and (99), this is the case if:

$$b_i > 1 + \left(\frac{\varrho_i}{1 - t_i}\right)\gamma_i. \quad (103)$$

The proof is completed by the observation that if the tax system is optimized, the welfare impact of the joint reform (increasing union power ρ_i, lowering T_i and raising T_f) is driven only by the increase in union power, as changes in the tax system have no impact on social welfare.

C Occupational choice

C.1 Optimal taxation

The total labor force consists of N workers who draw a vector $\boldsymbol{\varphi} \equiv (\varphi_0, \varphi_1, \cdots, \varphi_I) \in \Phi$ of participation costs according to some distribution function $G(\boldsymbol{\varphi})$. Based on this draw, each individual chooses the occupation $j \in \{0, 1, .., I\}$ according to equation (34), where occupation 0 refers to non-employment with $w_0 = \varphi_0 = 0$ and $T_0 = T_u$. Aggregate employment in sector i is denoted by E_i and total (voluntary and involuntary) unemployment is given by E_0, so that $\sum_{i=0}^{I} E_i = N$. This notation differs from what is used in the rest of the paper, where E_i is the employment *rate*. Another difference is that, unless stated otherwise, summation over i in this Appendix means summing over $i \in \{0, 1, \cdots, I\}$ instead of summing over $i \in \{1, \cdots, I\}$.

The Lagrangian for the maximization of social welfare is:

$$\mathcal{L} = N\sum_i \psi_i \int_{\Phi_i}\Big[u(-T_u) + p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)(u(w_i - (T_i - T_u) - T_u) - u(-T_u))\mathrm{d}G(\varphi)\Big]$$
$$+ \psi_f u(F(\cdot) - \sum_i w_i E_i - T_f) + \lambda\left[\sum_i E_i T_i + T_f - R\right]. \quad (104)$$

As in the previous cases, the first-order conditions with respect to T_u and T_f imply that the average social welfare weight of all workers and firm-owners equals one. The first-order condition with respect to the participation tax $T_i - T_u$ in sector i is:

$$E_i(\lambda - \psi_i \overline{u_i'}) + \lambda \sum_{j=1}^{I} E_j(\psi_j \overline{u_j'} - \psi_f u_f')\frac{\partial w_j}{\partial(T_i - T_u)} + \lambda\sum_{j=0}^{I} T_j \frac{\partial E_j}{\partial(T_i - T_u)} \quad (105)$$
$$+ \lambda N \sum_{j=1}^{I}\psi_j \int_{\Phi_j}\frac{\partial p_j(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)}{\partial(T_i - T_u)}(u(w_j - T_j - \varphi_j) - u(-T_u))\mathrm{d}G(\varphi) = 0.$$

Here, we used the property that $w_0 = 0$ and $p_0 = 1$, so they are not affected by taxation. The average marginal utility of employed workers in sector i is:

$$\overline{u'_i} = \frac{N}{E_i} \int_{\Phi_i} p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u) u'(w_j - T_j - \varphi_j) \mathrm{d}G(\varphi). \tag{106}$$

The first-order condition (105) can be simplified in a number of steps. First, because $\sum_{j=0}^{I} E_j(T_1 - T_u, .., T_I - T_u) = 1$ for all tax instruments, we can differentiate both sides with respect to $T_i - T_u$:

$$\sum_{j=0}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} = 0 \quad \Leftrightarrow \quad \sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} = -\frac{\partial E_0}{\partial(T_i - T_u)}. \tag{107}$$

Therefore, the third term on the first line of equation (105) can be simplified to:

$$\sum_{j=0}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} T_j = \sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} T_j + \frac{\partial E_0}{\partial(T_i - T_u)} T_u = \sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} (T_j - T_u). \tag{108}$$

Second, for all tax instruments, aggregate employment and the employment probabilities are related through

$$N \int_{\Phi_j} p_j(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u) \mathrm{d}G(\varphi) \equiv E_j(T_1 - T_u, \cdots, T_I - T_u). \tag{109}$$

