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# Optimal linear commodity taxation under optimal non-linear income taxation☆

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ABSTRACT

This paper analyzes optimal linear commodity taxes joint with non-linear income taxes. We provide optimal tax rules based on empirically observable elasticities, earnings and commodity demands. We demonstrate that commodities should be taxed/subsidized if – conditional on earnings – doing so boosts labor supply as the critical role of commodity taxation is to alleviate distortions on labor supply caused by income taxation. We extend the standard formula for optimal non-linear income taxation for the presence of optimal linear commodity taxes. We correct parts of the literature by showing that the optimal second-best allocations derived by Atkinson and Stiglitz (1976, 1980) cannot be supported by linear commodity taxes.


## 1. Introduction

The Atkinson–Stiglitz theorem is one of the cornerstones of normative public economics. Atkinson & Stiglitz (1976) demonstrated that when the government can optimize a fully non-linear income tax, indirect tax instruments are superfluous as long as individuals have homogeneous preferences that are weakly separable between leisure and commodities.[1] When preferences are not separable, differential commodity taxes should be deployed, but the analysis of the optimal tax structure remains opaque.[2]

There is a presumption, following the classical textbook treatment in Atkinson & Stiglitz (1980), and subsequently reported in Myles (1995) and Salanié (2011), that the second-best allocations derived by these authors can be decentralized using non-linear income and linear commodity taxes.[3] However, since quantities of taxed goods are used as control variables by Atkinson & Stiglitz (1980), it is implicitly assumed that the government can employ non-linear commodity taxes. The first contribution of this paper is to demonstrate that the marginal rates of substitution between commodities in the second-best allocation derived by Atkinson & Stiglitz (1976) are necessarily different among individuals, and can therefore not be supported by linear commodity taxes.

Although it is almost 40 years since the seminal contribution of Atkinson & Stiglitz (1976), it remains unclear how *linear* commodity taxes should be set, and how the optimal non-linear income tax should

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[1] Using the same preference assumptions, and starting with a differential commodity tax system and an arbitrary non-linear income tax, Laroque (2005) and Kaplow (2006) demonstrated that eliminating all commodity-tax differentiation, while adjusting the non-linear income-tax schedule such that distributional effects are neutralized, yields a Pareto improvement.

[2] Browning & Meghir (1991), Crawford et al. (2010), Gordon & Kopczuk (2013), and Pirttilä & Suoniemi (forthcoming) all empirically reject weak separability in the utility function, vindicating the relevance of analyzing non-separable preferences in optimal-tax studies.

[3] In their original paper, Atkinson & Stiglitz (1976, p. 67) explicitly note that they 'allow for the possibility that the tax rate on commodities may be a function of the level of consumption'. That is, they assume that commodity taxes are in fact non-linear.

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be modified when the government can only optimize linear commodity taxes. This is of theoretical interest, but it is also of practical importance. While the characterization of a fully non-linear system of income and commodity taxes is instructive, it is of limited policy relevance. Since most commodity transactions are anonymous, only linear commodity taxes can practically be levied for many commodities. The second contribution of this paper will be to provide a thorough analysis of the optimal non-linear income and linear commodity tax system when preferences are not weakly separable.

We demonstrate that the formulae for optimal commodity taxes resemble those from the Ramsey framework with homogeneous agents. In particular, we derive that the 'index of discouragement' of taxing commodities is directly related to the benefits of commodity taxes in reducing labor-market distortions. Non-uniform commodity taxes are employed to boost labor supply and thereby offset some of the distortions created by the income tax. Such a policy, however, comes at the cost of distorting commodity demands. This finding is the counterpart of the classic result that Corlett & Hague (1953) derived for a representative individual facing linear taxation.

We also provide a characterization of the optimal non-linear income tax system in the presence of optimal linear commodity taxes. Labor wedges in this setting include, besides the income tax rate, an additional term due to commodity taxation. That term cannot be unambiguously signed, hence it cannot be determined in general whether the optimal use of linear commodity taxation increases or decreases optimal marginal income taxes — ceteris paribus. Following Saez (2001), we derive optimal rules for income and commodity taxation that are dependent only on social welfare weights and empirically measurable variables: elasticities, the earnings distribution, and commodity demands. The zero marginal tax rates at the endpoints of the skill distribution in the absence of bunching reported in Sadka (1976) and Seade (1977) apply not to labor taxes but to the labor wedges, as in Edwards et al. (1994) and Nava et al. (1996). Moreover, the marginal cost of public funds in the optimal tax system always equals one, which confirms Jacobs (2013).

The approach we adopt has precursors in the literature, in particular Christiansen (1984), Edwards et al. (1994), Nava et al. (1996) and Saez (2002). Christiansen (1984) explores the desirability of introducing small linear commodity taxes alongside the optimal non-linear income tax. Small taxes on goods that are relatively more complementary with leisure than the untaxed numéraire commodity or small subsidies on those commodities that are relatively more substitutable for leisure will improve social welfare. Saez (2002) follows the approach of Christiansen (1984) to analyze the implications of heterogeneous preferences as in Mirrlees (1976) and demonstrates that goods for which high-income earners have a stronger taste should be taxed more.

The studies of Edwards et al. (1994) and Nava et al. (1996) are closest to ours. They adapt the two-type Stiglitz (1982) model to the case with multiple goods and leisure to analyze non-linear income taxes with linear commodity taxes. They show that the usefulness of commodity taxes lies in their ability to weaken the incentive constraints. By adopting the continuous-type Mirrlees (1971) framework, we are able to provide a complete characterization of optimal income and commodity taxes under non-separable preferences. Optimal linear commodity taxes can be depicted in terms of measurable elasticities, and non-linear income taxes follow a modified version of the *ABC*-formula of Diamond (1998).

The next Section sets up the model. The following Section demonstrates that the optimal allocations derived by Atkinson & Stiglitz (1976) require non-linear commodity taxes to be implemented. Then, we take up the case where commodity taxes are constrained to be linear. Optimal income taxes and optimal commodity taxes are derived and explained. The last section concludes.

## 2. Model

### 2.1. Individual behavior

We follow the setup of (Atkinson & Stiglitz, 1976). There is a continuum of individuals with mass one distributed by their skill level $n$ according to $F(n)$ with density $f(n)$, for $n \in \mathcal{N} \equiv [\underline{n}, \overline{n}]$, where $0 < \underline{n} < \overline{n} \leq \infty$. The skill level $n$ measures the number of efficiency units of labor of each individual. Assuming that all workers are perfect substitutes in production and that the wage rate per efficiency unit of labor is normalized to one, $n$ corresponds to the wage rate per unit of time worked for a worker of skill $n$. Individuals of skill level $n$ whose labor supply is $\ell_n$ produce output of $z_n \equiv n\ell_n$.

Individuals have an identical, strictly concave and twice differentiable utility function given by $u(c, x^1, x^2, \ldots, x^I, \ell)$, where $c$ is a numéraire commodity, $x^1, x^2, \ldots, x^I$ are other commodities $i = 1, \ldots, I$, and $\ell$ is labor supply.[4] For a type-$n$ individual, we rewrite utility as:

$$u_n \equiv u(c_n, \mathbf{x}_n, \ell_n), \ \forall n \in \mathcal{N}, \tag{1}$$

where $\mathbf{x}_n = (x_n^1, x_n^2, \ldots, x_n^I)$ is the vector of commodities consumed by a type-$n$ individual.

Let the income tax function be given by $T(z_n)$, where the derivative of the income tax function is assumed to be continuous and is denoted by $T'(z_n)$. The tax rate on commodity $x^i$ is $t_i$ for all $i = 1, \ldots, I$. Commodity $c$ remains untaxed, since one commodity tax is redundant. Producer prices for all commodities are constant and normalized to unity. The household budget constraint is thus given by

$$c_n + \sum_{i=1,\cdots,I} (1 + t_i)x_n^i = z_n - T(z_n), \ \forall n. \tag{2}$$

Maximizing utility $u(c_n, \mathbf{x}_n, z_n/n)$ subject to the household budget constraint yields the following necessary first-order conditions:

$$-\frac{u_\ell(c_n, \mathbf{x}_n, \ell_n)}{u_c(c_n, \mathbf{x}_n, \ell_n)} = (1 - T'(z_n))n, \ \forall n, \tag{3}$$

$$\frac{u_{x^i}(c_n, \mathbf{x}_n, \ell_n)}{u_c(c_n, \mathbf{x}_n, \ell_n)} = 1 + t_i, \ \forall n, i. \tag{4}$$

The marginal willingness to supply labor increases with the wage rate and decreases with the marginal tax rate on earnings. In addition, the marginal tax rate on commodity $x^i$ distorts the consumption choice away from $x^i$ toward $c$ if the marginal tax is positive (and vice versa if it is negative). In the remainder, we will write all optimal tax rules in terms of behavioral elasticities. Table 1 presents these elasticities. An asterisk will always be used to indicate compensated behavioral changes. In addition, the superscript $c$ denotes the elasticity of the *conditional* commodity demand function, which gives commodity demands as a function of commodity prices, net income and labor supply, as explained in detail later.

