Optimal Linear Income Taxation and Education Subsidies under Skill-Biased Technical Change

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Online Appendix

A Comparative statics

To obtain the analytical comparative statics for the optimal tax rate, we totally differentiate the first-order condition eq. (22) while keeping the subsidy rate s fixed and allowing the transfer b to adjust in response to changing A and t via the government budget constraint eq. (12) (once s and t are set, b is residually determined). Similarly, we obtain the analytical comparative statics for the optimal subsidy rate by totally differentiating the first-order condition eq. (23) with respect to A and s while keeping the income tax rate t fixed and allowing the transfer b to adjust in response to changes in A and s via the government budget constraint eq. (12).

We note that in our model, optimal taxes and subsidies are jointly optimized. In contrast, we obtain the comparative statics for t by holding s fixed, and vice versa. This approach simplifies the comparative statics. To ensure that fixing either the subsidy rate or the tax rate does not qualitatively change how optimal policy responds to SBTC, we plot in Figure 1 the optimal tax rate while fixing the subsidy rate, and the optimal subsidy rate while fixing the tax rate. Comparing this with Figure 2 reveals that the direction in

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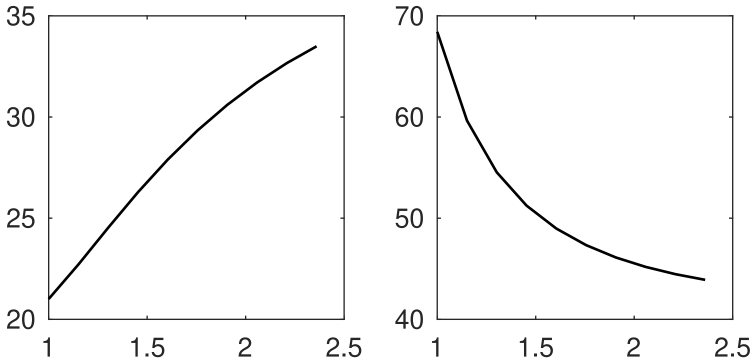
which SBTC impacts the optimal tax or subsidy rate is the same, irrespective of whether we optimize over both policies or keep one fixed. However, the magnitude by which policy changes with SBTC is affected.



(a) Opt. tax rate t (s fixed) (b) Opt. subsidy rate s (t fixed)

Note: Skill bias A on the horizontal axis. The respective values of s and t, are fixed at their optimum values at $A = 1$ as displayed in Figure 2.

Figure 1: Optimal policy under SBTC with a constant subsidy rate or tax rate

A.1 Effect on optimal tax rate

Totally differentiating eq. (22) while keeping the optimal subsidy s fixed and rearranging leads to

$$\frac{\mathrm{d}t}{\mathrm{d}A} = \frac{\frac{\partial \xi}{\partial A} - \frac{\partial}{\partial A}\left(\frac{\Delta}{(1-t)\bar{z}} f\left(\Theta\right)\Theta\varepsilon_{\Theta,t}\right) - \frac{\partial}{\partial A}\left((\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}\right)}{\frac{1}{(1-t)^2}\varepsilon - \frac{\partial \xi}{\partial t} + \frac{\partial}{\partial t}\left(\frac{\Delta}{(1-t)\bar{z}} f\left(\Theta\right)\Theta\varepsilon_{\Theta,t}\right) + \frac{\partial}{\partial t}\left((\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}\right)}. \tag{1}$$

We argue in Appendix B below that the denominator in eq. (1) is positive. To determine the sign of $\mathrm{d}t/\mathrm{d}A$ we can therefore focus on the numerator. The optimal tax rate increases with SBTC if the distributional benefits of income taxation increase more than tax-distortions and general-equilibrium effects taken together.

