Optimal Taxation of Human Capital and the Earnings Function

BAS JACOBS

Erasmus University Rotterdam

A. LANS BOVENBERG

Tilburg University

Abstract

This paper explores how the specification of the earnings function impacts optimal nonlinear taxes on human capital under optimal nonlinear income taxation. If education is complementary to labor effort, education should be subsidized to offset tax distortions on labor supply. However, if education is complementary to ability, education should be taxed in order to redistribute income. If education is weakly separable from labor and ability in the earnings function, these two effects cancel and education should be neither taxed nor subsidized.

1. Introduction

Should education be taxed or subsidized for redistributive reasons? Education subsidies are generally regressive due to the well-known ability bias in education (Card 1999, Heckman, Lochner, and Todd 2006). In the absence of other instruments aimed at redistribution, the government may thus want to

tax education to redistribute resources from high-ability to low-ability agents (see, e.g., Bovenberg and Jacobs 2005). However, if the government has also access to income taxation to redistribute incomes, it may optimally employ income taxes rather than education taxes to redistribute income. In fact, in such a setting, education subsidies may help to alleviate income-tax distortions on labor supply by raising after-tax wages. The government therefore faces a trade-off between equity and efficiency in setting educational policies. Education taxes directly redistribute resources towards low-ability agents, but at the same time worsen the labor-market distortions imposed by distortionary income taxes.

Bovenberg and Jacobs (2005) analytically characterized optimal nonlinear income tax and education policies by extending the Mirrlees (1971) optimal nonlinear income tax model with endogenous skill formation and verifiable human capital investments. They demonstrated that education is neither taxed nor subsidized if the government can optimize both income taxes and educational policy.[1] Education policies are thus used neither to redistribute directly nor to stimulate labor supply. However, Bovenberg and Jacobs (2005) did not note that the zero-tax result depends crucially on the assumed earnings function. In particular, they employed a log-additive earnings function, which implies that the elasticity of earnings with respect to education depends neither on labor supply nor ability. Maldonado (2008) subsequently showed that the zero education-tax result does not necessarily hold in the presence of more general earnings functions in which education and ability do not enter in a log-additive form so that the elasticity of earnings with respect to education may depend on ability. He demonstrated that education should be taxed (subsidized) if the education elasticity of earnings rises (falls) with ability. Intuitively, if education is more complementary to ability, taxes on education become a more (less) attractive proxy for taxing nonverifiable abilities.

This paper generalizes the earnings function further by allowing for a more flexible relationship between earnings and labor supply. Indeed, our earnings function does not impose any restrictions on the way in which ability, labor effort, and education determine labor earnings. Hence, the elasticity of earnings with respect to education may depend not only on ability but also on labor supply. Empirical evidence suggests that the elasticity of earnings with respect to education may indeed depend on labor supply as part-time work tends to pay lower wages than full-time work, which may be due to learning-by-doing effects.

Adopting a general earnings function, we identify two main factors determining the sign of the optimal education tax. In particular, we find that stronger complementarity between education and ability in the earnings

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[1] With linear tax instruments, this result continues to hold as long as the earnings function exhibits a constant elasticity in education (Jacobs and Bovenberg 2008).

function makes education taxes a more attractive instrument for redistributing resources to low-ability households. This is in line with the findings of Maldonado (2008). In addition, we extend Maldonado (2008) by identifying the degree of complementarity between labor supply and education as the second main factor determining optimal education policies. In particular, the case for education subsidies is strengthened if education and labor supply are stronger complements in the earnings function, so that education subsidies are a more powerful instrument for offsetting labor-supply distortions. This new element reveals the key trade-off in setting optimal education policy. If the elasticity of earnings with respect to education depends more strongly on labor supply than on ability, education is a stronger (weaker) complement to labor supply than to ability. Hence, education should be subsidized (taxed) on a net basis. The reason is that the efficiency gains of lower labor-supply distortions are larger (smaller) than the distributional losses resulting from their regressive incidence.

By allowing for a more general earnings function, we generalize the necessary and sufficient conditions on the earnings function derived by Maldonado (2008) for an optimal zero tax on human capital. In particular, whereas Maldonado (2008) stresses that the elasticity of earnings with respect to education should be independent of ability, we find that this elasticity should depend in the same way on labor supply as on ability. In particular, the change in the elasticity as a result of additional earnings produced by a higher ability level should exactly match the change in the elasticity as a result of additional earnings due to larger labor supply. This condition can be tested empirically.