### 2.2. Social objectives and aggregate resource constraint

The government is assumed to maximize the sum of concave social utilities, $\Psi(u_n)$:

$$\int_{\mathcal{N}} \Psi(u_n) \mathrm{d}F(n), \ \Psi'(u_n) > 0, \ \Psi''(u_n) \leq 0. \tag{5}$$

[4] The optimal allocation is independent of the choice of the numéraire commodity, and the structure of relative tax rates is as well. Moreover, one can always choose a positive tax rate for the numéraire commodity since the optimal mix of direct and indirect taxes is indeterminate.

**Table 1**
Behavioral elasticities.

| | | |
|---|---|---|
| Uncompensated elasticity of labor supply | $\varepsilon_{\ell n} \equiv \frac{\partial \ell_n}{\partial n}\frac{n}{\ell_n}$ | $= \frac{u_\ell/\ell + u_\ell u_{c\ell}/u_c - (u_\ell/u_c)^2 u_{cc} - n u_\ell T''/(1-T')}{u_{\ell\ell} + (u_\ell/u_c)^2 u_{cc} - 2u_\ell u_{c\ell}/u_c + n u_\ell T''/(1-T')}$ |
| Uncompensated elasticity of earnings supply | $\varepsilon_{zn} \equiv \frac{\partial z_n}{\partial n}\frac{n}{z_n} = 1 + \varepsilon_{\ell n}$ | $= \frac{u_\ell/\ell_n + u_{\ell\ell} - u_\ell u_{c\ell}/u_c}{u_{\ell\ell} + (u_\ell/u_c)^2 u_{cc} - 2u_\ell u_{c\ell}/u_c + n u_\ell T''/(1-T')}$ |
| Compensated tax elasticity of labor supply[a] | $\varepsilon^*_{\ell T'} \equiv -\frac{\partial \ell^*_n}{\partial \tau}\frac{(1-T'(z_n))}{\ell_n}$ | $= \frac{u_\ell/\ell_n}{u_{\ell\ell} + (u_\ell/u_c)^2 u_{cc} - 2u_\ell u_{c\ell}/u_c + n u_\ell T''/(1-T')}$ |
| Compensated tax elasticity of earnings supply[a] | $\varepsilon^*_{zT'} \equiv -\frac{\partial z^*_n}{\partial \tau}\frac{(1-T'(z_n))}{z_n} = \varepsilon^*_{\ell T'}$ | $= \frac{u_\ell/\ell_n}{u_{\ell\ell} + (u_\ell/u_c)^2 u_{cc} - 2u_\ell u_{c\ell}/u_c + n u_\ell T''/(1-T')}$ |
| Income elasticity of labor supply[b] | $\varepsilon^I_\ell \equiv (1-T'(z_n))n\frac{\partial \ell_n}{\partial \rho}$ | $= \frac{(u_{c\ell} - u_\ell u_{cc}/u_c)u_\ell/u_c}{u_{\ell\ell} + (u_\ell/u_c)^2 u_{cc} - 2u_\ell u_{c\ell}/u_c + n u_\ell T''/(1-T')}$ |
| Compensated tax elasticity of commodity demand | $\varepsilon^*_{ii} \equiv -\frac{\partial x^{*i}_n}{\partial t_i}\frac{(1+t_i)}{x^i_n}$ | $= \frac{u_{x^i}/x^i_n}{u_{x^i x^i} + (u_{x^i}/u_c)^2 u_{cc} - 2(u_{x^i}/u_c)u_{cx^i}}$ |
| Income elasticity of commodity demand[b] | $\varepsilon^I_i \equiv (1+t_i)\frac{\partial x^i_n}{\partial \rho}$ | $= \frac{(u_{x^i}u_{cc}/u_c - u_{cx^i})u_{x^i}/u_c}{u_{x^i x^i} + (u_{x^i}/u_c)^2 u_{cc} - 2(u_{x^i}/u_c)u_{cx^i}}$ |
| Conditional labor elasticity of commodity demand | $\varepsilon^c_{i\ell} \equiv \frac{\partial x^{i,c}_n}{\partial \ell_n}\frac{\ell_n}{x^i_n}$ | $= \frac{(u_{x^i}u_{c\ell}/u_c - u_{x^i\ell})\ell_n/x^i_n}{u_{x^i x^i} + (u_{x^i}/u_c)^2 u_{cc} - 2(u_{x^i}/u_c)u_{cx^i}}$ |

See Appendix A for derivations.
[a] $\tau$ denotes an exogenous increase in the marginal income tax rate.
[b] $\rho$ denotes an exogenous increase in unearned income.

Diminishing private marginal utility of income and non-increasing marginal social welfare $\Psi'(u_n)$ yields a social preference for redistribution.

Suppose the government requires an exogenous amount of resources $R$. Then, recalling that producer prices are in unity, the resource constraint of the economy is given by

$$\int_{\mathcal{N}} \left( z_n - c_n - \sum_{i=1,\cdots,I} x^i_n - R \right) \mathrm{d}F(n) = 0. \tag{6}$$

Satisfaction of this resource constraint and all the household budget constraints implies that the government budget constraint will hold by Walras' law.

## 3. Implementation with linear commodity taxes?

We begin first by supposing, following Atkinson & Stiglitz (1976), that the government can observe both individual incomes and the quantities of all commodities purchased by each individual. We characterize the second-best planning solution for this case using the revelation principle. Since the government can observe income and commodity demands but not skill levels, it cannot rule out individuals of one skill-type choosing a commodity-income bundle intended for another skill-type. The incentive-compatibility constraints imply that $u_n = \max_{n'} u(c_{n'}, x_{n'}, z_{n'}/n), \forall n, n' \neq n \in \mathcal{N}$. We can apply the envelope theorem with respect to $n$ to obtain the first-order incentive-compatibility constraint, as in Atkinson & Stiglitz (1976):

$$\dot{u}_n = -\frac{\ell_n u_\ell(c_n, \mathbf{x}_n, \ell_n)}{n}, \forall n, \tag{7}$$

where the dot denotes a derivative with respect to $n$.

The first-order approach is valid for characterizing the second-best optimum if the Spence–Mirrlees and monotonicity conditions are met. Lemma 1 provides these familiar conditions. We assume that Lemma 1 holds in the analyses that follow.[5]

**Lemma 1.** Let $U(c_n, \mathbf{X}_n, n) \equiv u(c_n, \mathbf{x}_n, z_n/n)$, where $\mathbf{X} \equiv (x_1, x_2, \ldots, x_I, z)$, then the following constraint on the Spence–Mirrlees and monotonicity conditions must hold at the optimal allocation:

$$\frac{\mathrm{d}(U_{\mathbf{X}}/U_c)}{\mathrm{d}n} \cdot \dot{\mathbf{X}}^{\mathrm{t}}_n \geq 0. \tag{8}$$

**Proof.** See Mirrlees (1976, 334–335).

The second-best allocation is obtained by maximizing social welfare (5) subject to the resource constraint (6) and the incentive constraint (7). We use $\ell_n$ and $x^i_n$ as controls, and $u_n$ as a state variable. The numéraire commodity $c_n$ is determined by the function $c_n(\mathbf{x}_n, \ell_n, u_n)$ obtained from inverting the utility function Eq. (1). Let $\theta_n$ be the co-state variable associated with the incentive constraint in Eq.(7). Multiplying Eq. (7) by $\theta_n$, and integrating by parts, we can write the Lagrangian for this optimal control problem as:

$$\begin{aligned} \mathcal{L} \equiv & \int_{\mathcal{N}} \left( \Psi(u_n) + \eta \left( n\ell_n - c_n(\mathbf{x}_n, \ell_n, u_n) - \sum_{i=1,\cdots,I} x^i_n - R \right) \right) f(n)\mathrm{d}n \\ & + \int_{\mathcal{N}} \left( \theta_n \frac{\ell_n u_\ell(c_n(\mathbf{x}_n, \ell_n, u_n), \mathbf{x}_n, \ell_n)}{n} - u_n \dot{\theta}_n \right) \mathrm{d}n + \theta_{\overline{n}} u_{\overline{n}} - \theta_{\underline{n}} u_{\underline{n}} \end{aligned} \tag{9}$$

where $\eta$ is the Lagrange multiplier on the resource constraint (6). The first-order conditions with respect to $\ell_n$, $x^i_n$, and $u_n$ are given by

$$\frac{\partial \mathcal{L}}{\partial \ell_n} = \eta \left( n - \frac{\partial c_n}{\partial \ell_n} \right) f(n) + \frac{\theta_n u_\ell}{n} \left( 1 + \frac{\ell_n u_{\ell\ell}}{u_\ell} + \frac{\ell_n u_{\ell c}}{u_\ell} \frac{\partial c_n}{\partial \ell_n} \right) = 0, \ \forall n, \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial x^i_n} = -\eta \left( 1 + \frac{\partial c_n}{\partial x^i_n} \right) f(n) + \frac{\theta_n \ell_n}{n} \left( u_{\ell c} \frac{\partial c_n}{\partial x^i_n} + u_{\ell x^i} \right) = 0, \ \forall i, n, \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial u_n} = \left( \Psi' - \eta \frac{\partial c_n}{\partial u_n} \right) f(n) + \frac{\theta_n u_\ell}{n} \frac{\ell_n u_{\ell c}}{u_\ell} \frac{\partial c_n}{\partial u_n} - \dot{\theta}_n = 0, \ \forall n \neq \overline{n}, \underline{n}, \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial u_{\underline{n}}} = -\theta_{\underline{n}} = 0, \ \frac{\partial \mathcal{L}}{\partial u_{\overline{n}}} = \theta_{\overline{n}} = 0. \tag{13}$$

These first-order conditions correspond to the characterization of the second-best optimum in Atkinson & Stiglitz (1976).

