

The Public Finance Approach to Optimal Stabilization Policy*

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Abstract

This paper studies optimal macro-economic stabilization policy by developing a New Keynesian model of a closed economy with three main macro-economic distortions: price rigidities, labor-market rigidities and the effective lower bound (ELB) on nominal interest rates. It is shown that optimal fiscal policy implementations should be geared towards solving the underlying market failures, not bang-for-the-buck calculations based on fiscal multipliers. A tax on wealth – coupled to an investment tax credit – is the optimal and foolproof way to eliminate the ELB. Furthermore, the first-best solution to involuntary unemployment is to remove the implicit tax (‘wedge’) on labor as a result of unemployment by means of an explicit subsidy on employment. Public goods are always provided according to the classical Samuelson rule and not to stabilize output. The effective lower bound on interest rates is a policy choice not a policy constraint. Central banks are superfluous if the government has a sufficiently rich instrument set. Optimal policy is fundamentally Pigouvian and not Keynesian in the New Keynesian model.

Keywords: business cycle stabilization, New Keynesian model, optimal fiscal policy, effective lower bound

JEL-codes: D6, E5, E6 H2, H4

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1 Introduction

During the Great Recession, public discussions on fiscal policy revealed deep and often ideologically charged divisions among economists, policy makers and politicians. Old battles among (New) Keynesians and (New) Classicals revived. Macro-economists have been sharply divided on simple questions regarding the usefulness and desirability of stabilization policy during (deep) recessions. Furthermore, fiscal policy recommendations have been based on multiplier arguments (Keynes, 1936; Woodford, 2011), confidence effects (Alesina and Ardagna, 2010)¹, debt thresholds (Reinhart and Rogoff, 2010)², hysteresis effects (DeLong and Summers, 2012) and, in the Eurozone, the presumed necessity to satisfy the budgetary rules of the Maastricht Treaty. However, none of these arguments rely on solid welfare analysis. One of the reasons why stabilization policy might be controversial is exactly that there is no commonly agreed framework to analyze the social costs and benefits of such policies.

Traditionally, stabilization policy was seen as one of the three fundamental branches of public economics, besides allocation and redistribution policy (Musgrave, 1959). This was long before stabilization policy was seen as a main part of macro-economists and before public finance economists had lost their interest in macro-economic issues. Indeed, there is a popular view that business cycle stabilization can simply be delegated to central banks, and is, therefore, a branch of monetary economics, see e.g., Blanchard and Galí (2007). This may perhaps be so in normal times. However, the effective lower bound on nominal interest rates became binding in recent years throughout the Western world. The Eurozone and Japan have been stuck on their lower bounds for many years and presumably will be for many years to come. As a result, central banks have lost their power to stabilize the business cycle, especially if they are unable or unwilling to let future inflation rise, e.g., due to credibility issues or by institutional rules that prohibit an increase in the inflation target.³

¹Jayadev and Konczal (2010) argue that the only real cases of ‘expansionary austerity’ are confounded due to concomitant depreciations or interest rate differentials for which Alesina and Ardagna (2010) do not control. Guajardo, Leigh, and Pescatori (2014) argue that the cyclically adjusted primary balance measures used by Alesina and Ardagna (2010) are endogenous. Using a narrative approach they identify arguably more exogenous cases of fiscal consolidation to show that there exists no expansionary austerity in the data.

²Herndon, Ash, and Pollin (2013) show that Reinhart and Rogoff (2010) selectively left out countries, made Excel coding errors and applied non-standard weighting of data. After correcting for this, the 90-percent debt threshold disappears. IMF research also finds no empirical evidence for a 90-percent debt threshold using longer time periods for (low) growth episodes (Pescatori, Sandri, and Simon, 2014). Finally, Lof and Malinen (2014) cast doubt on the direction of causality. If anything they find that the negative long-run correlation between the debt and growth is driven by the negative effect of growth on debt.