Differentiating both sides with respect to $T_i - T_u$ and imposing that employment probabilities are zero on the boundary of Φ_j allows us to rewrite the second line of equation (105):

$$N \int_{\Phi_j} \frac{\partial p_j(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)}{\partial(T_i - T_u)} \mathrm{d}G(\varphi) = \frac{\partial E_j(T_1 - T_u, \cdots, T_I - T_u)}{\partial(T_i - T_u)}. \tag{110}$$

Next, multiply and divide the final term in equation (105) by $\partial E_j / \partial(T_i - T_u)$ for each j and divide the entire expression by λ to find:

$$E_i(1 - b_i) + \sum_j E_j(b_j - b_f) \frac{\partial w_j}{\partial(T_i - T_u)} \tag{111}$$

$$\sum_{j=1}^{I} \frac{\partial E_j}{\partial(T_i - T_u)} \left[(T_j - T_u) + \psi_j N \int_{\Phi_j} \frac{\partial p_j / \partial(T_i - T_u)}{\partial E_j / \partial(T_i - T_u)} \left(\frac{u(w_j - T_j - \varphi_j) - u(-T_u)}{\lambda} \right) \mathrm{d}G(\varphi) \right] = 0.$$

The union wedge with an occupational choice is defined as follows:

$$\tau_j^o = \psi_j N \int_{\Phi_j} \frac{\partial p_j / \partial(T_i - T_u)}{\partial E_j / \partial(T_i - T_u)} \left(\frac{u(w_j - T_j - \varphi_j) - u(-T_u)}{\lambda w_j} \right) \mathrm{d}G(\varphi). \tag{112}$$

Using this notation, and the definitions of the labor shares (ω_i and ω_i) and wage and employment elasticities (κ_{ji} and η_{ji}), we obtain the final result from Proposition 5.

C.2 Desirability of unions

To study the desirability of labor unions, start from the Lagrangian

$$\mathcal{L} = N\sum_i \psi_i \int_{\Phi_i} \left[u(-T_u) + p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u)(u(w_i - (T_i - T_u) - T_u) - u(-T_u))\mathrm{d}G(\varphi)\right]$$
$$+ \psi_f u(F(\cdot) - \sum_i w_i E_i - T_f) + \lambda\left[\sum_i E_i T_i + T_f - R\right]. \tag{113}$$

Equilibrium wages and employment rates depend on the participation taxes $T_i - T_u$ and union power ρ_i in all sectors. As before, we analyze a reform where union power in sector i is increased: $\mathrm{d}\rho_i > 0$. This reform puts upward pressure on the wage w_i sector i and downward pressure on employment E_i. To off-set the impact on the equilibrium wage, the reform is combined by reduction in the income tax in sector i: $\mathrm{d}T_i < 0$. This reduction, in turn, is financed by an increase in the profit tax: $\mathrm{d}T_f > 0$. The combined welfare effect is

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = E_i(1 - b_i)\mathrm{d}T_i + (1 - b_f)\mathrm{d}T_f, \tag{114}$$

which is very similar to the equation (101) except there is no welfare loss due to an inefficient allocation of jobs over workers (i.e., there is no rationing wedge). Because the reform is budget-neutral, we have $E_i \mathrm{d}T_i = -\mathrm{d}T_f$. Moreover, the social welfare weight of firm-owners equals one if the tax system is optimized: $b_f = 1$. The increase in union power ρ_i combined with a reduction in the income tax T_i financed by a higher profit tax T_f increases the net incomes of workers in sector i. The prospects of a higher net wage could induce some individuals to switch from other sectors j (possibly non-employment) to sector i. However, this is only the case for workers who are *ex ante* indifferent between choosing occupation i and their second-best alternative. Under our assumption of efficient rationing, the employment probability of these individuals is zero: $p_i(\boldsymbol{\varphi}, T_1 - T_u, \cdots, T_I - T_u) = 0$ on the boundary of Φ_i. Hence, there is no welfare effect associated with such changes. According to equation (114), a higher union power then raises social welfare if and only if $b_i > 1$.