The above analysis uses $\ell_n$ (and thereby $z_n$) and $\mathbf{x}_n$ as controls, and so implicitly assumes that the government can observe them and can apply non-linear taxes to both income and commodity demands. Some authors suggest that the above formulation also applies when commodity taxes are linear and have the same rate for all individuals. In particular, they assume that the marginal tax rates implied by Eq. (11) can be interpreted as linear tax rates.[6] Unfortunately, this is not the case if preferences are not weakly separable as Proposition 1 shows.

[5] When we analyze linear commodity taxes, the first-order approach requires weaker conditions. Then, we need to assume that the Spence–Mirrlees condition and the monotonicity condition apply to gross labor earnings $z$ only, since linear commodity taxes are always incentive compatible.

[6] See Eqs. (14)–(39) in Atkinson & Stiglitz (1980, p. 435–7), Eq. (5.96) in Myles (1995, pp. 163–6), and Eq. (1) in Salanié (2011, 125–7).

**Proposition 1.** The allocation described by first-order conditions (10), (11), (12), and transversality conditions (13) cannot be implemented using linear commodity taxes.

**Proof.** Totally differentiating $u_n$ in Eq. (1) and using Eq. (4), we obtain $\partial c_n/\partial x_n^i = -u_{x^i}/u_c = -(1+t_i'(x_n^i))$. Substituting this into Eq. (11) and rearranging yields:

$$\frac{u_{x^i}(c_n,\mathbf{x}_n,\ell_n)}{u_c(c_n,\mathbf{x}_n,\ell_n)} = \left(1+\frac{u_c(c_n,\mathbf{x}_n,\ell_n)\theta_n/\eta}{nf(n)}\frac{\partial \ln(u_{x^i}/u_c)}{\partial \ln \ell_n}\right)^{-1}, \quad \forall n, i. \tag{14}$$

The right-hand side of Eq. (14) is not constant across skill types, except with weakly separable preferences, so that $u_{x^i}/u_c$ is independent of $\ell_n$. Therefore, in the absence of weak separability, the allocation described by Eqs. (10)–(13) and (6) cannot be implemented with constant tax rates on consumption of goods $x^i$.

This demonstrates that with non-separable preferences, non-linear commodity taxes are required in order to achieve the second-best optimum analyzed in Atkinson & Stiglitz (1976), contrary to the impression left by the literature.

## 4. Optimal linear commodity taxation under optimal non-linear income taxation

We now turn to the analysis of the case where the government can observe individuals' incomes, but commodity sales are anonymous. If the government is unable to observe individuals' commodity consumption levels, it can only levy linear commodity taxes with a common rate for all individuals. The assumption that consumption taxes are linear therefore implies an additional constraint on the set of admissible second-best allocations: the marginal rates of substitution between $c$ and $x^i$ must be identical for all agents. We follow Mirrlees (1976), who employs a 'mixed' primal-dual approach to determine the optimal income and commodity tax schedules. The non-linear income tax is found by choosing the optimal quantities of labor and utility, whereas the optimal commodity tax rates are found by choosing the optimal prices (rather than quantities) for each commodity.

### 4.1. Individual behavior

Following Mirrlees (1976), we disaggregate individual optimization into two stages. First, a type-$n$ person chooses labor supply $\ell_n$, which determines income $z_n$ and disposable income $y_n \equiv z_n - T(z_n)$, given the non-linear income tax function $T(z_n)$. Second, disposable income is allocated among the $I+1$ commodities, $c_n$ and $\mathbf{x}_n$. The individual anticipates the outcome of the second stage when choosing labor supply.

We start with the second-stage problem. In the second stage, the individual maximizes $u(c_n, \mathbf{x}_n, \ell_n)$ with respect to $c_n$ and $x_n^i$, given $\ell_n$, subject to

$$c_n + \sum_{i=1,\ldots,I} q_i x_n^i = y_n, \quad \forall n, \tag{15}$$

where $q_i \equiv 1+t_i$ is the consumer price of commodity $x^i$. The first-order conditions for this partial maximization problem are

$$\frac{u_{x^i}(c_n,\mathbf{x}_n,\ell_n)}{u_c(c_n,\mathbf{x}_n,\ell_n)} = q_i = 1+t_i, \quad \forall n, i. \tag{16}$$

The solution yields *conditional* commodity demands $c_n^c(\mathbf{q}, y_n, \ell_n)$ and $\mathbf{x}_n^c(\mathbf{q}, y_n, \ell_n)$, where $\mathbf{q} \equiv (q_1, q_2, \ldots, q_I)$ is the vector of consumer prices, and we use a superscript $c$ to denote conditional demand functions. Substitution in the utility function yields a conditional indirect utility function $v_n$:

$$v_n \equiv v(\mathbf{q}, y_n, \ell_n) \equiv u(c_n^c(\cdot), \mathbf{x}_n^c(\cdot), \ell_n), \quad \forall n. \tag{17}$$

It is useful for what follows to consider the dual to the above problem: the conditional expenditure-minimizing problem. Individual $n$ chooses $c_n$ and $x_n^i$ to minimize expenditures $c_n + \sum_i q_i x_n^i$, subject to $u(c_n, \mathbf{x}_n, \ell_n) = v_n$. The solution yields compensated conditional demands $c_n^{c*}(\mathbf{q}, \ell_n, v_n)$ and $x_n^{c*,i}(\mathbf{q}, \ell_n, v_n)$, and the expenditure function $e(\mathbf{q}, \ell_n, v_n)$. Note that these compensated demands are conditional in the sense that labor supply (gross income) is being held constant. In the individual's optimum, $y_n = e(\mathbf{q}, \ell_n, v_n)$, $c_n^{c*}(\mathbf{q}, \ell_n, v_n) = c_n^c(\mathbf{q}, y_n, \ell_n)$ and $x_n^{c*,i}(\mathbf{q}, \ell_n, v_n) = x_n^{c,i}(\mathbf{q}, y_n, \ell_n)$, since compensated and uncompensated conditional demands are the same.

In the first stage, a type-$n$ individual chooses labor supply $\ell_n$ to maximize the stage-2 partial utility function $v_n \equiv v(\mathbf{q}, y_n, \ell_n)$, subject to the budget constraint $y_n = n\ell_n - T(n\ell_n)$. This yields the first-order condition for labor supply:

$$-\frac{v_\ell(\mathbf{q},y_n,\ell_n)}{v_y(\mathbf{q},y_n,\ell_n)} = -\frac{u_\ell(c_n,\mathbf{x}_n,\ell_n)}{u_c(c_n,\mathbf{x}_n,\ell_n)} = (1-T'(n\ell_n))n, \quad \forall n. \tag{18}$$

### 4.2. The government's problem

The government takes as given individual behavior summarized in the conditional indirect utility functions $v_n \equiv v(\mathbf{q}, y_n, \ell_n) = v(\mathbf{q}, y_n, z_n/n)$ for all skill-types $n$. The incentive compatibility constraint can readily be formulated as earlier:

$$\dot{v}_n = -\frac{\ell_n v_\ell(\mathbf{q}, e(\mathbf{q},\ell_n,v_n),\ell_n)}{n}, \forall n. \tag{19}$$

Proceeding as above, we can write the Lagrangian for this optimal control problem as:

$$\mathcal{L} \equiv \int_{\mathcal{N}} \Psi(v_n) f(n) \mathrm{d}n + \int_{\mathcal{N}} \eta\left(n\ell_n - c_n^{c*}(\mathbf{q},\ell_n,v_n) - \sum_i x_n^{c*,i}(\mathbf{q},\ell_n,v_n) - R\right) f(n)\mathrm{d}n \tag{20}$$
$$+\int_{\mathcal{N}}\left(\theta_n \frac{\ell_n v_\ell(\mathbf{q}, e(\mathbf{q},\ell_n,v_n),\ell_n)}{n} - v_n\dot{\theta}_n\right)\mathrm{d}n + \theta_{\overline{n}} v_{\overline{n}} - \theta_{\underline{n}} v_{\underline{n}}.$$

The control variables are now $\ell_n$ and $\mathbf{q}$, and the state variable is $v_n$. The first-order conditions with respect to the control and state variables are:

$$\frac{\partial \mathcal{L}}{\partial \ell_n} = \eta\left(n - \frac{\partial c_n^{c*}}{\partial \ell_n} - \sum_i \frac{\partial x_n^{c*,i}}{\partial \ell_n}\right) f(n) + \frac{\theta_n}{n}(v_\ell + \ell_n v_{\ell\ell} + \ell_n v_{\ell e} e_\ell) = 0, \ \forall n, \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial q_j} = \int_{\mathcal{N}} -\eta\left(\frac{\partial c_n^{c*}}{\partial q_j} + \sum_i \frac{\partial x_n^{c*,i}}{\partial q_j}\right) f(n)\mathrm{d}n + \int_{\mathcal{N}} \frac{\theta_n \ell_n}{n}\left(v_{\ell q_j} + v_{\ell e} e_{q_j}\right)\mathrm{d}n = 0, \ \forall j, \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial v_n} = \left(\Psi' - \eta\frac{\partial c_n^{c*}}{\partial v_n} - \eta\sum_i \frac{\partial x_n^{c*,i}}{\partial v_n}\right) f(n) + \frac{\theta_n \ell_n v_{\ell e} e_u}{n} - \dot{\theta}_n = 0, \ \forall n \neq \underline{n}, \overline{n}, \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial v_{\underline{n}}} = -\theta_{\underline{n}} = 0, \quad \frac{\partial \mathcal{L}}{\partial v_{\overline{n}}} = \theta_{\overline{n}} = 0. \tag{24}$$