³The effectiveness of unconventional monetary policies (forward guidance or commitment, quantitative easing, qualitative easing, operation twist, etc.) remains controversial and heavily debated, even among New Keynesians, see for example Woodford (2012). Unconventional monetary policy can be effective only if central banks can credibly raise future inflation expectations by committing to a future boom (Krugman, 1998; Eggertsson and Woodford, 2003; Eggertsson, 2006; Eggertsson and Woodford, 2006; Werning, 2012).

Therefore, if monetary policy runs out of steam, fiscal policy may be the only available tool to stabilize the business cycle.

This paper aims to provide a first pass to develop a normative public finance framework to answer the following three fundamental questions of macro-economic stabilization policy: *i) Should fiscal policy be used to stabilize the business cycle? ii) If so, how much fiscal stimulus (contraction) is optimally needed during a recession (boom)? iii) Which type of tax and spending policies are needed to stabilize the business cycle?* To answer these most elementary normative questions this paper develops an analytically tractable New Keynesian model, which is a dynamic stochastic general equilibrium model of a closed economy that might be stuck at the ELB and in which the labor market has some rigidities leading to involuntary unemployment. This model is then used to explore the desirability of fiscal policy, the size of fiscal policy and what type of fiscal policy should be used to stabilize the business cycle. By making a first pass to evaluate all costs and all benefits of stabilization policy using welfare-economic principles, this paper hopes to help resolve the ideological macro-economic battles on stabilization policy.

The first question – *Should fiscal policy be used to stabilize the business cycle?* – asks: why is stabilization policy needed in the first place? More precisely, what are the exact macro-economic distortions that need to be addressed by fiscal policy? The workhorse New Keynesian models identify (at least) three main macro-economic distortions that may arise due to business-cycle fluctuations (Woodford, 2003; Galí, 2008). First, price rigidities result in production inefficiencies. Hence, an intertemporal distortion in consumer prices arises compared to the first-best allocation. Under normal conditions, this distortion can be perfectly offset with monetary policy (Woodford, 2003; Galí, 2008; Blanchard and Galí, 2007). Second, if monetary policy is constrained by the effective lower bound (ELB) on nominal short-term interest rates, the intertemporal distortion in the allocation of resources can no longer be fixed by monetary policy. The ELB then creates a wedge in capital markets, resulting in an implicit subsidy on the short-term nominal interest rate. As a result, goods-market equilibrium and – by Walras’ law – capital-market equilibrium will be restored by means of recession (if prices are fixed), deflation (if prices are fully flexible) or a combination of both (prices are rigid but not completely inflexible) (Krugman, 1998; Eggertsson and Woodford, 2003, 2006; Werning, 2012). At the ELB, fiscal policy can be extremely potent (Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson, 2011; Woodford, 2011). Third, some versions of the New Keynesian model furthermore allow for labor-market rigidities resulting in underemployment (if labor rationing occurs on the intensive margin) or unemployment (if rationing occurs on the extensive margin) (Erceg, Henderson, and Levin, 2000; Blanchard and Galí, 2007, 2010). Once more, rigidities give rise to an implicit tax on labor, which results in

welfare losses. Given that price rigidities, the ELB and labor-market rigidities result in a Pareto-inefficient market outcome, and macro-economic shocks generally exacerbate market distortions, there is in principle a useful role for stabilization policy.

It is demonstrated that fiscal instruments should be geared towards removing the underlying macro-economic distortions created by recessions (and booms). A sufficiently flexible set of fiscal instruments can in general remove *all* distortions in relative prices and restores first-best in the work-horse New Keynesian model. In particular, if the economic problem is the intertemporal wedge on the interest rate, then a *wealth tax* – coupled to an investment tax credit – is the best way to solve this intertemporal problem. Wealth taxes can remove all distortions in intertemporal consumption prices, irrespective of whether the ELB is binding or not. If the ELB is non-binding, a tax on capital income can control intertemporal consumption prices as well. The wealth tax is a relatively simple policy instrument. Although few OECD countries currently employ wealth taxes (France, Spain, The Netherlands, Switzerland and Norway), many countries have used wealth taxes in the past, see OECD (2018).⁴ The ELB is therefore a policy choice, not a policy constraint. This role of the wealth tax has barely received any attention so far in the macro-economics literature, but follows naturally once a public-finance approach is taken to fiscal policy. Moreover, the wealth tax makes central banks superfluous as it can effectively replicate or undo any action of the central bank. The wealth tax (or capital income tax) thus upsets the standard monetary-fiscal dichotomy in macro-economics. Furthermore, if the economic problem is involuntary unemployment, then a first-best solution is to remove the implicit tax (‘wedge’) on labor as a result of unemployment by means of an explicit subsidy on employment. Consequently, only three tax instruments are needed to fix a recession: wealth taxes, investment tax credits and wage subsidies.