D Bargaining over multiple wages

D.1 Labor-market equilibrium

We assume that there is one union with a utilitarian objective and denote union power by $\delta \in [0, 1]$. The union bargains with the firm-owners over the wages all sectors i. Hence, the union affects the entire wage distribution. Under Nash-bargaining, the solution for wages and employment in all sectors i follow from solving the following maximization problem:

$$\begin{aligned}\max_{\{w_i,E_i\}_{i\in\mathcal{I}}} \Omega \;=\; & \delta\log\left(\sum_i N_i\int_{\underline{\varphi}}^{G_i^{-1}(E_i)}(u(w_i-T_i-\varphi)-u(-T_u))\mathrm{d}G_i(\varphi)\right)\\ & +(1-\delta)\log\left(u(F(K,E_1N_1,\cdots,E_IN_I)-\sum_i w_iN_iE_i-T_f)-u(F(K,0,\cdots,0)-T_f)\right)\\ \text{s.t. } & w_i-F_i(K,E_1N_1,\cdots,E_IN_I)=0,\quad \forall i,\\ & G_i(w_i-T_i+T_u)-E_i\geq 0,\quad \forall i.\end{aligned} \tag{115}$$

The payoffs of both parties are taken in deviation from the payoff associated with the disagreement outcome. The Lagrangian is:

$$\begin{aligned}\mathcal{L}=&\delta\log\left(\sum_i N_i\int_{\underline{\varphi}}^{G_i^{-1}(E_i)}(u(w_i-T_i-\varphi)-u(-T_u))\mathrm{d}G_i(\varphi)\right)\\ &+(1-\delta)\log\left(u(F(K,E_1N_1,\cdots,E_IN_I)-\sum_i w_iN_iE_i-T_f)-u(F(K,0,\cdots,0)-T_f)\right)\\ &+\sum_i\vartheta_i(w_i-F_i(K,E_1N_1,\cdots,E_IN_I))+\sum_i\mu_i(G_i(w_i-T_i+T_u)-E_i).\end{aligned} \tag{116}$$

The first-order conditions are:

$$w_i:\frac{\delta}{\sum_j N_jE_j(\overline{u_j}-u_u)}N_iE_i\overline{u_i'}-\frac{1-\delta}{u_f-\underline{u}_f}N_iE_i\overline{u_f'}+\vartheta_i+\mu_iG_i'=0, \tag{117}$$

$$E_i:\frac{\delta}{\sum_j N_jE_j(\overline{u_j}-u_u)}N_i(\hat{u}_i-u_u)-N_i\sum_j\vartheta_jF_{ji}-\mu_i=0, \tag{118}$$

$$\vartheta_i: w_i-F_i=0, \tag{119}$$

$$\mu_i: G_i-E_i=0. \tag{120}$$

where $\underline{u}_f\equiv u(F(K,0,\cdots,0)-T_f)$. These conditions characterize labor-market equilibrium, which has the following properties.

First, if the union has zero bargaining power ($\delta=0$), the equilibrium coincides with the competitive outcome (i.e., $G_i=E_i$ and $w_i=F_i$ for all i). To see why, substitute $\delta=0$ in the first-order conditions for w_i and E_i in equations (117) and (118). Next, use (117) to substitute for ϑ_i in equation (118) and rearrange:

$$\mu_i\underbrace{(N_iG_i'F_{ii}-1)}_{<0}+\;N_i\sum_{j\neq i}\mu_j\underbrace{G_j'F_{ji}}_{\geq 0}=N_i\underbrace{\frac{u_f'}{u_f-\underline{u}_f}}_{>0}\underbrace{\sum_j N_jE_jF_{ji}}_{=-F_{Ki}K<0}. \tag{121}$$

The inequalities follow from the assumptions of co-operant factors of production and constant returns to scale. Non-increasing marginal productivity and co-operant factors of production imply $F_{ii}\leq 0\leq F_{ji}$, whereas constant returns to scale implies $\sum_j N_jE_jF_{ji}=-F_{Ki}K\leq 0$.[29]

[29] This follows from differentiating $F(\cdot)=F_K(\cdot)K+\sum_j N_jE_jF_j(\cdot)$ with respect to E_ℓ.