Distributional benefits of income taxes ξ. Recall that ξ is minus the normalized covariance between income and the social welfare weights. By raising the ratio of wage rates w^H/w^L, SBTC directly affects gross incomes. However, incomes are affected indirectly via changes in labor supply. The direct effect increases the income gap between skill-groups. Moreover, since labor supply increases more strongly with the wage rate the higher an individual's ability, income inequality within skill-groups also increases. To see this, use eq. (4) to write income as

$$z_\theta^j = l_\theta^j w^j \theta = \left[(1-t)w^j\theta\right]^{\varepsilon} w^j\theta = (w^j\theta)^{1+\varepsilon}(1-t)^{\varepsilon}. \tag{2}$$

An increase in w^j thus has a stronger effect on income z^j_θ if θ is higher. Both the increase of between- and within-group inequality contribute to an increase in ξ. At the same time, SBTC affects social welfare weights. Consumption, and thus utility, of the high-skilled increases more than the consumption of the low-skilled. Whether, as a result, social welfare weights decline more or less steeply with θ depends on the curvature of the social welfare function. Since a strictly concave social welfare function is steeper at low θ and flatter at high θ, the same increase in utility changes social marginal utility more at low θ and less at high θ individuals. Therefore, there are two counteracting effects: at high θ individuals, a larger change in utility goes along with social welfare weights being less responsive to such a change, while the opposite is true at low θ individuals. The effect of SBTC on social welfare weights is therefore ambiguous. As a consequence, $\partial\xi/\partial A$ cannot be unambiguously signed.

Education distortions of income taxes $\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\,\Theta\varepsilon_{\Theta,t}$. To analyze the partial impact of SBTC on the tax distortions of education, write

$$\begin{aligned}&\frac{\partial}{\partial A}\left(\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\,\Theta\varepsilon_{\Theta,t}\right)\\ =&\frac{1}{1-t}\left[\frac{\partial(\Delta/\bar{z})}{\partial A} f(\Theta)\,\Theta\varepsilon_{\Theta,t} + \frac{\partial f(\Theta)\,\Theta}{\partial A}\frac{\Delta}{\bar{z}}\varepsilon_{\Theta,t} + \frac{\partial\varepsilon_{\Theta,t}}{\partial A}\frac{\Delta}{\bar{z}} f(\Theta)\,\Theta\right].\end{aligned} \tag{3}$$

The sign of $\frac{\partial(\Delta/\bar{z})}{\partial A}$ is ambiguous. On the one hand, SBTC raises the income differential between the marginally high-skilled and the marginally low-skilled, which raises Δ – ceteris paribus. On the other hand, the costs of higher education for the marginal graduate $p(\Theta)$ (weakly) increase, since Θ falls. If the subsidy rate is positive, education subsidies for the marginal graduate (weakly) increase, which lowers Δ. If in contrast $s < 0$, the net tax Δ unambiguously increases with SBTC. However, SBTC also raises $\bar{z}$. If aggregate income increases relatively more than Δ, $\Delta/\bar{z}$ falls nevertheless. The sign of $\frac{\partial f(\Theta)\Theta}{\partial A}$ is again ambiguous. SBTC lowers Θ, but if $f'(\Theta) < 0$, the density increases as Θ falls, making the overall impact ambiguous. If in contrast, $f'(\Theta) > 0$, SBTC unambiguously decreases $f(\Theta)\,\Theta$. Finally, consider $\partial\varepsilon_{\Theta,t}/\partial A$. We have that $\partial\alpha/\partial A > 0$ and $\partial\beta/\partial A < 0$.[1] Moreover, we cannot sign the impact of SBTC on δ. Hence, it is unclear whether SBTC raises or lowers $\varepsilon_{\Theta,t}$. Overall, we conclude that whether tax distortions on education increase or decrease with SBTC is theoretically ambiguous.

General-equilibrium effects of income taxes $(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$. How does SBTC affect general-equilibrium effects? First, we focus on the effect on the income-weighted social

[1] To verify this, write $\alpha = (\frac{H}{L}\frac{w^H}{w^L})/(\frac{H}{L}\frac{w^H}{w^L}+1)$. SBTC increases $\frac{H}{L}\frac{w^H}{w^L}$, and thus the numerator increases relatively more than the denominator. Write $\beta = (w^H/w^L)^{1+\varepsilon}/((w^H/w^L)^{1+\varepsilon}-1)$, where now the numerator increases relatively less with SBTC than the denominator.