Our paper is related to Ulph (1977) and Hare and Ulph (1979). They were not able to obtain clear-cut interpretations of optimal tax and education policies for earnings functions similar to those adopted by Maldonado (2008). Our paper, in contrast, provides intuitive characterizations of optimal nonlinear tax and education policies for general specifications of earnings functions. Our findings are also relevant for the "new public finance" literature that analyzes optimal income insurance rather than income redistribution. For example, Grochulski and Piskorsi (2006), da Costa and Maestri (2007), and Anderberg (2009) adopt a similar earnings function as Maldonado (2008), but concentrate mainly on the desirability of capital income and wealth taxes. In particular, Grochulski and Piskorsi (2006) do not optimize education policy, since education is assumed to be nonverifiable, whereas da Costa and Maestri (2007) do not explicitly elaborate on the role of education policy to alleviate the distortions of the labor income tax. Anderberg (2009) strengthens our findings by demonstrating that the earnings function is critical for the design of optimal education policies in risky environments with *ex ante* homogeneous individuals. Indeed, by adopting the same earnings function as Maldonado (2008), he finds that efficiency in human capital investments requires that the earnings function should be weakly separable between (stochastic) ability and education. Our paper helps to gain a deeper understanding about the interaction between

optimal tax and education policies in the presence of these and more complex earnings functions adopted in this literature.

The rest of this paper is structured as follows. Section 2 formalizes the model. Subsequently, Section 3 explores optimal education taxes for nonlinear instruments. Section 4 concludes.

2. The Model

This section briefly summarizes the main features of the model, which extends Bovenberg and Jacobs (2005) with a general earnings function. $n \in [\underline{n}, \overline{n}]$ is individual ability, $f(n)$ is the density of ability, and $F(n)$ is the cumulative distribution of ability. e_n denotes investment in education, l_n represents labor effort, c_n is consumption, and z_n stands for gross labor earnings.

Households exhibit identical utility functions and derive utility from consumption c_n and suffer disutility from work effort l_n:

$$u_n \equiv u(c_n, l_n), \ u_c > 0, \ u_l < 0, \ u_{cc}, \ u_{ll} < 0, \qquad (1)$$

where subscripts refer to an argument of differentiation (except where it signifies ability n). This specification generalizes the separable utility function in Bovenberg and Jacobs (2005).

In addition, we specify a general earnings function. In particular, gross earnings z_n are a function $\Phi(.)$ of ability n, education e_n, and labor effort l_n:

$$z_n \equiv \Phi(n, l_n, e_n), \ \Phi_n, \Phi_l, \Phi_e > 0, \ \Phi_{ee} < 0, \ \Phi_{ne}, \Phi_{nl} \geqslant 0. \qquad (2)$$

Ability, education, and labor effort increase earnings. Marginal returns to education diminish with the level of education, which ensures an interior solution for human capital investment. More able workers feature a (weakly) higher marginal return to both work and education effort. These latter restrictions ensure single crossing of the utility functions under nonlinear policies.

Mirrlees (1971) and Maldonado (2008) assume that the effect of work effort on earnings does not depend on the level of labor supply (i.e., $\Phi_{ll} = 0$). We allow for a more general specification. On the one hand, the first hours of work may be more productive than the last hours of work on a day, a month or a year, so that the marginal productivity of labor may decline with hours worked. On the other hand, learning by doing may cause labor productivity to rise with work effort (Heckman, Lochner, and Cossa 2002). Indeed, the empirical literature finds a part-time wage penalty (see, e.g., Manning and Petrongolo 2008, Gregory and Connolly 2008, Connolly and Gregory 2009). This part-time penalty may be due to learning-by-doing effects and missed career opportunities (see Russo and Hassink 2008).