We use these conditions to characterize first the optimal non-linear income tax, and then the optimal linear commodity tax system.[7]

### 4.3. Optimal non-linear income taxation

In order to facilitate comparison with the earlier literature, we express the optimal income tax both in the traditional way as in Atkinson & Stiglitz (1976) and in terms of measurable variables following Saez (2001) and Jacquet et al. (2013). We do so using a variant of the *ABC* formula introduced by Diamond (1998). Denote the cumulative distribution of earnings by $\widetilde{F}(z_n)$, which by definition equals the cumulative distribution of ability $\widetilde{F}(z_n) \equiv F(n)$. The density of earnings then satisfies $\widetilde{f}(z_n)\mathrm{d}z_n/\mathrm{d}n = f(n)$. Next, define the net expenditure share of individual $n$ on good $x^i$ as $\gamma_n^i \equiv \frac{(1+t_i)x_n^i}{(1-T_a(z_n))n\ell_n}$, where $T_a(z_n) \equiv T(z_n)/z_n$ is the average income tax rate. And, let the coefficient of residual income progression for individual $n$ be $\sigma_n \equiv \frac{1-T'(z_n)}{1-T_a(z_n)}$. Using these definitions and the elasticities in Table 1, Proposition 2 gives a traditional and a modified *ABC*-type formula for the optimal non-linear income tax under optimal linear commodity taxation.

**Proposition 2.** The optimal non-linear marginal income wedges $\mathcal{W}_n$ under optimal linear commodity taxes are given by

$$\mathcal{W}_n \equiv \frac{T'(z_n)}{1-T'(z_n)} + \frac{1}{\sigma_n}\sum_{i=1,\ldots,I}\frac{t_i}{1+t_i}\gamma_n^i\varepsilon_{i\ell}^c = A_nB_nC_n, \;\; \forall z_n \neq z_{\overline{n}}, z_{\underline{n}}, \tag{25}$$

$$A_n \equiv \frac{1}{\varepsilon_{zT'}^*}, \tag{26}$$

$$B_n \equiv \frac{u_{c,n}}{1-F(n)}\int_n^{\overline{n}}\frac{1}{u_{c,m}}\left(1-\frac{\Psi' u_{c,m}}{\eta}-\sum_i\frac{t_i}{1+t_i}\varepsilon_i^I\right) \times\exp\left[-\int_n^m\left(\frac{\partial \ln u_{c,s}}{\partial \ln \ell_s}\right)\frac{\mathrm{d}s}{s}\right]f(m)dm \tag{27}$$

$$= \frac{\int_{z_n}^{z_{\overline{n}}}(1-g_m)\widetilde{f}(z_m)\mathrm{d}z_m}{1-\widetilde{F}(z_n)}, \; g_m \equiv \frac{\Psi' u_{c,m}}{\eta} + \mathcal{W}_m\varepsilon_\ell^I + \sum_{i=1,\ldots,I}\frac{t_i}{1+t_i}\varepsilon_i^I, \tag{28}$$

$$C_n \equiv \frac{\varepsilon_{zn}(1-F(n))}{nf(n)} = \frac{1-\widetilde{F}(z_n)}{z_n\widetilde{f}(z_n)}. \tag{29}$$

**Proof.** See Appendix B.

Eq. (25) generalizes the Diamond *ABC*-formula to include optimal linear commodity taxes. The interpretation of the right-hand side of the optimal income tax structure is familiar. The $A_n$ and $C_n$ terms are equivalent to those in the formulation of the Mirrlees model in Diamond (1998) and its restatement in terms of the earnings distribution by Saez (2001). These terms need not be discussed further. The integral term in $B_n$ of both the traditional and Saez formulations are slightly modified by the term involving changes in commodity tax revenue, $\sum_i \frac{t_i}{1+t_i}\varepsilon_i^I$.

$\mathcal{W}_n$ on the left-hand-side of Eq. (25) represents the total tax wedge on labor income. As Eq. (25) shows, the total tax wedge $\mathcal{W}_n$ includes not only the direct marginal tax on labor earnings $T'(z_n)$, but also indirect taxes multiplied with their expenditure shares $\gamma_n^i$. Intuitively, if an individual spends $\gamma_n^i$ of his net earnings on consumption of $x^i$, then the tax on $x^i$ creates an additional marginal tax burden on labor of $\gamma_n^i t_i/(1+t_i)$. In addition, there is a correction for the rate of tax progression through $\sigma_n$. The more is the marginal tax rate above the average tax rate, the lower is the coefficient of residual income progression $\sigma_n$. Consequently, the larger is the additional impact of the indirect tax on the total tax wedge on labor effort. Finally, the tax wedge on labor is determined by the conditional cross elasticity $\varepsilon_{i\ell}^c$ of commodity demand $x^i$ with respect to labor $\ell$. From Table 1, it follows that $\varepsilon_{i\ell}^c > 0$ if $u_{x^i\ell}\ell/u_{x^i} - u_{c\ell}\ell/u_c = \partial\ln(u_{x^i}/u_c)/\partial\ln\ell > 0$, that is, if commodity $x^i$ is more complementary with work than the numéraire commodity $c$ is. See also Jacobs & Boadway (2013). Thus, when conditional commodity demand for $x^i$ is associated with a larger labor supply ($\varepsilon_{i\ell}^c > 0$), a higher tax on $x^i$ imposes a larger distortion on labor supply, implying a larger total tax wedge on labor (and vice versa if $\varepsilon_{i\ell}^c < 0$).

Without imposing structure on the utility function we cannot make unambiguous statements as to whether optimal non-linear income taxes are higher or lower under optimal linear commodity taxes for any given desire to redistribute income — represented by the right-hand side of Eq. (25). The reason is that $\varepsilon_{i\ell}^c$ and $t_i$ can be of the same or opposite signs depending on the structure of preferences, so the commodity-tax term on the left-hand side of Eq. (25) can be either positive or negative (Jacobs & Boadway, 2013). These terms will only disappear if utility is weakly separable, i.e. if $\varepsilon_{i\ell}^c = 0$ for all $i$.

Turning to the $B_n$ term, we see that it equals the conditional average of $1-g_m$ above $z_n$. The term $g_m$ corresponds to Diamond (1975)'s social marginal value of income, and measures the social marginal value, in monetary equivalents, of transferring one unit of resources to individual $m$. $g_m$ can be interpreted as the social marginal welfare weight of a type-$m$ individual. The same $g_m$ term would also be obtained by using the tax-perturbation approach of Saez (2001) and Jacquet et al. (2013). Note that $g_m$ includes the income effects on taxed bases. As extracting revenue from all taxpayers above $n$ makes them poorer, they supply more labor if leisure is normal. As a result, the government receives additional tax revenue if labor income is taxed. Similarly, the income effect in commodity demands results in a reduction (increase) in revenue if commodities are taxed (subsidized). Thus, $B_n$ measures the average marginal social value of redistributing one unit of revenue from all individuals above skill level $n$ to the government, where 1 represents the social value of an additional unit of government revenue and $g_m$ captures the net utility losses (in monetary units) of individuals above $n$ as they need to pay an additional unit income tax.

From the transversality conditions (13) it follows that the average social marginal value of income $g_n$ equals one: $\int_{z_{\underline{n}}}^{z_{\overline{n}}} g_m\widetilde{f}(z_m)\mathrm{d}z_m = 1$. Intuitively, by optimizing the intercept $T(0)$ of the tax function, the government ensures that the social marginal value of resources in the public sector equals the average social marginal value of resources in the private sector. The marginal cost of public funds equals the ratio of the social marginal value of resources (in utils) in the public sector ($\eta$), and the average social marginal value of resources (in utils) in the private sector, which includes the income effects on taxed bases $\left(\int_{z_{\underline{n}}}^{z_{\overline{n}}}\eta g_m\widetilde{f}(z_m)\mathrm{d}z_m\right)$. Hence, we confirm Jacobs (2013) that the marginal cost of public funds equals one at the optimal tax system: raising a marginal unit of revenue by raising the marginal tax at some income level $z_n$ thus produces offsetting distributional gains and excess burdens if the tax system is optimized.

[7] A formal proof that the tax schedules indeed implement the optimal second-best allocation is generally missing in the literature. However, recently Renes & Zoutman (2013) have demonstrated that, in economies with a one-dimensional type-space and no externalities, the Spence–Mirrlees and monotonicity conditions of Lemma 1 are sufficient conditions to implement the second-best optimal allocation using separate tax schedules on labor income and commodity demands. Consequently, our separate tax schedules are implementable.