welfare weights $\tilde{g}^L$ and $\tilde{g}^H$ defined in eq. (19). An increase in A changes these terms via three channels: by affecting incomes, by affecting social welfare weights, and by affecting Θ. We discuss each of them in turn. SBTC increases incomes for both low- and high-skilled workers (though the high-skilled workers benefit more). Moreover, according to eq. (2), an increase in the wage rate w^j raises income more if θ is higher. As a result, the income weight z_θ^j in $\tilde{g}^L$ and $\tilde{g}^H$ increases for all g_θ, but more so if θ is higher. After normalizing by aggregate income per skill-group, g_θ at low θ are weighted relatively less within skill-groups, whereas g_θ at high θ are weighted relatively more within skill-groups. Since social welfare weights are declining in θ, the impact on $\tilde{g}^L$ and $\tilde{g}^H$ is ambiguous. Add to this that the impact of SBTC on the social welfare weights themselves is ambiguous, as has already been discussed. Finally, consider the effect of SBTC lowering Θ. As the marginal individual becomes high-skilled, both the numerator and the denominator of $\tilde{g}^L$ decrease. However, if $g_\Theta < \tilde{g}^L$, the numerator decreases relatively less than the denominator – and $\tilde{g}^L$ increases.[2] In contrast, a lower Θ raises both the numerator and denominator of $\tilde{g}^H$. If $g_\Theta > \tilde{g}^H$, the numerator increases relatively more, and $\tilde{g}^H$ rises with SBTC. Numerically, we find $\tilde{g}^L > g_\Theta > \tilde{g}^H$. SBTC thus contributes to an increase in both $\tilde{g}^L$ and $\tilde{g}^H$ via a lower Θ. The overall effect on $\tilde{g}^L$ and $\tilde{g}^H$, and thus on $(\tilde{g}^L - \tilde{g}^H)$, is theoretically ambiguous. Next, we turn to the impact of SBTC on ε_{GE}. Whether skill bias increases or decreases ε_{GE} depends on its impact on α, β and δ. Moreover, we have $\partial\alpha/\partial A > 0$, and $\partial\beta/\partial A < 0$ and the sign of $\partial\delta/\partial A$ is ambiguous, prohibiting us to clearly sign the effect on ε_{GE} . We conclude that the theoretical impact of SBTC on general-equilibrium effects is ambiguous.

Combined effect. Since we cannot sign the effect of SBTC on the different determinants of the optimal tax rate, the theoretical effect of SBTC on the optimal tax rate is ambiguous.

A.2 Effect on optimal subsidy rate

Totally differentiating eq. (23), while keeping t fixed, leads to

$$\frac{\mathrm{d}s}{\mathrm{d}A} = \frac{-\frac{\pi}{(1-t)} s \frac{\partial}{\partial A}\left(\frac{\zeta}{\bar{z}}\right) + \frac{\partial}{\partial A}\left(\frac{\Delta}{(1-t)\bar{z}}\Theta f\left(\Theta\right)\varepsilon_{\Theta,s}\right) + \rho\frac{\partial}{\partial A}\left(\left(\tilde{g}^L - \tilde{g}^H\right)\varepsilon_{GE}\right)}{\frac{\pi}{(1-t)}\left(\frac{\zeta}{\bar{z}} + \frac{\partial}{\partial s}\left(\frac{\zeta}{\bar{z}}\right)s\right) - \frac{\partial}{\partial s}\left(\frac{\Delta}{(1-t)\bar{z}}\Theta f\left(\Theta\right)\varepsilon_{\Theta,s}\right) - \frac{\partial}{\partial s}\left(\rho\left(\tilde{g}^L - \tilde{g}^H\right)\varepsilon_{GE}\right)}. \tag{4}$$

As we argue in Appendix B, the denominator of eq. (4) is positive. To determine the sign of $\mathrm{d}s/\mathrm{d}A$ we can therefore focus on the numerator.