Suppose that there is a sector in which $G_i > E_i$, i.e., the wage is above the market-clearing level. Then, from the Kuhn-Tucker conditions, it must be that $\mu_i = 0$. Because of the non-negativity of all multipliers, however, equation (121) cannot be satisfied unless all labor types would be perfect substitutes, i.e., $F_{ii} = F_{ij} = F_{Ki} = 0$ for all i, j. This is a contradiction. Therefore, $G_i = E_i$ for all i if $\delta = 0$.

Second, if the union has sufficiently high bargaining power δ, there is at least one sector i for which the wage exceeds the market-clearing level, i.e., there exists a sector i such that $G_i > E_i$. To see why, suppose $\delta = 1$. In this case, the union is a monopoly union, and sets wages in order to maximize the expected utility of all workers, subject to the labor-demand equations $w_i = F_i(K, E_1N_1, \cdots, E_IN_I)$. Consequently, the union objective can be written as:

$$\Lambda = \sum_i N_i \int_{\underline{\varphi}}^{G_i^{-1}(E_i)} (u(F_i(K, E_1N_1, \cdots, E_IN_I) - T_i - \varphi) - u(-T_u))\mathrm{d}G_i(\varphi). \tag{122}$$

Now, suppose that, starting from the competitive equilibrium where $G(F_i - T_i - T_u) = E_i$ for all i, the union considers reducing the employment rate in the sector ℓ where the marginal utility of workers' consumption is highest (i.e., $\overline{u'_\ell} > \overline{u'_j}$ for all $j \neq \ell$). This reduction in employment increases the wage of the workers with the highest marginal utility of consumption and reduce the wages for all other workers. The impact of a reduction in employment in sector ℓ on the union's objective is:

$$\mathrm{d}\Lambda = N_\ell \sum_j N_j E_j \overline{u'_j} F_{j\ell} \times \mathrm{d}E_\ell = N_\ell \left(N_\ell E_\ell F_{\ell\ell} \overline{u'_\ell} + \sum_{j \neq \ell} N_j E_j F_{j\ell} \overline{u'_j} \right) \mathrm{d}E_\ell. \tag{123}$$

This expression can be thought of as summing a weighted average of marginal utilities, with weights $N_j E_j F_{j\ell}$. The first term in brackets is negative (because $F_{\ell\ell} < 0$), whereas the second term in brackets is positive (because $F_{j\ell} \geq 0$ for all $j \neq \ell$). The first term unambiguously dominates the second term. This is because the weights sum to less than zero (constant returns to scale implies $\sum_j N_j E_j F_{j\ell} = -F_{Ki}K \leq 0$) and the only negative component (i.e., $N_\ell E_\ell F_{\ell\ell}$) is multiplied by the largest marginal utility (i.e., $\overline{u'_\ell} > \overline{u'_j}$ for all $j \neq \ell$). Consequently, the union objective unambiguously increases if – starting from the competitive equilibrium – the rate of employment for workers in the sector with the lowest wage is reduced (i.e., $\mathrm{d}E_\ell < 0$). Hence, a monopoly union ($\delta = 1$) always demands a wage above the market-clearing level in at least one sector.

D.2 Optimal taxation

In the absence of income effects and under the assumption that firm-owners are risk-neutral, the first-order conditions in equations (117) and (120) characterize equilibrium wages and employment rates as a function the participation tax rates: $w_i = w_i(T_1 - T_u, .., T_1 - T_u)$ and $E_i = E_i(T_1 - T_u, .., T_1 - T_u)$.[30] These reduced-form equations can be used to derive the optimal tax formulas. This case is identical to the one with multiple unions, which is analyzed

[30]Risk-neutrality of firm-owners ensures that equilibrium wages and employment rates do not depend on the profit tax.

in the main text. The optimal tax formulas (written in terms of elasticities) therefore remain unaffected.

D.3 Desirability of unions

To study the desirability of a national union, we analyze the welfare effects of a joint marginal increase in union power δ combined with a tax reform, such that all labor-market outcomes are unaffected. If the tax system is optimized, any change in social welfare must then necessarily be the result of the change in union power.