Sadka (1976) and Seade (1977) showed that the marginal tax rates are zero at the endpoints of the skill distribution (in case of a finite upper bound in the skill distribution and no bunching at the bottom skill level). The above results imply that a zero marginal tax rate at the endpoints applies only to the total tax wedge $\mathcal{W}_n$ on labor. Formally, $\theta_{\underline{n}} = \theta_{\overline{n}} = B_{\underline{n}}C_{\underline{n}} = B_{\overline{n}}C_{\overline{n}} = 0$ by Eq. (24), so $\mathcal{W}_{\underline{n}} = \mathcal{W}_{\overline{n}} = 0$ in Eq. (25), as also noted by Edwards et al. (1994) and Nava et al. (1996). Consequently, marginal income tax rates at the endpoints are non-zero and satisfy:

$$\frac{T'(z_n)}{1-T'(z_n)} = -\frac{1}{\sigma_n}\sum_{i=1,\ldots,I}\frac{t_i}{1+t_i}\gamma_n^i\varepsilon_{i\ell}^c, \quad n = \underline{n}, \overline{n}. \tag{30}$$

The reason that optimal rates at the endpoints are non-zero is that the indirect tax part of the labor wedge is always non-zero. Hence, a direct tax wedge is needed to keep the total labor wedge at zero at the endpoints.[8]

### 4.4. Optimal linear commodity taxation

To characterize optimal commodity taxes, denote the compensated demand response of commodity $i$ with respect to commodity price $j$ by $s_n^{ij} \equiv \partial x_n^{*i}/\partial q_j$. Compensated elasticities of demand for $x_n^j$ are then defined as $\varepsilon_{ji}^* \equiv -s_n^{ji}(1+t_i)/x_n^j$. Further, let a 'bar' denote a commodity demand-weighted variable, for example: $\overline{\varepsilon_{ji}^*} \equiv \left[\int_{\mathcal{N}}\varepsilon_{ji}^* x_n^j \mathrm{d}F(n)\right]\left[\int_{\mathcal{N}} x_n^j \mathrm{d}F(n)\right]^{-1}$. Armed with these definitions, the optimal commodity tax structure in given by Proposition 3.

**Proposition 3.** The optimal linear commodity tax structure at the optimal non-linear income tax satisfies

$$\frac{\int_{\mathcal{N}}\sum_i t_i s_n^{ji} f(n)\mathrm{d}n}{\int_{\mathcal{N}} x_n^j f(n)\mathrm{d}n} = \frac{t_j}{1+t_j}\overline{\varepsilon_{jj}^*} + \sum_{i\neq j}\frac{t_i}{1+t_i}\overline{\varepsilon_{ji}^*} = -\overline{\left(\mathcal{W}_n\varepsilon_{zT'}^*\frac{\varepsilon_{j\ell}^c}{\varepsilon_{zn}}\right)}, \quad \forall j. \tag{31}$$

**Proof.** See Appendix B.

The left-hand side of Eq. (31) is similar to that found when all taxes are linear (Atkinson & Stiglitz, 1980, Eqs. (12)–(55) and is analogous to the so-called index of discouragement of commodity $j$ (Mirrlees, 1976, Eq. (86)). Roughly speaking, it represents the proportional reduction in the compensated aggregate demand for commodity $j$ when all commodity taxes are marginally increased. Alternatively, it captures the marginal excess burden of distorting the demand of commodity $j$ by marginally increasing all commodity taxes $t_i$. From the second term it can be seen that commodity taxes are less attractive for alleviating labor-supply distortions the more responsive is the demand for commodity $x^j$ to tax rates $t_i$, that is, the larger are $\varepsilon_{ji}^*$.

The right-hand side of Eq. (31) measures the marginal reduction in distortions on labor supply by discouraging (encouraging) the demand of commodity $x^j$. If $\varepsilon_{j\ell}^c > 0$ ($< 0$) then conditional demand of commodity $x^j$ boosts (reduces) labor supply, as $x^j$ is then more (less) complementary with work than the numéraire commodity $c$ is. By encouraging (discouraging) the consumption of this commodity, the government indirectly stimulates labor supply, and thereby alleviates the distortions of the income tax on work effort. Commodity taxes or subsidies reduce labor-supply distortions more the more elastic is labor supply to a change in the conditional commodity demand for $x^j$, that is, the larger is $\varepsilon_{j\ell}^c$ in absolute value. The structure of commodity taxes relates in a complex way to the compensated own and cross-elasticities of commodity demands with respect to the commodity taxes ($\varepsilon_{ji}^*$) and the cross-elasticity of conditional commodity demands with respect to labor supply ($\varepsilon_{j\ell}^c$).

The term $\mathcal{W}_n\varepsilon_{zT'}^*$ represents the total distortion created by the labor wedge at skill level $n$, defined by Eq. (25). The larger is the labor wedge ($\mathcal{W}_n$), and the higher is the labor-supply elasticity ($\varepsilon_{zT'}^*$), the larger should commodity taxes (or subsidies) be to alleviate the labor-supply distortions created by the income tax. Starting from an equilibrium without indirect taxes, the distortion in consumption choices due to the indirect tax is second-order, whereas the reduction of distortions in labor supply is first-order. At the optimum, the marginal reduction in labor-market distortions equals the marginal increase in goods-market distortions.

The term $\varepsilon_{zn}$ is the earnings elasticity with respect to the wage rate. The larger this elasticity, the stronger labor earnings correlate with ability. Moreover, the conditional elasticity of commodity demands with respect to labor effort equals minus the elasticity of conditional commodity demand with respect to ability: $\varepsilon_{i\ell}^c = -\varepsilon_{in}^c \equiv \frac{\partial x_n^i}{\partial n}\frac{n}{x_n^i}\Big|_{\overline{y}}$.[9] Therefore, the ratio $-\frac{\varepsilon_{i\ell}^c}{\varepsilon_{zn}} = \frac{\varepsilon_{in}^c}{\varepsilon_{zn}}$ implicitly determines which goods are more useful to tax in order to redistribute income. If labor earnings correlate more heavily with ability than conditional commodity demands do ($\varepsilon_{zn}$ increases relative to $\varepsilon_{in}^c$), the government relies more on distorting labor supply and less on distorting commodity demands for redistribution (and vice versa).[10]

Following Edwards et al. (1994) and Nava et al. (1996), the role of commodity taxes can be interpreted in terms of relaxing the incentive constraints. If $\varepsilon_{i\ell}^c < 0$, we have $\partial\ln(u_{x^i}/u_c)/\partial\ln\ell_n < 0$, so that a higher-skilled individual who is mimicking a lower-skilled individual has the same net income, but derives a higher net benefit $u_{x^i}/u_c$ from $x^i$, while supplying less labor. Increasing the tax on $x^i$ and redistributing the revenue so as to keep the utility of the mimicked individual unchanged makes the mimicking individual worse off. Since such a policy relaxes the incentive constraint, the government can increase redistribution through the income tax. The reverse argument holds true for $\varepsilon_{i\ell}^c > 0$.

The optimal structure of commodity taxes depends neither on the particular social welfare function nor on the distribution of skills. Commodity taxes play primarily an efficiency role as they are targeted at alleviating the distortions of the income tax. Intuitively, conditional on observing (and taxing) income directly, commodity-tax differentiation does not help to redistribute more income than can be achieved with the income tax alone. The reason is that, conditional on observing earnings, commodity demands do not reveal any more information on ability than is already available from observing earnings. Hence, direct redistribution through income taxation is superior, because it avoids distortions in commodity demands, while generating the same distortions in labor supply for the same redistribution of income. Naturally, by reducing the distortions of the income tax – for a given desire to redistribute income – commodity-tax differentiation indirectly helps to redistribute more income by allowing for a more progressive income tax system. Our findings thus establish the close connection between the Corlett–Hague rule for optimal linear commodity taxation in a revenue-raising setting with homogeneous agents and optimal linear commodity taxation in the Mirrlees framework with optimal non-linear income taxes.

[8] Naturally, if the skill distribution does not have a finite upper bound, asymptotic tax wedges converge to a constant if the skill distribution is Pareto, as shown by Diamond (1998) and Saez (2001). In particular, if the right-hand side of Eq. (25) converges to a constant, then total labor wedges will asymptotically converge to a constant. Provided that the indirect tax term also asymptotically converges to a constant, top tax rates become constant as well.

[9] Conditional commodity demands are given by $x_n^{c,i}(\mathbf{q}, y_n, \ell_n) = x_n^{c,i}(\mathbf{q}, y_n, z_n/n)$. Differentiating $x_n^{c,i}$ with respect to $n$ (at constant $y_n$), yields $\frac{\partial x_n^{c,i}}{\partial n} = -\frac{\partial x_n^{c,i}}{\partial \ell_n}\frac{\ell_n}{n}$. Consequently we have $\varepsilon_{in}^c = \frac{\partial x_n^{c,i}}{\partial n}\frac{n}{x_n^i} = -\frac{\partial x_n^{c,i}}{\partial \ell_n}\frac{\ell_n}{x_n^i} = -\varepsilon_{i\ell}^c$.

[10] $\varepsilon_{j\ell} = -\varepsilon_{jn}$ is the result of assuming identical utility functions across individuals. In that case, there is a perfect mapping between the conditional cross elasticities. However, this relationship breaks down when there is also heterogeneity in the utility function. See also Mirrlees (1976) and Saez (2002). Then, even by assuming weak separability, non-uniform commodity taxation is optimal for direct redistribution, but not for alleviating labor-market distortions.