[2] To see this, note that sign of the impact of A on $\tilde{g}^L$ via Θ is given by $\mathrm{sgn}[\partial\Theta/\partial A g_\Theta z_\Theta^L f(\Theta)\bar{z}^L - \partial\Theta/\partial A z_\Theta^L f(\Theta)\int_{\underline{\theta}}^{\Theta} g_\theta z_\theta^L f(\Theta)\mathrm{d}\theta] = \mathrm{sgn}(\tilde{g}^L - g_\Theta)$, where we use $\partial\Theta/\partial A < 0$. The derivations for the effect on $\tilde{g}^H$ are analogous.

Distributional losses of education subsidies $\frac{s\pi}{(1-t)\bar{z}}\zeta$. For given s and t, only $\zeta/\bar{z}$ is affected by SBTC. To analyze the sign of $\partial\zeta/\partial A$, write

$$\frac{\partial \zeta}{\partial A} = -\int_{\Theta}^{\theta} \theta^{-\psi}\frac{\partial g_{\theta}}{\partial A}\mathrm{d}F(\Theta) - \frac{\partial \Theta}{\partial A}\Theta^{-\psi}(1-g_{\Theta})f(\Theta). \tag{5}$$

SBTC thus affects ζ via two channels: by changing the social welfare weights g_θ, and by lowering the threshold Θ. We have already argued above that the impact of SBTC on social welfare weights is ambiguous. The drop in Θ corresponds to more individuals becoming high-skilled. If the social welfare weight attached to the newly high-skilled is lower than one, as one would expect, ζ increases. Intuitively, as more individuals with lower-than-average social welfare weights become high-skilled, it becomes more beneficial to raise revenue from the high-skilled by taxing education. In addition, SBTC unambiguously increases $\bar{z}$, and with $\partial\zeta/\partial A > 0$ the theoretical impact on $\zeta/\bar{z}$ is unclear.

Education distortions of education subsidies $\frac{\Delta}{(1-t)\bar{z}}\Theta f(\Theta)\varepsilon_{\Theta,s}$. Turning to the distortions of education, note that the tax distortions and subsidy distortions on education only differ by a factor ρ. Since ρ is not affected by A, the effect of SBTC on the subsidy distortions of education is ρ times the impact of SBTC on the tax distortions of education, which – as argued above – is theoretically ambiguous.

General-equilibrium effects education subsidies $\rho(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$. We have already discussed the effect of an increase in skill bias on general-equilibrium effects when analyzing the response of the optimal tax rate given by eq. (1). The only difference is that now the effect is multiplied by ρ, which is unaffected by A. As a consequence, the impact of SBTC on general-equilibrium effects is theoretically ambiguous.

Combined effect. Since we cannot sign the effect of SBTC on the different determinants of the optimal subsidy rate, the theoretical effect of SBTC on the optimal subsidy rate is ambiguous.

B Comparative statics: Denominators

In this Section, we discuss the impact of an increase in skill bias on the denominators in eq. (1) and eq. (4). Combining analytical and numerical insights, we argue that in both cases, the denominator is positive.

B.1 Denominator of eq. (1)

Distributional benefits of income taxes ξ. An increase in t affects gross incomes and social welfare weights. Gross incomes fall as higher taxes distort labor supply downwards. Since this distortion is larger for individuals with high ability, the income distribution becomes more equal, which contributes to a drop in ξ. Social welfare weights change for two reasons. First, a drop in gross income directly lowers consumption of each individual, thereby lowering utility. Second, the increased tax revenue is redistributed lump sum, increasing everyone's utility. Individuals of low ability on net gain utility relative to individuals of high ability. This leads to a decrease of social welfare weights at the bottom and an increase at the top. In other words, social welfare weights become flatter. With incomes that are more equal, and social welfare weights declining less steeply, the benefits of redistributing with the income tax decline, that is $\partial\xi/\partial t < 0$. This is also confirmed by our numerical results in Table 1.