Which tax reform offsets any impact of the increase in union power on equilibrium wages and employment? First, the tax reform cannot include a change in the participation tax for workers whose wage is at the market-clearing level. To see why, consider the labor-market equilibrium condition in a sector i where the wage is at the market-clearing level:

$$G_i(F_i(\cdot) - (T_i - T_u)) = E_i. \tag{124}$$

A change in the participation tax in this sector needs to be accompanied by a change in either $F_i(\cdot)$ or E_i. For this to be the case, employment in at least one sector i needs to adjust. However, the tax change is intended keep employment in all sectors unaffected. Hence, in sectors where $G_i = E_i$ it must be the case that $\mathrm{d}(T_i - T_u) = 0$. The tax reform thus changes income taxes in all sectors j where the wage is set above above the martket-clearing level, i.e., where $G_i > E_i$. The marginal tax reform should then satisfy:

$$\forall i \in k(\delta): \quad \sum_{j\in k(\delta)} \frac{\partial w_i(T_1 - T_u, \cdots, T_I - T_u, \delta)}{\partial T_j}\mathrm{d}T_j^* + \frac{\partial w_i(T_1 - T_u, \cdots, T_I - T_u, \delta)}{\partial \delta}\mathrm{d}\delta = 0. \tag{125}$$

Here, $k(\delta) \equiv \{i : G_i > E_i\}$ is the set of sectors where the wage is raised above the market-clearing level. As before, assume that the government adjusts the profit tax to keep the budget balanced. Since the combined increase in union power δ and the tax reform $\mathrm{d}T_j^*$ for all j leaves all labor-market outcomes unaffected, there is only a transfer of resources from firm-owners to the workers whose wage is higher than the market-clearing level (i.e., for whom $G_i > E_i$). The welfare effect is thus equal to:

$$\frac{\mathrm{d}\mathcal{W}}{\lambda} = \sum_{i\in k(\delta)} N_i E_i (1 - b_i)\mathrm{d}T_i^*, \tag{126}$$

where λ is the multiplier on the government budget constraint. Divide the latter by $\sum_i N_i > 0$. The remaining term is positive if and only if

$$\sum_{i\in k\delta} \omega_i (1 - b_i)\mathrm{d}T_i^* > 0. \tag{127}$$

E Efficient bargaining

E.1 Derivation elasticities

Partial equilibrium in labor market i is obtained by combining the contract curve from equation (42) and the rent-sharing rule from equation (43):

$$\overline{u'(w_i(1-t_i)-T_u-\varphi)}(w_i-F_i(E_i)) = u(w_i(1-t_i)-T_u-G_i^{-1}(E_i))-u(-T_u), \tag{128}$$

$$w_i = (1-\upsilon_i)F_i(E_i)+\upsilon_i\phi_i(E_i). \tag{129}$$

Unlike before, here we directly express our results in terms of participation tax rates t_i, as opposed to levels $T_i - T_u$. This has no implications for the main insights. In the absence of income effects, these equations define $E_i = E_i(t_i)$ and $w_i = w_i(t_i)$. As before, the absence of income effects implies a change in T_u does not affect equilibrium wages and employment if the participation tax rate t_i remains constant. Hence, the derivative of equation (128) with respect T_u, while keeping t_i constant, is zero:

$$-\overline{u''(w_i(1-t_i)-T_u-\varphi)}(w_i-F_i(E_i)) = -u'(w_i(1-t_i)-T_u-G_i^{-1}(E_i))+u'(-T_u). \tag{130}$$

To derive the elasticities of employment and wages with respect to the participation tax rate, we first linearize the rent-sharing rule:

$$\frac{\mathrm{d}w_i}{w_i} = -\left((1-m_i)\frac{(1-\upsilon_i)}{\varepsilon_i}+m_i\right)\frac{\mathrm{d}E_i}{E_i}, \tag{131}$$

where $m_i \equiv (w_i - F_i)/w_i = 1 - F_i/w_i$ is the implicit subsidy on labor demand, as a fraction of the wage. If union power is zero, $\upsilon_i = 0$, $m_i = 0$, and equation (131) reduces to the linearized labor-demand equation.