Second, if fiscal policy is to be used as a stabilization tool, *how much fiscal stimulus is optimally needed during a recession?* In most academic work, policy debates, and popular discourse, the fiscal multiplier is taken as the sufficient statistic to judge the desirability of fiscal policy. The public-finance approach in this paper reveals that fiscal policy should be geared towards solving the underlying market failures, not bang-for-the-buck calculations based on fiscal multipliers: optimal tax policy is distinctly Pigouvian and optimal spending policy is Samuelsonian, and not Keynesian in the New Keynesian macro-model. Importantly, the standard New Keynesian model does not explain why optimal business cycle stabilization should not reach first-best outcomes.

⁴OECD (2018) does not consider The Netherlands as a country with a wealth tax. The Dutch government is able to convince the OECD (and also the European Commission) that it employs a tax on capital income. This is false, since tax is levied on a *fictitious* return on total wealth. The tax liability thus ultimately only depends on total wealth, not on capital income. See also Jacobs (2013).

Third, *which type of tax and spending policies do governments need to use to stabilize the business cycle?* Keynes (1936, p.129) once (in)famously argued in *The General Theory* that the government might pay people to dig deep holes in the ground to bury bottles filled with bank notes and then to fill up these holes with dirt. Keynes mentioned that such a policy would obviously be less attractive than public investments in housing. Such examples of ‘vulgar’ Keynesianism have been an open invitation to criticism or even ridicule from (Neo-)Classical economists. This paper shows that the optimal fiscal policy implementation requires a differentiated set of policy instruments, including wealth taxes, investment tax credits and wage subsidies. More importantly, this paper demonstrates that there is *no* role for Keynesian stimulus during recessions via expanding public goods. Indeed, with sufficient fiscal instruments, the desirability of public spending – and thereby the size of the state – should be based exclusively on classical principles, i.e., public spending should always follow the traditional Samuelson (1954)-rule for public goods. Once more, the New Keynesian model thus delivers classical and not Keynesian policy prescriptions.

After having developed the main theoretical insights the model is used to conduct a back-of-the-envelope calculation for optimal policies for the Eurozone during the Great Recession. Optimal stabilization then required a 2 percent wealth tax raising 10 percent of GDP in revenue, a wage subsidy of 12 percent costing 8.4 percent of GDP and an investment tax credit of 25 percent, costing 5 percent of GDP. These numbers suggest that fiscal policy should be much more aggressive than commonly observed. Moreover, these policies contrast heavily with actual policies implemented throughout the Eurozone, suggesting that policies were far from optimal.

The remainder of this paper is structured as follows. Section 2 discusses earlier literature. Section 3 introduces the model. Section 4 presents the main results for optimal stabilization policy. Section 5 provides several robustness checks. Section 6 presents a back of the envelope calculation of optimal stabilization policy during the Great Recession. Section 7 concludes. An appendix contains all derivations and proofs of propositions.