Education distortions of income taxes $\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\,\Theta\varepsilon_{\Theta,t}$. The term $\frac{\partial(\Delta/\bar{z})}{\partial t}$ is likely to be positive. For given incomes z_Θ^H and z_Θ^L, a higher tax rate leads to a larger increase in tax revenue if the marginal individual becomes high-skilled, contributing to an increase of Δ. Still, a change in the tax rate lowers incomes, as it distorts labor supply downwards, and more so for the high-skilled than the low-skilled workers, partly counteracting the increase in tax revenue.[3] Moreover, by increasing Θ, expenditures on education subsidies are affected. If education is subsidized ($s > 0$), expenditures on education subsidies fall, since $p(\Theta)$ (weakly) decreases in Θ, thereby contributing to an increase in Δ. In contrast, if education is taxed ($s < 0$), revenue from the education tax falls, which lowers Δ – ceteris paribus. Still, we expect an increase in Δ unless the latter effect is very strong. In addition, $\bar{z}$ decreases with t due to labor-supply distortions. Hence, we expect $\Delta/\bar{z}$ to increase with t. Numerically, we confirm that both Δ and $\Delta/\bar{z}$ increase with t (Table 1). The impact of a higher tax on $\Theta f(\Theta)$ is less clear. While Θ increases, $f(\Theta)$ may increase or decrease, depending on the shape of the density and the location of Θ. In our simulations, we find a decrease in $f(\Theta)$. Numerically, $\Theta f(\Theta)$ falls with t whereas there is no impact on $\varepsilon_{\Theta,t} = \varsigma$. Overall, distortions on education rise as t becomes larger.

General-equilibrium effects of income taxes $(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$. Finally, consider the effect of t on general-equilibrium effects. First, focus on the terms $\tilde{g}^L$ and $\tilde{g}^H$. Due to distorting labor supply, incomes z_θ^j are reduced, and more so the higher is θ. After normalizing by aggregate incomes per skill-group, social welfare weights g_θ at low θ receive relatively more weight, whereas the income weighting of social welfare weights at high

[3] It is unlikely that, at the optimum, an increase in the tax rate leads to lower tax revenue from the marginal graduate. For that to be the case, the optimal tax rate would have to maximize revenue from the marginal graduate.

θ decreases. Since social welfare weights are decreasing in θ – and thus in income – $\tilde{g}^L$ and $\tilde{g}^H$ increase, ceteris paribus. However, so far, we have not taken into account the change in social welfare weights themselves and the increase in Θ. With higher taxes, and thus more redistribution, we expect g_θ to flatten, which ceteris paribus lowers $\tilde{g}^L$ and increases $\tilde{g}^H$. Finally, for given incomes and social welfare weights, the increase in Θ leads to lower $\tilde{g}^L$ if $g_\Theta < \tilde{g}^L$ and to lower $\tilde{g}^H$ if $g_\Theta < \tilde{g}^H$. Due to decreasing g_θ, we expect $\tilde{g}^H < g_\Theta < \tilde{g}^L$, and thus – ceteris paribus – an decrease in $\tilde{g}^L$ and an increase in $\tilde{g}^H$. Numerically, we indeed find that $\tilde{g}^L$ falls, while $\tilde{g}^H$ increases. As a consequence, $\tilde{g}^L - \tilde{g}^H$ declines. The impact of t on ε_{GE} works again via α, β, and δ. While higher taxes decrease α, they increase β via general-equilibrium effects. Still, the impact on δ remains ambiguous, making the theoretical impact on ε_{GE}, and on general-equilibrium effects overall, ambiguous as well. Numerically, we find an increase in ε_{GE}. However, the drop in $(\tilde{g}^L - \tilde{g}^H)$ dominates, such that general-equilibrium effects become less important as t increases.

Combined effect. Quantitatively, the decline in general-equilibrium effects is small compared to the drop in ξ and the increase in education distortions. As a consequence, the denominator in eq. (1) is positive.