The government can verify both gross labor incomes and educational expenditures at the individual level.[2] Accordingly, the government can levy a

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[2] Bovenberg and Jacobs (2005) also explore the effects of nonverifiable investments—besides verifiable investments—for the setting of the optimal net tax on education.

nonlinear income tax $T(z_n)$ on gross incomes $z_n \equiv \Phi(n, l_n, e_n)$. The marginal income tax rate is $T'(z_n) \equiv dT(z_n)/dz_n$. Furthermore, the government employs a nonlinear subsidy on resources e_n invested in education. The subsidy is denoted as $S(e_n)$, where $S'(e_n) \equiv dS(e_n)/de_n$ represents the marginal subsidy rate on e_n.

For notational convenience, education requires only resources and the unit cost of education is normalized to one. If education would require also forgone labor time, the results would continue to hold as long as both time and resources invested in education are verifiable, and can therefore be subsidized (see Bovenberg and Jacobs 2005). The household budget constraint can thus be written as

$$c_n = \Phi(n, l_n, e_n) - T(\Phi(n, l_n, e_n)) - e_n + S(e_n). \tag{3}$$

Utility maximization yields the first-order conditions for the optimal choices of educational investment and labor supply[3]

$$(1 - T'(z_m))\Phi_e(n, l_n, e_n) = 1 - S'(e_n), \tag{4}$$

$$\frac{-u_l(c_n, l_n)}{u_c(c_n, l_n)} = (1 - T'(z_n))\Phi_l(n, l_n, e_n). \tag{5}$$

Expression (4) reveals that the net marginal returns to education (the left-hand side) should be equal to net marginal costs (the right-hand side); taxes reduce net returns while subsidies reduce costs. Equation (5) indicates that the marginal rate of substitution in utility between leisure and consumption should equal the net real wage, which is reduced by the marginal tax rate on earnings.

Incentive compatibility requires that each individual n prefers the bundle $\{c_n, z_n, e_n\}$ over the bundles $\{c_m, z_m, e_m\}$ intended for all other individuals m:

$$U(c_n, z_n, e_n, n) \geq U(c_m, z_m, e_m, n), \quad \forall m \in [\underline{n}, \overline{n}], \forall n \in [\underline{n}, \overline{n}], \tag{6}$$

where $U(c_n, z_n, e_n, n) \equiv u(c_n, \vartheta(n, z_n, e_n)) = u(c_n, l_n)$. The function $l_n \equiv \vartheta(n, z_n, e_n)$ is derived by inverting the gross earnings function $z_n \equiv \Phi(n, l_n, e_n)$, so that its derivatives are given by $\vartheta_n = -\frac{\Phi_n}{\Phi_l} < 0$, $\vartheta_z = \frac{1}{\Phi_l} > 0$, and $\vartheta_e = -\frac{\Phi_e}{\Phi_l} < 0$.

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[3] We assume that second-order conditions for a maximum are satisfied. However, a positive feedback between working and learning may violate these second-order conditions. In particular, higher levels of human capital investment raise wages, which provides stronger incentives to supply labor. This, in turn, boosts the returns to learning, so that investments in human capital expand. To ensure that the feedback between learning and working dampens out so that an interior solution is obtained, we assume that the utility function exhibits sufficiently increasing disutility of labor and the earnings function features sufficiently decreasing returns to education. See also Bovenberg and Jacobs (2005).

These global incentive-compatibility constraints can be replaced by the (first-order) incentive-compatibility constraint (see, e.g., Mirrlees 1971)[4]

$$\frac{du_n}{dn} = u_l(c_n, l_n)\vartheta_n(n, l_n, e_n). \tag{7}$$

The government maximizes the following social welfare function, which is concave in individual utilities:

$$\int_{\underline{n}}^{\overline{n}} \Psi(u_n)\,dF(n), \quad \Psi'(u_n) > 0, \quad \Psi''(u_n) \le 0, \tag{8}$$

subject to the incentive compatibility constraint (Equation (7)) and the economy's resource constraint[5]

$$\int_{\underline{n}}^{\overline{n}} (\Phi(n, l_n, e_n) - e_n - c_n)\,dF(n) = E, \tag{9}$$

where E represents the exogenous government revenue requirement.