2 Earlier literature

This paper relates in a number of ways to the existing literature. First, although modern macro-economic analyses are abundant in micro-foundations, they are scarce in analyzing the welfare-economic consequences of fiscal policy, with the exception of a small number of papers to be discussed next. The standard operating procedure is to use New Keynesian models to calculate fiscal policy multipliers or to calculate the optimal fiscal policy response using an ad hoc loss function that does not fully take into account all relevant macro-economic

distortions, e.g., Eggertsson and Woodford (2006) and Eggertsson (2011).⁵

This paper is conceptually closest to Correia et al. (2013). These authors also analyze a standard New Keynesian model to analyze optimal fiscal policy at the ELB. However, the instrument set of the government is restricted so that only taxes on labor and capital income, consumption and subsidies on investment are allowed for. They show that it is then also possible to eliminate the ELB by setting an exponential ramp of consumption taxes jointly with an exponentially growing path of labor subsidies. However, such a policy implementation may be less realistic from a practical point of view. It is shown that taxes on wealth can achieve exactly the same result and are practically feasible.

Related is Mankiw and Weinzierl (2011) who analyze a two-period model with complete price rigidity in the first period to analyze optimal fiscal policy at the ELB with a limited set of fiscal instruments. They find that public spending could indeed be geared to eliminate output gaps, but this results in welfare losses because the public sector becomes inefficiently large compared to the first-best Samuelson rule. An investment tax credit might then be preferred over spending increases to stabilize output. Moreover, Mankiw and Weinzierl (2011) show that multiplier calculations are not informative of the welfare effects of fiscal policy.

DeLong and Summers (2012) make highly stylized back-of-the-envelope calculations of the social costs and benefits of a fiscal expansion in an environment with low interest rates and hysteresis effects. They show that fiscal expansions might pass a social cost-benefit test, especially if hysteresis is important.

Michaillat and Saez (2018) also study optimal fiscal policy in a model with matching frictions on the goods market, like in the Diamond-Mortenson-Pissarides model. They show that optimal stabilization policy requires that the government expands spending beyond the first-best Samuelson rule to compensate for demand shortfalls, since it is not possible to directly eliminate the frictions in the goods market.

Finally, Bouakez, Guillard, and Roulleau-Pasdeloup (2019) analyze optimal public investment in a standard New Keynesian model with limited government instruments: the government can only set interest rates, public consumption and public investment. They show that if the ELB binds, the government may optimally increase public investments.

Mankiw and Weinzierl (2011), Michaillat and Saez (2018) and Bouakez, Guillard, and Roulleau-Pasdeloup (2019) all analyze second-best policy responses in macro-models with demand shortfalls and a potentially binding ELB. However, in the New Keynesian model

⁵Clarida, Galí, and Gertler (1999), Woodford (2003) and Galí (2008) discuss optimal monetary policy in the New Keynesian model. The literature typically assumes that the welfare function can be approximated with a second-order Taylor expansion around an undistorted steady state, so that welfare can be written as a function of the squares of the inflation and output gaps.

of this paper there are no reasons why the government cannot achieve overall efficiency given that it has sufficient fiscal instruments. Optimal government policies ensure that inefficiencies are directly targeted. If the central bank is bound by the ELB, a wealth tax is sufficient to overcome the ELB combined with an investment tax credit to offset distortions in investment. If (involuntary) unemployment is too high owing to rigid wages, then a wage subsidy can fully offset this distortion. Hence, public spending, tax credits or investment do not need to be geared towards stabilizing aggregate demand, government spending always satisfies the first-best Samuelson rule and government investment should follow the standard cost-benefit rule.

3 Model

This paper analyzes a standard New Keynesian dynamic, stochastic, general-equilibrium model (DSGE) of a closed economy.⁶ The model consists of a representative household, a set of representative producers, the government, and a central bank. To study classical public finance questions, the model explicitly allows for public goods in the utility function. Moreover, labor supply is modeled on the extensive margin – as in Diamond (1980) and Saez (2002) – in contrast to the canonical New Keynesian framework, which analyzes labor supply on the intensive margin, cf. Woodford (2003) and Galí (2008). Economists and policy makers are arguably more concerned about the extensive margin. First, business cycle fluctuations in employment are more concentrated on the extensive than the intensive margin, see e.g., Cho and Cooley (1994). Second, in models with an intensive margin unemployment takes the form of ‘underemployment’: reductions in hours worked. However, policy makers typically want to reduce involuntary unemployment rather than than involuntary underemployment of hours.