Finally, we can derive that the Atkinson and Stiglitz theorem applies under optimal linear commodity taxes with weakly separable preferences. That is, linear commodity taxes are superfluous if utility is weakly separable between commodities and labor, so that $u_n \equiv u(h(c_n, \mathbf{x}_n), \ell_n)$. In this case, first-order conditions for commodity demands in Eq. (16) are independent of $\ell_n$: $h_{x^i}/h_c = q_i,\ \forall i$. Therefore, the conditional commodity demands are functions only of prices $\mathbf{q}$ and net income $y_n$, and not of labor supply $\ell_n$. Hence, $\partial x_n^i/\partial \ell_n = 0$, so that $\varepsilon_{i\ell}^c = 0, \forall\, i$. Substitution of $\varepsilon_{j\ell}^c = 0, \forall\, j$, in Eq. (31) yields $t_j = 0, \forall\, j$.

## 5. Conclusions

This paper analyzed optimal linear commodity taxes combined with non-linear income taxes. We have demonstrated that the optimal second-best allocation derived by Atkinson & Stiglitz (1976, 1980) cannot be implemented with linear commodity taxes and non-linear income taxes. In addition to clarifying that, we have provided a full characterization of the optimal linear commodity and income tax structure in terms of empirically measurable elasticities.

Our results demonstrate that there is a close link between the classical results of Corlett & Hague (1953) and those of Atkinson & Stiglitz (1976, 1980). Indeed, the intuition that goods should be taxed/subsidized if they are more/less complementary with leisure than the untaxed numéraire good fully carries over to the case with optimal non-linear income taxes. Intuitively, commodity taxes are used for efficiency reasons to offset distortions of the income tax on labor supply by boosting labor supply. Hence, analyzing optimal income redistribution with heterogeneous agents does not change the nature of the Corlett–Hague conclusions. Moreover, we have adjusted Diamond's *ABC*-formula for optimal non-linear income taxation to take into account the presence of optimal linear commodity taxes. We have shown that optimal commodity taxes are employed only to reduce the distortions of the income tax, but not to directly redistribute incomes.

Our theoretical results have policy-relevant implications. First, as long as individuals have the same utility function and governments can employ non-linear income taxes, commodity-tax differentiation is desirable only if it boosts labor supply. This implication can readily be tested empirically. However, despite the central importance of the Atkinson–Stiglitz theorem in the optimal-tax literature, it is disappointing that not more evidence is available on its empirical validity. Browning & Meghir (1991) and Crawford et al. (2010) directly estimate the conditional commodity demand functions that are key in our analysis. Gordon & Kopczuk (2013) and Pirttilä & Suoniemi (forthcoming) follow a different route and use commodity demands to predict wage rates, while controlling for disposable income.[11] All these studies reject weak separability, but none of these studies presents robust evidence that (groups of) commodities are strongly associated with labor supply, except, perhaps, for child-care facilities, housing expenditures and capital incomes. More research regarding the complementarity of conditional commodity demand and labor supply is needed, given the importance of differentiated commodity-tax structures in the real world. Our optimal-tax formulae could then readily be applied.

Second, we can recast our model in an intertemporal, life-cycle setting where commodities are consumption levels at different dates. The results would imply that capital-income taxation is desirable only when doing so stimulates labor supply. This is a condition that can also be tested empirically. Estimates by Pirttilä & Suoniemi (forthcoming) indeed suggest that labor supply falls when capital incomes are larger while controlling for income, which implies that capital income should be taxed for efficiency reasons. Recent research in the new dynamic public finance literature suggests that there could be a role for capital-income taxes to alleviate the distortions in labor supply, but that restricting tax codes to zero capital taxes entails only small welfare losses (Fahri & Werning, 2012). More research could be done to explore this issue further, especially by empirically grounding the utility functions that underlie such analyses.

[11] To obtain unbiased estimates of labor supply on conditional commodity demands one requires some exogenous variation in net income as it is typically endogenous. Browning & Meghir (1991) use asset incomes and levels of schooling (as a proxy for wage rates) to instrument net income. Crawford et al. (2010) use education, total household income and demographic variables (mean household age of parents and children, number of children) to instrument net income. Gordon & Kopczuk (2013) and Pirttilä & Suoniemi (forthcoming) do not instrument net income to correct for potential endogeneity problems.

## Appendix A. Derivation of elasticities in Table 1

### A.1. Elasticities of labor and earnings supply

As in Jacquet et al. (2013), define the following *shift function*:

$$\begin{aligned}L(\ell,\mathbf{x},n,\tau,\rho) \equiv\ & n\left(1-T'(n\ell)-\tau\right)\\ &\times u_c\left(n\ell-T(n\ell)-\tau(n\ell-n\ell_n)+\rho-\sum_{i=1,\cdots,I}(1+t_i)x^i,\ell,\mathbf{x}\right)\\ &+u_\ell\left(n\ell-T(n\ell)-\tau(n\ell-n\ell_n)+\rho-\sum_{i=1,\cdots,I}(1+t_i)x^i,\ell,\mathbf{x}\right).\end{aligned} \tag{A.1}$$

$L(\ell, \mathbf{x}, n, \tau, \rho)$ measures a *shift* in the first-order condition for labor supply when one of the variables $\ell, \mathbf{x}, n, \tau$ or $\rho$ changes. $\tau$ captures an exogenous increase in the marginal tax rate (i.e. for any level of earnings). $\rho$ is introduced to retrieve the income effect when the individual receives an exogenous amount of income $\rho$, irrespective of the amount of labor supplied. The first-order condition for labor supply of the individual $n$ is thus equivalent to $L(\ell, \mathbf{x}, n, 0, 0) = 0$.

We find the following partial derivatives, using the first-order condition $-u_\ell = n(1-T')u_c$:

$$L_\ell(\ell,\mathbf{x},n,0,0) = u_{\ell\ell}+\left(\frac{u_\ell}{u_c}\right)^2u_{cc}-2\frac{u_\ell}{u_c}u_{c\ell}+nu_\ell\frac{T''}{1-T'}, \tag{A.2}$$

$$L_n(\ell,\mathbf{x},n,0,0) = \left(\frac{-u_\ell}{\ell}+nu_\ell\frac{T''}{1-T'}+\left(\frac{u_\ell}{u_c}\right)^2u_{cc}-\left(\frac{u_\ell}{u_c}\right)u_{\ell c}\right)\frac{\ell}{n}, \tag{A.3}$$

$$L_\tau(\ell,\mathbf{x},n,0,0) = -nu_c, \tag{A.4}$$

$$L_\rho(\ell,\mathbf{x},n,0,0) = \frac{u_{\ell c}u_c-u_\ell u_{cc}}{u_c}. \tag{A.5}$$

Now, by applying the implicit function theorem, i.e. $\frac{\partial\ell}{\partial a} = -\frac{L_a}{L_\ell}$ for $a = n, \tau, \rho$, and using $z = n\ell$, we obtain the elasticities $\varepsilon_{\ell n}$, $\varepsilon_{zn}$, $\varepsilon_\ell^I$, $\varepsilon_{\ell T'}^*$, and $\varepsilon_{zT'}^*$ in Table 1.

### A.2. Elasticities of commodity demands

Next, define the following *shift function* for the commodity demands:

$$\begin{aligned}X^i(\ell,\mathbf{x},\mathbf{t},n,\rho) \equiv\ & (1+t_i)u_c\left(n\ell-T(n\ell)+\rho-\sum_{i=1,\ldots,I}(1+t_i)x^i,\ell,\mathbf{x}\right)\\ &-u_{x^i}\left(n\ell-T(n\ell)+\rho-\sum_{i=1,\ldots,I}(1+t_i)x^i,\ell,\mathbf{x}\right).\end{aligned} \tag{A.6}$$

$X^i(\ell, \mathbf{x}, \mathbf{t}, n, \rho)$ measures a *shift* in the first-order condition for commodity demand $i$ when one of the variables $\ell$, $\mathbf{x}$, $\mathbf{t}$, $n$, or $\rho$ changes. We find the following partial derivatives, using the first-order condition $u_{x^i} = u_c(1+t_i)$:

$$X^i_{x^i}(\ell, \mathbf{x}, \mathbf{t}, n, 0) = -u_{x^i x^i} - \left(\frac{u_{x^i}}{u_c}\right)^2 u_{cc} + 2\left(\frac{u_{x^i}}{u_c}\right) u_{cx^i}, \tag{A.7}$$

$$X^i_{t_i}(\ell, \mathbf{x}, \mathbf{t}, n, 0) = u_c + x^i\left(u_{x^i c} - \left(\frac{u_{x^i}}{u_c}\right)\right) u_{cc}, \tag{A.8}$$

$$X^i_p(\ell, \mathbf{x}, \mathbf{t}, n, 0) = \frac{u_{x^i} u_{cc} - u_{x^i} u_c}{u_c}. \tag{A.9}$$

Now, by applying the implicit function theorem, i.e. $\frac{\partial x^i}{\partial a} = -\frac{X^i_a}{X^i_{x^i}}$ for $a = t_i$, $\rho$, we obtain the elasticities $\varepsilon^*_{ii}$ and $\varepsilon^I_i$ in Table 1, where we used the Slutsky equation ($\varepsilon^*_{ii} = \varepsilon_{ii} + \varepsilon^I_i$) to derive the compensated tax/price elasticity of commodity demand $\varepsilon^*_{ii}$.