3. Optimal Policies

We solve for the optimal allocation by applying the maximum principle and setting up a Hamiltonian $\mathcal{H}$, with l_n and e_n as control variables, u_n as state variable, and θ_n as costate variable (which is defined negatively) for the incentive-compatibility constraint (Equation (7)):

$$\max_{\{l_n, e_n, u_n\}} \mathcal{H} = \Psi(u_n)f(n) - \theta_n u_l(c_n, l_n)\vartheta_n(n, l_n, e_n)$$

$$+ \lambda(\Phi(n, l_n, e_n) - e_n - c_n - E)\,f(n), \tag{10}$$

where λ stands for the shadow value of the resource constraint.[6] The Appendix derives the optimal nonlinear income tax, which essentially reproduces the expression derived by Mirrlees (1971).

The optimal net tax on education—when the income tax is optimally set—follows from the first-order condition for e_n, and is given by (see the Appendix)

$$\frac{(T'(z_n) - S'(e_n))}{(1 - T'(z_n))(1 - S'(e_n))} = \frac{u_c \theta_n/\lambda}{nf(n)} \frac{\omega_n}{\omega_e} \frac{\partial \ln(\Phi_n/\Phi_l)}{\partial \ln e_n} = \frac{u_c \theta_n/\lambda}{nf(n)} \omega_n (\rho_{ne} - \rho_{le}),$$

$$\tag{11}$$

[4] We assume that the first-order approach is valid and that no bunching occurs due to either binding nonnegativity constraints or the violation of monotonicity conditions.

[5] If all individuals respect their budget constraints, and the economy's resource constraint is met, the government budget constraint is automatically satisfied by Walras' law.

[6] The transversality conditions for this control problem are as follows: $\lim_{n \to \overline{n}} \theta_n = 0$ and $\lim_{n \to \underline{n}} \theta_n = 0$.

where $\omega_n \equiv \frac{\Phi_n n}{\Phi}$ and $\omega_e \equiv \frac{\Phi_e e}{\Phi}$ denote the shares in gross earnings of, respectively, ability and education. $u_c \theta_n / \lambda$ denotes the marginal value—expressed in monetary units—of redistributing one unit of income from individuals with ability larger than n to individuals with ability smaller than n. The Appendix provides the solution of θ_n / λ. The more valuable redistribution is, the higher will be the net tax (or subsidy) on education (*ceteris paribus*). $\rho_{ne} \equiv \frac{\Phi_{ne} \Phi}{\Phi_n \Phi_e}$ represents Hicks's (1963, 1970) partial elasticity of complementarity between ability and education. The partial elasticity of complementarity measures the extent to which ability and education are gross complements in generating earnings (Bertoletti 2005). Similarly, $\rho_{le} \equiv \frac{\Phi_{le} \Phi}{\Phi_l \Phi_e}$ stands for Hicks's partial elasticity of complementarity between labor and education.[7]

To interpret Equation (11), we consider some hypothetical limiting cases. In the limiting case in which education (almost) does not affect the wage rate per hour worked (i.e., $\Phi_{le} \downarrow 0$, so that $\rho_{le} \downarrow 0$), education should be taxed on a net basis for redistributive reasons as long as education raises the additional earnings from ability (i.e., $\Phi_{ne} > 0$, so that $\rho_{ne} > 0$). If Φ_{ne} becomes larger, investments in education result in more substantial rents from ability, and optimal net taxes on education should be larger in order to combat inequality (*ceteris paribus*). In this hypothetical limiting case, ability but not labor supply augments the productivity of education. An example of such an earnings function would be $\Phi(n, l_n, e_n) = n(\phi(e_n) + l_n)$.

Another limiting case involves ability not affecting the productivity of education (i.e., $\Phi_{ne} \downarrow 0$, so that $\rho_{ne} \downarrow 0$). In this hypothetical limiting case, education boosts the productivity of only labor supply, but not the rents from ability. In this case, education should be subsidized as long as education raises the additional earnings from labor supply (i.e., $\Phi_{le} > 0$ and $\rho_{le} > 0$). By raising the marginal reward of labor supply, an education subsidy offsets the distortionary impact of a redistributive labor tax on labor supply. More educated workers typically feature higher participation rates, work more, and retire later than less educated agents do, which suggests that education and labor effort are complementary (so that $\Phi_{le} > 0$ and $\rho_{le} > 0$). An example of such an earnings function is $(n, l_n, e_n) = (n + \varphi(e_n)) l_n$.