The model contains three main economic distortions that are relevant for the business cycle. First, prices are rigid due to staggered Calvo pricing, which results in production distortions. Second, rigid nominal wages generate involuntary unemployment. Third, monetary policy can be constrained by the lower bound on nominal short-term interest rates.

3.1 Households

The model is closely related to Correia et al. (2013), which is based on Eggertsson and Woodford (2003, 2006) and Eggertsson (2009). Time is discrete and denoted by $t \in [0, \infty)$. Uncertainty in period t is captured by random variable $s_t \in \mathcal{S}_t$, where \mathcal{S}_t denotes the set of

⁶See also Woodford (2003), Eggertsson and Woodford (2003, 2006), Galí (2008) and Eggertsson (2011).

possible states of the economy at time t . $s^t \in \mathcal{S}^t$ is the entire history of realizations of s_t up to t , that is: $s^t \equiv [s_0, s_1, \dots, s_t]$. For notational simplicity, the dependence of all model variables – including all policy variables – on the state history s^t is suppressed. $\pi(s_t)$ is the probability state s_t occurs in period t . Households have rational expectations, where the expectations operator over variable x_{t+1} as of time t is defined as $E_t[x_{t+1}] \equiv \sum_{\mathcal{S}_{t+1}} x_{t+1} \pi(s_{t+1})$. In line with the notational convention in Woodford (2003) it's assumed that the interest rate in period t is equal to i_{t-1} . That is, the interest rate is determined before state s_t is realized.

There is a representative household that consumes, supplies labor on the extensive margin, invests in shares of firms and holds risk-free and state-contingent government bonds. The economy is cashless.⁷ The household consists of a continuum of individuals of unit mass that differ in their participation costs θ_t . The household optimally decides how many of its members participate on the labor market.⁸ Participation costs are private information and reflect the value of leisure, home production, commuting costs and the value of immaterial benefits from work, such as social inclusion and avoiding the stigma of unemployment. The time-invariant cumulative distribution of participation costs θ_t is denoted by $H(\theta_t)$ and the corresponding probability density function is denoted by $h(\theta_t)$, which has support $\theta_t \in (-\infty, \infty)$.⁹

The representative household maximizes the expected discounted value of utilities $U(\cdot)$ from private consumption C_t , public goods G_t , and disutility $V(\cdot)$ from participation costs $\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t$:

$$\mathcal{U} \equiv E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[U(C_t, G_t, \xi_t) - V \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \right] \right], \quad (1)$$

$$\beta^t < 1, \quad U_C, U_G, V' > 0, \quad U_{CC}, U_{GG}, V'' < 0,$$

where β^t is the discount rate and ξ_t designates a potentially stochastic preference shock. The household takes public goods G_t in each period and state as given. By explicitly allowing for public goods in the utility function, government spending may serve both a classical, Samuelsonian role – public goods provision – and a Keynesian role – stabilizing the business cycle. If household members work, they incur participation costs, where $\bar{\theta}_t$ is the cut-off value of participation costs below which individuals supply their labor (see below). For simplicity

⁷The assumption that the economy is cashless is the limit of a model with cash. This assumption is also made by Correia et al. (2013).

⁸This setup allows for heterogeneity within the household, but not across households. Therefore, participation taxes feature no distributional benefits and would be optimally zero in the absence of business cycle fluctuations.

⁹A time-varying distribution of participation costs can be allowed for without loss of generality. Since this yields no additional insights, the main text assumes time-invariant participation costs.

it's assumed that participation costs enter utility separably.