B.2 Denominator of eq. (4)

Distributional losses of education subsidies $\frac{s\pi}{(1-t)\bar{z}}\zeta$. An increase in s affects ζ via its impact on social welfare weights, as well as by lowering Θ:

$$\frac{\partial \zeta}{\partial s} = -\int_{\Theta}^{\theta} \theta^{-\psi} \frac{\partial g_\theta}{\partial s} \mathrm{d}F(\Theta) - \frac{\partial \Theta}{\partial s} \Theta^{-\psi}(1 - g_\Theta) f(\Theta). \tag{6}$$

The first term is expected to be positive. The second term is positive if $g_\Theta < 1$, that is, if the social welfare weight attached to the marginally high-skilled is below one, as we would expect as well. In this case, raising the subsidy distributes income from low-skilled to high-skilled individuals – thereby increasing the benefits of taxing – rather than subsidizing – education on a net basis. Numerically, we find $g_\Theta < 1$, and consequently $\partial\zeta/\partial s > 0$ (Table 2). The impact of s on $\bar{z}$ works via raising H/L due to a reduction in Θ, and depends on the specific production function. For example, if the high-skilled contribute more to output than the low-skilled, output can increase with the subsidy rate. Table 2 reports that $\bar{z}$ increases in s. However, the relative increase in ζ is larger, so that $\zeta/\bar{z}$ rises with the subsidy rate.

Education distortions of education subsidies $\frac{\Delta}{(1-t)\bar{z}}\Theta f(\Theta)\varepsilon_{\Theta,s}$. Next, we analyze the impact on the distortions of education:

$$\begin{aligned} &\frac{\partial}{\partial s}\left(\frac{\Delta}{(1-t)\bar{z}}f\left(\Theta\right)\Theta\varepsilon_{\Theta,s}\right) \\ =&\frac{1}{1-t}\left[\frac{\partial(\Delta/\bar{z})}{\partial s}f\left(\Theta\right)\Theta\varepsilon_{\Theta,t}+\frac{\partial f\left(\Theta\right)\Theta}{\partial s}\frac{\Delta}{\bar{z}}\varepsilon_{\Theta,t}+\frac{\partial\varepsilon_{\Theta,t}}{\partial s}\frac{\Delta}{\bar{z}}f\left(\Theta\right)\Theta\right]. \end{aligned} \tag{7}$$

First, consider the effect of s on Δ. Using $z_\theta^j=(w^j\theta)^{1+\varepsilon}(1-t)^\varepsilon$, we arrive at

$$\frac{\partial\Delta}{\partial s}=-p(\Theta)+(1+\varepsilon)\frac{\partial\Theta}{\partial s}\Theta^\varepsilon t(1-t)(w^H-w^L)-sp'(\Theta)\frac{\partial\Theta}{\partial s}<0 \tag{8}$$

$-p(\Theta)$ is the direct effect of a lower Θ on subsidy expenditures, which lowers Δ. In addition, an increase in s has indirect effects on Δ. Due to the lower Θ, the income differential between the marginally high- and low-skilled decreases. Moreover, expenditures on education subsidies increase further, since $-p(\Theta)$ (weakly) increases as Θ falls. This adds to the drop in Δ. Numerically, we confirm $\partial\Delta/\partial s<0$ (Table 2). Moreover, since $\bar{z}$ increases, we see a drop in $\Delta/\bar{z}$. As with the tax rate, the impact of the subsidy on $\Theta f(\Theta)$ is theoretically ambiguous. Θ decreases, whereas the impact on $f(\Theta)$ depends on the density. Numerically, we find that the increase in $f(\Theta)$ more than compensates the drop in Θ, so that $\Theta f(\Theta)$ increases. Finally, consider the response of the elasticity $\varepsilon_{\Theta,s}$ to an increase in s. Note that with exogenous wages, $\varepsilon_{\Theta,s}=\varsigma\rho$, with $\rho=\frac{s}{(1-s)(1+\varepsilon)}$. Since ς is not affected by s, and $\partial\rho/\partial s>0$, $\varepsilon_{\Theta,s}$ increases with s. Still the overall impact on education distortions is theoretically ambiguous. Numerically, we find that education distortions decrease with s.