In the general case both labor supply and ability raise the marginal productivity of education. Therefore, education subsidies can be either positive or negative depending on whether education generates more effects on the reward to labor supply than on the rents of ability. Positive net subsidies on education are optimal if the efficiency gains of education subsidies brought about by boosting labor supply dominate the regressive distributional impact of education subsidies, and *vice versa*. Which of these two factors determining optimal education policy is dominant remains an open empirical question.

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[7] For classical contributions on the elasticity of complementarity and how it relates to the elasticity of substitution, see Hicks (1963, 1970) and Samuelson (1947, 1973). More recent contributions include Broer (2004), Bertoletti (2005), and Blackorby, Priment, and Russell (2007).

As regards the complementarity between education and ability, education subsidies are regressive, in view of the well-documented ability bias in education (Card 1999, Heckman *et al.* 2006). At the same time, education appears to be complementary to labor supply, as work effort and delayed retirement boost the incentives for human capital investments (initial education and OJT) (Heckman and Jacobs 2011). Also, empirical evidence indicates that education and labor effort are complements, since better-educated workers exhibit larger participation rates, retire later, and work more hours (OECD 2006). However, the observed correlation between labor supply and education may not be causal. A higher ability may boost both education and labor supply. However, the empirical literature on education (e.g., Card 1999, Heckman *et al.* 2006) shows that the returns to education are substantial even if one corrects for ability bias. At the same time, labor supply does respond to exogenous variations in wages. The review of estimates in Blundell and McCurdy (1999) and the meta-analysis of Evers, de Mooij, and van Vuuren (2008) show that uncompensated wage elasticities of labor supply are small but positive for men (around 0.1), but larger for women (around 0.5). Accordingly, the substitution effect of higher wages dominates the income effect. With education raising wages and higher wages stimulating labor supply, education causally raises labor supply.[8]

For education policies not to be employed in an optimal redistributive program, the incentive compatibility constraint (see Equation (7)) reveals that $-\vartheta_n = \frac{\Phi_n(n, l_n, e_n)}{\Phi_l(n, l_n, e_n)}$ should not depend on education (i.e., $\frac{\partial \ln(\Phi_n/\Phi_l)}{\partial \ln e_n} = 0$). This condition implies that the earnings function should exhibit the following weakly separable form:

$$\Phi(n, l_n, e_n) \equiv \phi(\psi(n, l_n), e_n). \tag{12}$$

With this earnings function, education policies do not affect the incentive compatibility constraint. Hence, the benefits of education subsidies in terms of fewer labor supply distortions exactly offset the distributional losses on account of the regressive incidence of education subsidies. Bovenberg and Jacobs (2005) adopt a special case of Equation (12), namely, $\Phi(n, l_n, e_n) \equiv nl_n\phi(e_n)$, where $\rho_{ne} = \rho_{le} = 1$. Accordingly, the efficiency gains from education subsidies in terms of larger labor supply exactly offset the equity losses due to regressive education subsidies. Consequently, education policies ensure efficiency in education choices. The public finance literature on

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[8] The observed empirical correlation between labor and education is an indication of their complementarity in the earnings function. To see this, suppose that there would be no complementarity in the earnings function. In that case, returns to education would be independent from labor supply, see Equation (4). Conditional upon ability (which presumably correlates both with labor and education), educational investments would be identical for all levels of labor supply. Thus, after controlling for ability biases, there would be no correlation between learning and working, irrespective of the structure of preferences.

education and taxation typically adopts earnings functions that satisfy Equation (12).[9]

The result of a zero net tax on human capital is in similar spirit as the optimal zero commodity result by Atkinson and Stiglitz (1976) (see also Gauthier and Laroque 2009). However, our finding stresses the trade-off between the direct gain of net education taxes in reducing inequality across abilities and the loss of these taxes in exacerbating tax distortions on labor supply. The intuition for the Atkinson and Stiglitz (1976) theorem relies exclusively on the question whether distorting commodity markets helps to reduce the distortions in the labor market, whereas distortions in commodity markets are not introduced for redistributional reasons.[10]

Bovenberg and Jacobs (2005) invoked the Diamond and Mirrlees (1971) production efficiency theorem to explain their findings. If education is absent in the incentive compatibility constraints, consumption and investment choices can be separated. Therefore, all investments should be efficient, whereas distortions arising from redistribution involve only the consumption choices of households. However, in the absence of weak separability in the earnings function, incentive constraints do depend on human capital investments. Net taxes or subsidies on education should be employed to relax the incentive constraints and thereby help to improve the equity-efficiency trade-off. Since investment choices can no longer be separated from consumption choices, the production efficiency theorem breaks down.