Second, linearizing the contract curve yields:

$$\frac{\mathrm{d}\overline{u'_i}}{\overline{u'_i}}+\frac{\mathrm{d}(w_i-F_i)}{w_i-F_i} = \frac{\mathrm{d}(\hat{u}_i-u_u)}{\hat{u}_i-u_u}. \tag{132}$$

Using equation (130), the linearized sub-parts are given by:

$$\frac{\mathrm{d}\overline{u'_i}}{\overline{u'_i}} = \frac{\overline{u''_i}w_i(1-t_i)}{\overline{u'_i}}\left(\frac{\mathrm{d}w_i}{w_i}-\frac{\mathrm{d}t_i}{1-t_i}\right)+\frac{(\hat{u}'_i-\overline{u'_i})}{\overline{u'_i}}\frac{\mathrm{d}E_i}{E_i}, \tag{133}$$

$$\frac{\mathrm{d}(w_i-F_i)}{w_i-F_i} = \frac{1}{m_i}\left(\frac{\mathrm{d}w_i}{w_i}+\frac{(1-m_i)}{\varepsilon_i}\frac{\mathrm{d}E_i}{E_i}\right), \tag{134}$$

$$\frac{\mathrm{d}(\hat{u}_i-u_u)}{\hat{u}_i-u_u} = \frac{\hat{u}'_i w_i(1-t_i)}{(\hat{u}_i-u_u)}\left(\frac{\mathrm{d}w_i}{w_i}-\frac{\mathrm{d}t_i}{1-t_i}\right)-\frac{\hat{u}'_i E_i}{G'_i(\hat{\varphi}_i)(\hat{u}_i-u_u)}\frac{\mathrm{d}E_i}{E_i}. \tag{135}$$

Solving for the relative changes in employment and wages yields:

$$\frac{\mathrm{d}E_i}{E_i} = \frac{-u_u' w_i(1-t_i)}{\frac{\hat{u}_i' E_i}{G_i'(\hat{\varphi}_i)} + u_u' w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right) + (\hat{u}_i - u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i} - 1 + \frac{(\hat{u}_i' - \overline{u_i'})}{\overline{u_i'}}\right)}\frac{\mathrm{d}t_i}{1-t_i}, \tag{136}$$

$$\frac{\mathrm{d}w_i}{w_i} = \frac{u_u' w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right)}{\frac{\hat{u}_i' E_i}{G_i'(\hat{\varphi}_i)} + u_u' w_i(1-t_i)\left(\frac{(1-m_i)(1-\upsilon_i)}{\varepsilon_i} + m_i\right) + (\hat{u}_i - u_u)\left(\frac{(1-m_i)}{m_i}\frac{(1-\upsilon_i)}{\varepsilon_i} - 1 + \frac{(\hat{u}_i' - \overline{u_i'})}{\overline{u_i'}}\right)}\frac{\mathrm{d}t_i}{1-t_i}. \tag{137}$$

The elasticities are now as given in Proposition 9.

E.2 Optimal taxation

Start with the Lagrangian for the maximization of social welfare if the government has utilitarian preferences:[31]

$$\begin{aligned}\mathcal{L} = &\sum_i N_i \left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u)\mathrm{d}G_i(\varphi)\right) \\ &+u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda\left(\sum_i N_i(T_u + E_i t_i w_i) + T_f - R\right).\end{aligned} \tag{138}$$

Differentiating with respect to T_u, T_f, and t_i yields:

$$T_u : -\sum_i N_i E_i \overline{u_i'} - \sum_i N_i(1-E_i)u_u' + \lambda\sum_i N_i = 0, \tag{139}$$

$$T_f : -u_f' + \lambda = 0, \tag{140}$$

$$\begin{aligned} t_i : &-N_i E_i w_i(\overline{u_i'} - \lambda) + \frac{\partial E_i}{\partial t_i}\left(N_i(\hat{u}_i - u_u) + u_f' N_i(F_i - w_i) + \lambda N_i t_i w_i\right) \\ &+\frac{\partial w_i}{\partial t_i}\left(N_i E_i \overline{u_i'}(1-t_i) - N_i E_i u_f' + \lambda N_i E_i t_i\right) = 0. \end{aligned} \tag{141}$$