General-equilibrium effects education subsidies $\rho(\tilde{g}^L-\tilde{g}^H)\varepsilon_{GE}$. Finally, we turn to the impact of s on general-equilibrium effects. A higher subsidy affects the income-weighted social welfare weights $\tilde{g}^L$ and $\tilde{g}^H$ via three channels: by changing the social welfare weights, by changing incomes, and by lowering Θ. A higher subsidy redistributes from the low-skilled to the high-skilled. The direct consequence is that consumption rises most for the marginally high-skilled individual (who faces the highest cost of higher education). Larger utility leads to a decline of social welfare weights for the high-skilled around Θ, due to the concavity of the social welfare function. In addition, the subsidy also affects consumption – and thus utility and social welfare weights – by changing incomes. As Θ falls, H/L increases and the wage differential w^H/w^L is reduced. These general-equilibrium effects raise consumption of the low-skilled workers, while they decrease consumption of the high-skilled workers. For the low-skilled workers, the increase in w^L runs against the direct loss in consumption due to the higher subsidy. As a consequence, social welfare weights for the low-skilled increase less than if there were no general-equilibrium effects on wages. The decrease in w^H partly offsets the gains of the high-skilled workers due to the larger subsidy. Moreover, the high-skilled workers with the highest ability benefit less from the larger subsidy, since they have low direct costs of higher-education. The same individuals experience the largest drop in consumption due to the decreased wage w^H. As a result, we expect social welfare weights to increase at high θ. Hence, taking all effects together, we expect an increase in $\tilde{g}^L$, whereas the effect on $\tilde{g}^H$ is unclear. The income weighting of the social welfare weights suggests that the lower social welfare weights at the top compensate for the decrease around Θ, hence $\tilde{g}^H$ might increase as well. However, the income weights are also affected. As w^H falls, the income distribution among the high-skilled features more equality, and more so at the top. This raises $\tilde{g}^H$, since social welfare weights decline, and social welfare weights for workers with lower ability θ now receive relatively more weight. In contrast, among the low-skilled income dispersion increases with w^L, which raises g_θ at higher θ. This contributes to a drop in $\tilde{g}^L$. Finally, the drop in Θ affects $\tilde{g}^L$ and $\tilde{g}^H$ in the same way as SBTC, i.e., $\tilde{g}^L$ and $\tilde{g}^H$ increase if $\tilde{g}^L > g_\Theta > \tilde{g}^H$, which we find to be satisfied numerically. Overall, we find that the higher subsidy raises both $\tilde{g}^L$ and $\tilde{g}^H$, and since the increase in $\tilde{g}^H$ is more pronounced, $\tilde{g}^L - \tilde{g}^H$ decreases. The impact on the general-equilibrium elasticity ε_{GE} is theoretically ambiguous, since we cannot sign $\partial\delta/\partial s$. Numerically, we find that ε_{GE} decreases with s. Finally, the general-equilibrium term also changes with ρ, which increases in s. Numerically, we find this effect to dominate, such that $\rho(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$ becomes larger as s increases.

Combined effect. If the positive impact on general-equilibrium effects is large, the denominator of eq. (4) might become negative. However, we find quantitatively that distortions on education decrease by more than the increase in general-equilibrium effects, and hence, the denominator is positive (compare the respective terms in Table 2).

Table 1: Ceteris paribus impact of changing t

Expression	Initial value	Change
Distributional benefits of the income tax and education tax		
ξ	806.75	-32.83
ζ	0.08	-0.01
Tax and subsidy distortions on education		
$\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\Theta\varepsilon_{\Theta,t}$	-52.83	-1.07
$\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\Theta\varepsilon_{\Theta,s}$	-88.07	-1.78
General-equilibrium effects		
$(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$	61.90	-1.24
$\rho(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$	103.19	-2.07

Note: All table entries have been multiplied by 1e+04.

Table 2: Ceteris paribus impact of changing s

Expression	Initial value	Change
Distributional benefits of the income tax and education tax		
ξ	806.75	-131.38
ζ	0.08	0.02
Tax and subsidy distortions on education		
$\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\Theta\varepsilon_{\Theta,t}$	-52.83	33.61
$\frac{\Delta}{(1-t)\bar{z}} f(\Theta)\Theta\varepsilon_{\Theta,s}$	-88.07	-835.29
General-equilibrium effects		
$(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$	61.90	-54.89
$\rho(\tilde{g}^L - \tilde{g}^H)\varepsilon_{GE}$	103.19	233.82

Note: All table entries have been multiplied by 1e+04.