Maldonado (2008) adopts a special case of our general earnings function in which education and ability are weakly separable from labor effort: $\Phi(n, l_n, e_n) \equiv \phi(n, e_n) l_n$. With this particular earnings function, we have $\rho_{ne} = \frac{\phi_{ne}\phi}{\phi_n\phi_e} \neq 1$ and $\rho_{le} = 1$, so that education is taxed (subsidized) on a net basis if $\rho_{ne} = \frac{\phi_{ne}\phi}{\phi_n\phi_e} > (<)1$.[11] In this case, the elasticity of complementarity between education and labor supply is fixed at unity. Thus, whether education should be subsidized or taxed depends on whether the elasticity of complementarity between ability and education is smaller or larger than unity.

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[9] See, for example, Nielsen and Sørensen (1997), Brett and Weymark (2003), Wigger (2004), Jacobs (2005, 2007), Blumkin and Sadka (2008), Bohacek and Kapicka (2008), Richter (2009), Jacobs, Schindler, and Yang (Forthcoming), Jacobs and Schindler (2009), Jacobs and Bovenberg (2010), and others. Separability of human capital and labor in labor earnings is also adopted in classical papers on life-cycle models with education; see, for example, Heckman (1976), Kotlikoff and Summers (1979), Eaton and Rosen (1980); or in modern articles on growth with endogenous human capital; see, for example, Jones, Manuelli, and Rossi (1993, 1997), Trostel (1993), Judd (1999), and Hendricks (1999).

[10] As long as preferences are identical for all individuals, distortions in commodity markets are not introduced for redistributional reasons if preferences are weakly separable. Saez (2002) has shown that the Atkinson and Stiglitz (1976) no-commodity tax-differentiation result disappears with heterogeneous, weakly separable preferences.

[11] Anderberg (2009) derives a similar condition on the earnings function in a model with earnings risk, where *ex ante* identical individuals have stochastic abilities n, so that individuals differ *ex post*. The condition for a positive, zero or negative tax on human capital is also that $\rho_{ne} \gtreqless 1$, given that the earnings function is the same as in Maldonado (2008).

With this particular earnings function, Maldonado (2008) demonstrated that a zero optimal tax on education requires that the education elasticity of earnings,

$$\eta(n, l_n, e_n) \equiv \frac{e_n \Phi_e(n, e_n, l_n)}{\Phi(n, e_n, l_n)}, \tag{13}$$

should be independent of ability, i.e., $\frac{\partial \eta}{\partial n} = 0$. Moreover, the optimal education tax is positive (negative) if the elasticity η rises (falls) with ability.

We can express Equation (11) also in terms of the derivatives of the education elasticity η. In particular, the elasticities of complementarity are related to the derivatives of η with respect to e_n and l_n in the following way: $\omega_e \rho_{ne} = \frac{\Phi}{\Phi_n} \frac{\partial \eta}{\partial n} + \omega_e$ and $\omega_e \rho_{el} = \frac{\Phi}{\Phi_l} \frac{\partial \eta}{\partial l} + \omega_e$. Consequently, we can write the optimal net tax on education (Equation (11)) as

$$\frac{(T'(z_n) - S'(e_n))}{(1 - T'(z_n))(1 - S'(e_n))} = \frac{u_c \theta_n / \lambda}{n f(n)} \frac{\omega_n}{\omega_e} \Phi \left(\frac{\partial \eta / \partial n}{\Phi_n} - \frac{\partial \eta / \partial l}{\Phi_l} \right). \tag{14}$$