The first two expressions from Proposition 9 are obtained by dividing equation (139) by $\lambda\sum_i N_i$ and equation (140) by λ, and imposing the definitions of the social welfare weights $b_i \equiv \overline{u'(c_i)}/\lambda$, $b_u \equiv u'(c_u)/\lambda$ and the employment shares $\omega_i \equiv N_i E_i/\sum_j N_j$ and $\omega_u \equiv \sum_i N_i(1-E_i)/\sum_j N_j$. The second result can be found by dividing equation (140) by λ and using $b_f \equiv u'(c_f)/\lambda$. The expression for the optimal participation tax rate t_i is obtained by substituting $u_f' = \lambda$ in equation (141) and dividing the expression by $N_i E_i \lambda w_i$. After imposing the definitions of the union wedge $\tau_i \equiv \frac{u(\hat{c}_i) - u(c_u)}{\lambda w_i}$, the mark-up $m_i = \frac{w_i - F_i}{w_i}$ and the elasticities κ_{ii} and η_{ii} as defined in equations (48)–(49), we arrive at the final expression stated in Proposition 9.

[31] It is straightforward to add Pareto weights ψ_i and adjust the definitions of the social welfare weights accordingly. This has no implications for our results.

E.3 Desirability of unions

To determine how a change in union power υ_i affects social welfare, we formulate the Lagrangian by taking the labor-market equilibrium conditions explicitly into account:

$$\begin{aligned}
\mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi)\mathrm{d}G_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}} u(-T_u)\mathrm{d}G_i(\varphi) \right) \\
& + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i(T_u + E_i t_i w_i) + T_f - R \right) \\
& + \sum_i \vartheta_i N_i (w_i - (1-\upsilon_i)F_i(\cdot) - \upsilon_i \phi_i(\cdot)) \\
& + \sum_i \mu_i N_i \bigg(\int_{\underline{\varphi}}^{G_i^{-1}(E_i)} u'(w_i(1-t_i) - T_u - \varphi)\mathrm{d}G_i(\varphi)(F_i(\cdot) - w_i) \\
& + E_i(u(w_i(1-t_i) - T_u - G_i^{-1}(E_i)) - u(-T_u)) \bigg). \qquad (142)
\end{aligned}$$

To determine how a change in the union power affects social welfare, differentiate the Lagrangian with respect to υ_i, and apply the envelope theorem:

$$\frac{\partial \mathcal{W}}{\partial \upsilon_i} = \frac{\partial \mathcal{L}}{\partial \upsilon_i} = N_i \vartheta_i (F_i - \phi_i). \qquad (143)$$

Because the production function $F(\cdot)$ is concave in E_i, $w_i - F_i = \upsilon_i(\phi_i(\cdot) - F_i(\cdot)) > 0$ if $\upsilon_i > 0$. Hence, $\frac{\partial \mathcal{L}}{\partial \upsilon_i}$ is positive if and only if $\vartheta_i < 0$. To determine the sign of ϑ_i ,use the first-order conditions of the Lagrangian with respect to t_i, w_i and T_f:

$$\begin{aligned}
t_i :& - w_i N_i E_i(\overline{u_i'} - \lambda) - \mu_i w_i N_i E_i \left(\overline{u_i''}(F_i - w_i) + \hat{u}_i' \right) = 0, \qquad (144) \\
w_i :& (1-t_i) N_i E_i \overline{u_i'} - N_i E_i u_f' + \lambda t_i N_i E_i + \vartheta_i N_i \\
& + \mu_i (1-t_i) N_i \left(E_i \overline{u_i''}(F_i - w_i) + E_i \hat{u}_i' \right) - \mu_i N_i E_i \overline{u_i'} = 0, \qquad (145) \\
T_f :& - u_f' + \lambda = 0. \qquad (146)
\end{aligned}$$

Combining equations (144) and (145) and substituting equation (146) yields:

$$\vartheta_i = \mu_i E_i \overline{u_i'}. \qquad (147)$$

Substituting for μ_i using equation (144) and simplifying gives:

$$\vartheta_i = E_i \left(\frac{\lambda \overline{u_i'}(1-b_i)}{\overline{u_i''}(F_i - w_i) + \hat{u}_i'} \right). \qquad (148)$$

From equations (143) and (148), it follows that an increase in υ_i increases social welfare if and only if the term on the right-hand side of expression (148) is negative:

$$b_i > 1. \qquad (149)$$