Finally, we evaluate the first-order conditions for $x^i$ and $c$ for a *given* level of net income $\bar{y}$ to obtain the *conditional* commodity demand elasticities with respect to labor supply. First, the shift function is modified to:

$$\begin{aligned} X^i(\ell, \mathbf{x}, \mathbf{t}, n, 0|\bar{y}) \equiv (1+t_i) u_c\left(\bar{y} - \sum_{i=1,\ldots,I}(1+t_i)x^i_n, \ell, \mathbf{x}\right) \\ -u_{x^i}\left(\bar{y} - \sum_{i=1,\ldots,I}(1+t_i)x^i_n, \ell, \mathbf{x}\right). \end{aligned} \tag{A.10}$$

Therefore, we find:

$$X^i_{x^i}(\ell, \mathbf{x}, \mathbf{t}, n, 0|\bar{y}) = -u_{x^i x^i} - \left(\frac{u_{x^i}}{u_c}\right)^2 u_{cc} + 2\left(\frac{u_{x^i}}{u_c}\right) u_{cx^i}, \tag{A.11}$$

$$X^i_\ell(\ell, \mathbf{x}, \mathbf{t}, n, 0|\bar{y}) = \frac{u_{x^i}}{u_c} u_{c\ell} - u_{x^i \ell}. \tag{A.12}$$

Hence, by applying the implicit function theorem, i.e. $\frac{\partial x^i}{\partial \ell} = -\frac{X^i_\ell}{X^i_{x^i}}$, we find the conditional elasticity of commodity demands with respect to labor supply $\varepsilon^c_{i\ell}$ in Table 1. We note, for later reference, that the price elasticities of the unconditional and conditional commodity demand functions are the same.

## Appendix B. Proofs of Propositions 2 and 3

### B.1. Properties of conditional demands, utility and expenditure

Some properties of conditional demand, utility and expenditure functions are useful in the derivations below. Applying the envelope theorem, we obtain Roy's identity for the conditional indirect utility function:

$$v_{q_i}(\mathbf{q}, y_n, \ell_n) = -v_y(\mathbf{q}, y_n, \ell_n) x^{c,i}_n(\mathbf{q}, y_n, \ell_n), \ \forall n, i, \tag{B.1}$$

where $v_y(\mathbf{q}, y_n, \ell_n)$ is the private marginal utility of income and is equal to $u_c(\cdot)$. Differentiating this with respect to $\ell_n$ gives:

$$\begin{aligned} v_{q_i \ell}(\mathbf{q}, y_n, \ell_n) = -v_y \frac{\partial x^{c,i}_n(\mathbf{q}, y_n, \ell_n)}{\partial \ell_n} \\ -v_{y\ell}(\mathbf{q}, y_n, \ell_n) x^{c,i}_n(\mathbf{q}, y_n, \ell_n), \ \forall n, i. \end{aligned} \tag{B.2}$$

The type-$n$ individual's conditional budget constraint can be expressed in terms of conditional compensated demands $c^{c*}_n(\mathbf{q}, \ell_n, v_n)$ and $x^{c*,i}_n(\mathbf{q}, \ell_n, v_n)$:

$$c^{c*}_n(\mathbf{q}, \ell_n, v_n) + \sum_i (1+t_i) x^{c*,i}_n(\mathbf{q}, \ell_n, v_n) = e(\mathbf{q}, \ell_n, v_n), \ \forall n. \tag{B.3}$$

By using the envelope theorem (Shephard's lemma) we obtain the following properties of the expenditure function $e(\mathbf{q}, \ell_n, v_n)$:

$$\begin{aligned} e_{q_i}(\mathbf{q}, \ell_n, v_n) &= x^{c*,i}_n(\mathbf{q}, \ell_n, v_n), \\ e_v(\mathbf{q}, \ell_n, v_n) &= \frac{1}{u_c}, \\ e_\ell(\mathbf{q}, \ell_n, v_n) &= -\frac{u_\ell}{u_c}, \ \forall n, i. \end{aligned} \tag{B.4}$$

Differentiating Eq. (B.3) with respect to prices $q_j$ (recall $q_j \equiv 1 + t_j$), labor $\ell_n$, and utility $v_n$ and using the envelope properties of the expenditure function (B.4) yields:

$$\frac{\partial c^{c*}_n}{\partial q_j} + \sum_i (1+t_i) \frac{\partial x^{c*,i}_n}{\partial q_j} = 0, \ \forall n, j, \tag{B.5}$$

$$\frac{\partial c^{c*}_n}{\partial \ell_n} + \sum_i (1+t_i) \frac{\partial x^{c*,i}_n}{\partial \ell_n} = -\frac{u_\ell}{u_c}, \ \forall n, \tag{B.6}$$

$$\frac{\partial c^{c*}_n}{\partial v_n} + \sum_i (1+t_i) \frac{\partial x^{c*,i}_n}{\partial v_n} = e_v = \frac{1}{u_c}, \ \forall n. \tag{B.7}$$

Differentiating Eq. (B.3) with respect to $v_n$ and $\ell_n$, and using the envelope properties (B.4) gives:

$$\frac{\partial x^{c*,i}_n}{\partial v_n} = \frac{1}{u_c} \frac{\partial x^{c,i}_n}{\partial y_n}, \ \forall n, i \tag{B.8}$$

$$\frac{\partial x^{c*,i}_n}{\partial \ell_n} = \frac{\partial x^{c,i}_n}{\partial \ell_n} + \frac{u_\ell}{u_c} \frac{\partial x^{c,i}_n}{\partial y_n}, \ \forall n, i. \tag{B.9}$$

### B.2. Rewriting the first-order condition for $\ell_n$

Substitute Eqs. (B.6) and (18) into Eq. (21) to find:

$$nT'(n\ell_n) + n\sum_i \frac{t_i}{1+t_i} \frac{(1+t_i)x^i_n}{n\ell_n} \frac{\partial x^{c*,i}_n}{\partial \ell_n} \frac{\ell_n}{x^i_n} = -\frac{\theta_n/\eta}{nf(n)} (v_\ell + \ell_n v_{\ell\ell} + \ell_n v_{\ell e} e_\ell). \tag{B.10}$$

Use $\varepsilon^c_{i\ell} = \varepsilon^{c*}_{i\ell}$ since compensated and uncompensated conditional demand functions are identical. Further, note that $v_\ell = u_\ell$ and $v_y = u_c$. Use these results to find:

$$\frac{T'(z_n)}{1-T'(z_n)} + \frac{1}{\sigma_n} \sum_i \frac{t_i}{1+t_i} \gamma^i_n \varepsilon^c_{i\ell} = \left(1 + \frac{\ell_n v_{\ell\ell}}{v_\ell} + \frac{\ell_n v_{\ell e} e_\ell}{v_\ell}\right) \frac{u_c \theta_n/\eta}{f(n)n}. \tag{B.11}$$

Next, recall that $v_\ell = u_\ell$ and use $e_\ell = -\frac{u_\ell}{u_c}$ from Eq. (B.4). Note that net income equals expenditure on commodities ($y_n = e_n$), hence $v_{\ell e} = v_{\ell y} = u_{c\ell}$. By using the definitions for the elasticities $\varepsilon_{\ell n}$, $\varepsilon_{zn}$, and

$\varepsilon_{zT'}$ from Table 1 we find $1+\frac{\ell_n v_{\ell\ell}}{v_\ell}+\frac{\ell_n v_{\ell e} e_\ell}{v_\ell}=1+\frac{\ell_n u_{\ell\ell}}{u_\ell}-\frac{\ell_n u_{c\ell}}{u_c}=\frac{1+\varepsilon_{\ell n}}{\varepsilon^*_{\ell T'}}=\frac{\varepsilon_{zn}}{\varepsilon^*_{zT'}}$. This gives the optimal income tax:

$$\mathcal{W}_n \equiv \frac{T'(z_n)}{1-T'(z_n)}+\frac{1}{\sigma_n}\sum_i \frac{t_i}{1+t_i}\gamma_n^i \varepsilon^c_{i\ell}=\frac{(1+\varepsilon_{\ell n})}{\varepsilon^*_{\ell T'}}\frac{u_c\theta_n/\eta}{f(n)n}=\frac{\varepsilon_{zn}}{\varepsilon^*_{zT'}}\frac{u_c\theta_n/\eta}{nf(n)}. \tag{B.12}$$

*B.3. Rewriting the first-order condition for $q_j$*

Substitute Eqs. (B.5) and (B.2) in Eq. (22) and note that $v_{y\ell}=v_{\ell e}$, $x_n^{c*,j}=e_{q_j}$, and $v_y=u_c$. Then, rewrite the first-order condition for $q_j$ as:

$$\int_{\mathcal{N}}\left[\sum_i t_i \frac{\partial x_n^{c*,i}}{\partial q_j}-\frac{u_c\theta_n/\eta}{nf(n)}\frac{\partial x_n^{c,j}}{\partial \ell_n}\frac{\ell_n}{x_n^j}x_n^j\right]f(n)dn=0. \tag{B.13}$$

Use the symmetry of the Slutsky matrix $\left(\frac{\partial x_n^{c*,i}}{\partial q_j}=\frac{\partial x_n^{c*,j}}{\partial q_i}\right)$ and rewrite the expression so that the optimal commodity taxes satisfy:

$$\sum_i \frac{t_i}{1+t_i}\overline{\varepsilon^*_{ji}}=-\overline{\left(\frac{u_c\theta_n/\eta}{nf(n)}\varepsilon^c_{j\ell}\right)}, \tag{B.14}$$

where we used the fact that the elasticities of conditional and unconditional commodity demands with respect to commodity prices are the same.