Education should therefore be taxed (subsidized) if the education elasticity of earnings η responds more (less) to changes in earnings produced by ability (i.e., $\frac{\partial \eta / \partial n}{\Phi_n}$) than to changes in earnings produced by labor supply (i.e., $\frac{\partial \eta / \partial l_n}{\Phi_l}$). The condition for efficiency in educational investment can thus be stated as

$$\frac{\partial \eta / \partial n}{\Phi_n} = \frac{\partial \eta / \partial l_n}{\Phi_l}. \tag{15}$$

If, as in Maldonado (2008), $\partial \eta / \partial l_n = 0$, then the condition for efficiency in human capital investments boils down to $\partial \eta / \partial n = 0$. However, if $\partial \eta / \partial l_n \neq 0$, then $\partial \eta / \partial n = 0$ is neither a sufficient nor a necessary requirement for efficiency in human capital investment. Maldonado (2008) discusses various papers on the effect of ability on the education elasticity and he finds that the empirical evidence is mixed. We are not aware of empirical research directly testing whether human capital elasticities depend on labor effort. Note that a sufficient—but not necessary—condition for efficiency in human capital investments is $\frac{\partial \eta}{\partial n} = \frac{\partial \eta}{\partial l_n} = 0$. This condition is in fact implied by the earnings function employed by Bovenberg and Jacobs (2005).

Our results uncover the elasticities that should be estimated in order to determine the sign of the optimal education tax. In particular, one can regress the log of earnings on measures for ability, education and labor effort, and check whether there are cross-effects among these explanatory variables. For example, if log-earnings are defined as $\Omega(n, e_n, l_n) \equiv \ln \Phi(n, e_n, l_n)$, then the empirical condition for efficiency in educational choices would be equivalent to testing whether the following holds:

$$\frac{\Omega_{en}}{\Omega_n} = \frac{\Omega_{el}}{\Omega_l}. \tag{16}$$

Thus, our efficiency condition can be directly related to a large empirical literature using log-earnings as a dependent variable.

Jacobs and Bovenberg (2008) demonstrate that the main conclusions of this paper carry over to the case of linear tax instruments. In particular, they demonstrate that education policy is again determined by a trade-off between efficiency effects on labor-supply distortions and distributional effects as a result of the ability bias in education. They show that the optimal education tax is zero if the earnings function takes the following weakly separable form:

$$\Phi(n, l_n, e_n) \equiv \psi(n, l_n) e_n^{\beta}, \quad 0 < \beta < 1. \tag{17}$$

With this specification, the positive efficiency impacts of education subsidies on labor supply exactly offset their regressive distributional effects. Intuitively, labor earnings and education are related in a linear fashion across different ability levels if the earnings function is weakly separable and features a constant elasticity in education. Compared to labor income taxes, education taxes therefore imply both the same distortions on labor supply and the same effects on the income distribution. In contrast to labor taxes, however, taxes on education distort the education decision. Consequently, the government does not employ net taxes on education and adopts only labor income taxes as a redistributive instrument.

4. Conclusions

This paper has contributed to the literature on optimal education subsidies in models of labor supply and human capital formation. Using general earnings functions, we show that education decisions are generally not efficient in a second-best optimum. Net subsidies on education are optimal if sufficiently large efficiency gains of lower labor supply distortions, which are due to complementarities between learning and working, dominate the regressive incidence of education subsidies. Efficiency in human capital formation is obtained only if the earnings function is weakly separable in ability and labor, on the one hand, and in education, on the other. In that case, the positive efficiency gains of smaller labor supply distortions exactly offset the negative distributional impact of education subsidies. Thus, with weakly separable earnings functions, education policy is aimed exclusively at ensuring production efficiency in human capital investments.

Appendix: Optimal Policies

The first-order condition for l_n is given by

$$\frac{\partial \mathcal{H}}{\partial l_n} = \lambda \left(\Phi_l - \frac{dc_n}{dl_n} \Big|_{\bar{u}, \bar{e}} \right) f(n) - \theta_n \vartheta_n \left(u_{ll} + u_{lc} \frac{dc_n}{dl_n} \Big|_{\bar{u}, \bar{e}} \right) - \theta_n u_l \vartheta_{nl} = 0. \tag{A1}$$