*B.4. Composite multiplier $\Theta_n$*

Introduce a composite multiplier on the incentive-compatibility constraint $\Theta_n \equiv \frac{v_y\theta_n}{\eta}$, and substitute $\ell_n \equiv z_n/n$:

$$\Theta_n \equiv \frac{v_y(\mathbf{q},y_n,\ell_n)\theta_n}{\eta}=\frac{v_y(\mathbf{q},y_n,z_n/n)\theta_n}{\eta}. \tag{B.15}$$

This composite multiplier has a total derivative:

$$\dot{\Theta}_n=\dot{\theta}_n\frac{v_y}{\eta}-\frac{\theta_n}{\eta}\frac{v_{y\ell}\ell_n}{n}+\frac{\theta_n}{\eta}u_{yy}\dot{y}_n+\frac{\theta_n}{\eta}\frac{v_{y\ell}}{n}\dot{z}_n. \tag{B.16}$$

Totally differentiate the household budget constraint $y_n \equiv z_n - T(z_n)$ and use the first-order conditions (3) to find:

$$\dot{y}_n=(1-T'(z_n))\dot{z}_n=-\frac{v_\ell}{nv_y}\dot{z}_n. \tag{B.17}$$

Substitute this result into Eq. (B.16), use $v_y=u_c$ and $v_{\ell y}=u_{c\ell}$ so that we find:

$$\dot{\Theta}_n=\dot{\theta}_n\frac{v_y}{\eta}-\frac{\theta_n}{\eta}\frac{v_{y\ell}\ell_n}{n}+\frac{\theta_n}{\eta n}\left(u_{c\ell}-\frac{u_{cc}u_\ell}{u_c}\right)\dot{z}_n. \tag{B.18}$$

Next, use the income elasticity of labor supply $\varepsilon^I_\ell$ in Table 1 to find an expression for $u_{c\ell}-\frac{u_{cc}u_\ell}{u_c}$:

$$u_{c\ell}-\frac{u_\ell}{u_c}u_{cc}=\frac{\varepsilon^I_\ell}{\frac{u_\ell}{u_c}}\left[u_{\ell\ell}+\left(\frac{u_\ell}{u_c}\right)^2u_{cc}-2\left(\frac{u_\ell}{u_c}\right)u_{c\ell}+nu_\ell\frac{T''}{1-T'}\right]. \tag{B.19}$$

Use the elasticity for $\varepsilon^*_{\ell T'}$ in Table 1 to find an expression for the term in brackets:

$$\left[u_{\ell\ell}+\left(\frac{u_\ell}{u_c}\right)^2u_{cc}-2\left(\frac{u_\ell}{u_c}\right)u_{c\ell}+nu_\ell\frac{T''}{1-T'}\right]=\frac{u_\ell/\ell}{\varepsilon^*_{\ell T'}}. \tag{B.20}$$

Hence, we derive

$$u_{\ell c}-\frac{u_\ell u_{cc}}{u_c}=\frac{\varepsilon^I_\ell}{n(1-T')}\frac{-u_\ell}{\ell\varepsilon^*_{\ell T'}}. \tag{B.21}$$

Thus, substituting Eq. (B.21) into Eq. (B.18) results in:

$$\dot{\Theta}_n=\dot{\theta}_n\frac{v_y}{\eta}-\frac{\theta_n}{\eta}\frac{v_{y\ell}\ell_n}{n}+\frac{\varepsilon^I_\ell}{(1-T')}\frac{-u_\ell}{n}\frac{\theta_n}{\eta n}\frac{\varepsilon_{zn}}{\varepsilon^*_{\ell T'}}. \tag{B.22}$$

Use the first-order condition for $\ell_n$ (B.12) to derive:

$$(1-T'(z_n))\mathcal{W}_n f(n)=\frac{-u_\ell}{n}\frac{\theta_n}{n\eta}\frac{\varepsilon_{zn}}{\varepsilon^*_{zT'}}. \tag{B.23}$$

Substituting Eq. (B.21) in Eq. (B.18) yields:

$$\dot{\Theta}_n=\dot{\theta}_n\frac{v_y}{\eta}-\frac{\theta_n}{\eta}\frac{v_{y\ell}\ell_n}{n}+\varepsilon^I_\ell\mathcal{W}_n f(n). \tag{B.24}$$

*B.5. Rewriting the first-order condition for $u_n$*

The first-order condition for the level of utility $u_n$ in Eq. (23) can be rewritten as:

$$\left(\Psi'-\eta\left(\frac{\partial c_n^{c*}}{\partial v_n}+\sum_i\frac{\partial x_n^{c*,i}}{\partial v_n}\right)\right)f(n)+\frac{\theta_n\ell_n v_{\ell e}e_u}{n}=\dot{\theta}_n. \tag{B.25}$$

From Eqs. (B.7), (B.8), and (B.9), it follows that

$$\frac{\partial c_n^{c*}}{\partial v_n}+\sum_i\frac{\partial x_n^{c*,i}}{\partial v_n}=\frac{1}{u_c}-\sum_i t_i\frac{\partial x_n^{c*,i}}{\partial y_n}\frac{1}{u_c}. \tag{B.26}$$

Using this result in Eq. (B.25), and noting that $e_u=1/v_y$, $y_n=e(\cdot)$, and $\frac{\partial x_n^{c*,i}}{\partial y_n}=\frac{\partial x_n^{c,i}}{\partial y_n}$ we find:

$$\left(\frac{\Psi' u_c}{\eta}-1+\sum_i t_i\frac{\partial x_n^{c,i}}{\partial y_n}\right)f(n)+\frac{v_y\theta_n/\eta}{n}\frac{\ell_n v_{\ell y}}{v_y}=\frac{v_y}{\eta}\dot{\theta}_n. \tag{B.27}$$

In order to retrieve the traditional expression for the optimal income tax, rewrite the latter expression to find:

$$\dot{\theta}_n=\left(\Psi'-\eta\left(1-\sum_i t_i\frac{\partial x_n^{c,i}}{\partial y_n}\right)\right)f(n)+\frac{\theta_n\ell_n v_{\ell y}}{nv_y}. \tag{B.28}$$

Next, substitute $\ell v_{\ell y}/v_y=\partial \ln u_c/\partial\ln\ell$, integrate by parts, and use Eq. (24) to find:

$$\frac{\theta_n}{\eta}=\int_n^{\overline{n}}\frac{1}{u_{c,m}}\left(1-\frac{\Psi' u_{c,m}}{\eta}-\sum_i t_i\frac{\partial x_m^{c,i}}{\partial y_m}\right)\exp\left[-\int_n^m\left(\frac{\partial\ln u_c}{\partial\ln\ell_s}\right)\frac{ds}{s}\right]f(m)dm. \tag{B.29}$$

Finally, use the equivalence $\frac{\partial x^i_m}{\partial\rho}=\frac{\partial x^{c,i}_m}{\partial y_m}$, and the income elasticity of commodity demands $\varepsilon^I_i$ from Table 1, to retrieve the optimal income tax expression.

To obtain the alternative representation for the optimal income tax, substitute Eq. (B.24), use the equivalence $\frac{\partial x^i_m}{\partial\rho}=\frac{\partial x^{c,i}_m}{\partial y_m}$, and the income elasticity of commodity demands $\varepsilon^I_i$ from Table 1, to rewrite the first-order condition for $u_n$ as:

$$\dot{\Theta}_n=-\left(1-\frac{\Psi' u_{c,n}}{\eta}-\mathcal{W}_n\varepsilon^I_\ell-\sum_i\frac{t_i}{1+t_i}\varepsilon^I_i\right)f(n). \tag{B.30}$$

Integrating the last expression, while using a transversality condition from Eq. (24), yields

$$\Theta_n = \frac{u_c \theta_n}{\eta} = \int_n^{\overline{n}} \left( 1 - \frac{\Psi'(u_m) u_{c,m}}{\eta} - \mathcal{W}_m \varepsilon_\ell^I - \sum_i \frac{t_i}{1+t_i} \varepsilon_i^I \right) f(m) dm. \tag{B.31}$$

*B.6. Optimal taxes*

Finally, to find the traditional formulation of the income tax, substitute Eq. (B.29) in Eq. (B.12) and rearrange to find Eq. (25). The Saez-formulation is obtained by writing the optimal tax formula in terms of earnings densities. Given that $F(n) \equiv \widetilde{F}(z_n)$, we have $nf(n) = \varepsilon_{zn} z_n \widetilde{f}(z_n)$ and $\int_n^{\overline{n}} f(n) \mathrm{d}n = \int_{z_n}^{z_{\overline{n}}} \widetilde{f}(z_n) \mathrm{d}z_n$. Consequently, we can derive that $\int_n^{\overline{n}} a_n f(n) \mathrm{d}n = \int_{z_n}^{z_{\overline{n}}} a_n \widetilde{f}(z_n) \mathrm{d}z_n$ for any variable $a_n$. Using these results and substituting Eq. (B.13) in Eq. (B.12) yields Eq. (25). Finally, use the expression for the optimal income tax (B.12) in Eq. (B.14) to establish Eq. (31).

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