Next, substitute $\frac{dc_n}{dl_n}|_{\bar{u},\bar{e}} = (1 - T')\Phi_l$[12] and $\vartheta_{nl} = \frac{\Phi_{ll}\Phi_n - \Phi_{nl}\Phi_l}{(\Phi_l)^2}$ in Equation (A1) to find the optimal nonlinear income tax at optimal nonlinear education subsidies

$$\frac{T'(z_n)}{1 - T'(z_n)} = \frac{u_c\theta_n/\lambda}{nf(n)}\omega_n\left(\rho_{nl} + \frac{1}{\omega_l\varepsilon^*}\right), \tag{A2}$$

where $\rho_{nl} \equiv \frac{\Phi_{nl}\Phi}{\Phi_n\Phi_l}$ is Hicks's partial elasticity of complementarity between ability and work effort in earnings. $\varepsilon^* \equiv \left(\frac{u_{ll}l_n}{u_l} - \frac{u_{lc}l_n}{u_c} - \frac{\Phi_{ll}l_n}{\Phi_l}\right)^{-1} > 0$ is a measure for the compensated wage elasticity of labor supply, which depends on the curvature of both the utility function and the earnings function. As before, we also find here that marginal taxes increase if ability rents increase with labor effort (ρ_{nl} is higher). If the earnings function is linear in ability and labor ($\omega_n = \omega_l = \rho_{nl} = 1$), the expression found by Mirrlees (1971) results.[13]

The first-order condition for u_n is

$$\frac{\partial \mathcal{H}}{\partial u_n} = \left(\Psi'(u_n) - \lambda \frac{dc_n}{du_n}\Big|_{\bar{l},\bar{e}}\right)f(n) - \theta_n\vartheta_n u_{lc}\frac{dc_n}{du_n}\Big|_{\bar{l},\bar{e}} = \frac{d\theta_n}{dn}. \tag{A3}$$

Since we defined θ_n negatively, there is no minus sign on the right-hand side. Substitution of $\frac{dc_n}{du_n}\Big|_{\bar{l},\bar{e}} = \frac{1}{u_c}$ yields a first-order differential equation in θ_n. This equation can be solved analytically to find the marginal value of redistribution θ_n/λ:

$$\frac{\theta_n}{\lambda} = \int_n^{\bar{n}}\left(\frac{1}{u_c(.)} - \frac{\Psi'(u_n)}{\lambda}\right)\exp\left(\int_n^m -\frac{\Phi_s(.)}{\Phi_l(.)}\frac{u_{lc}(.)}{u_c(.)}ds\right)f(m)\,dm. \tag{A4}$$

The expressions for the nonlinear income tax and the marginal value of redistribution are virtually the same as the ones found in the optimal tax literature. We refer the reader to Mirrlees (1971), Seade (1977), Atkinson and Stiglitz (1976), Saez (2001), and Bovenberg and Jacobs (2005) for the interpretation.

The first-order condition for e_n is

$$\frac{\partial \mathcal{H}}{\partial e_n} = \lambda\left(\Phi_e - 1 - \frac{dc_n}{de_n}\Big|_{\bar{u},\bar{l}}\right)f(n) - \theta_n\vartheta_n u_{lc}\frac{dc_n}{de_n}\Big|_{\bar{u},\bar{l}} - \theta_n u_l\vartheta_{ne} = 0. \tag{A5}$$

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[12] This expression can be found by taking the total derivative of utility at constant utility and education, and substituting the first-order condition for labor supply (Equation (5)).
[13] We note here that the elasticities of gross income with respect to the marginal tax rates are higher than in the case where human capital formation is exogenous. Optimal marginal income taxes are consequently lower. In order to show this, one needs to write the optimal tax formula in terms of the density of gross earnings. See Bovenberg and Jacobs (2005).

Substitution of $\frac{dc_n}{de_n}\big|_{\bar{u},\bar{l}} = 0$,[14] the first-order condition for learning (Equation (4)), the first-order condition for labor supply (Equation (5)), and $\vartheta_{ne} = \frac{\Phi_{le}\Phi_n - \Phi_{ne}\Phi_l}{(\Phi_l)^2}$ in Equation (A5) yields the optimal net tax on education in the main text.

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[14] We find $\frac{dc_n}{de_n}\big|_{\bar{u},\bar{l}} = 0$ by totally differentiating the household budget constraint and substituting the individuals' first-order condition for learning (Equation (4)) at constant utility and labor supply.

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