Financial markets are complete. The representative household can trade risk-free government bonds and state-contingent government bonds. Moreover, it can buy shares of firms. The budget constraint of the household is given by:

$$\begin{aligned}
& \bar{B}_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] + (1 - \sigma_t^K)P_tK_{t+1} & (2) \\
& = (1 + \rho_{t-1})(\bar{B}_t + B_{t-1,t}) + (1 - \sigma_t^K)(1 - \delta)P_tK_t \\
& + (w_t - T_t)H(\bar{\theta}_t) + (1 - \tau_t^K)R_tK_t - \tau_t^A P_tK_t \\
& + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_tC_t,
\end{aligned}$$

where \bar{B}_t denotes nominal holdings of riskless government bonds in period t . i_{t-1} is the nominal one-period interest rate on the risk-free bond in period t . $B_{t,t+1}$ is the nominal quantity of state-contingent bonds at t in state s_{t+1} . $Q_{t,t+1}$ is the price of a state-contingent bond. The return of the state contingent bond $B_{t,t+1}$ is harmlessly normalized such that it pays out gross return $1 + i_{t-1}$ if state s_t occurs and zero otherwise. This is done for expositional ease and to avoid notational clutter later on. K_{t+1} denotes holdings of shares in firms, which are claims on the physical capital stock. R_t is dividend income on shares. Alternatively, K_{t+1} can be viewed as physical capital supplied by the household with R_t as its rental rate. Initial assets are normalized to zero. Moreover, a no-Ponzi-game condition on terminal assets is imposed.

The government taxes the returns from risk-free bonds and state contingent bonds symmetrically. τ_t^K is the tax on capital income, which is levied on nominal capital income (interest on risk free bonds, returns on state-contingent bonds and rental income). τ_t^A denotes the wealth tax on nominal total wealth (risk-free bonds, state-contingent bonds and capital). For later reference, $\rho_t \equiv (1 - \tau_t^K)i_{t-1} - \tau_t^A$ is the *net* nominal interest rate on bonds. Shares are treated differently than bonds. σ_t^K is the subsidy on investment in physical capital. Due to complete financial markets, the model features Ricardian equivalence in the absence of taxes, subsidies or transfers.

w_t is the nominal wage rate. T_t is the tax on labor income. And τ_t^C is the consumption tax. $H(\bar{\theta}_t)$ denotes effective labor supply, i.e., the fraction of the household members that work. The remainder, i.e., $1 - H(\bar{\theta}_t)$, of household members are unemployed and incur unemployment benefits b_t . The income tax is not a lump-sum tax, since it distorts the decision of individuals to participate in the labor market. Non-distortionary lump-sum taxes T_t are part of the instrument set of the government if it sets the unemployment benefit equal to the tax on workers and zero consumption taxes, i.e., $b_t = -T_t$, and $\tau_t^C = 0$. Π_t denotes profit income from firms – discussed below – which are taxed at rate τ_t^Π .

Since complete markets are assumed, the no-arbitrage condition (NAC) for state-contingent bonds implies that returns on risk-free and state-contingent government bonds are equalized – see Appendix A:

$$\mathbb{E}_t[Q_{t,t+1}] = 1. \quad (3)$$

Since the state-contingent bond pays out gross return $1 + i_{t-1}$ if state s_t occurs and zero otherwise the expected state-contingent bond price over all contingencies $\mathbb{E}_t[Q_{t,t+1}]$ must be equal to the price of a risk-free bond that yields the same nominal return of $1 + i_{t-1}$. Note that the tax system does not distort the portfolio of risk-free and state-contingent bonds, since the returns on both assets are taxed symmetrically.

Moreover, there is a NAC for bonds and investments in physical capital – see Appendix A:

$$\mathbb{E}_t \left[\frac{Q_{t,t+1}}{1 + \rho_t} \left[\frac{1 - \sigma_{t+1}^K}{1 - \sigma_t^K} (1 - \delta) P_{t+1} + \frac{(1 - \tau_{t+1}^K) R_{t+1} - \tau_{t+1}^A P_{t+1}}{1 - \sigma_t^K} \right] \right] = P_t, \quad (4)$$

This NAC ensures that the return on physical capital is equal to that of risk-free government bonds. The investment tax credit on investment in physical capital makes investment more attractive compared to holding bonds. In the absence of stochastics, the NAC can be reduced to: $(1 - \tau_{t+1}^K) R_{t+1} = (1 + \rho_t)(1 - \sigma_t^K) P_t - [(1 - \sigma_{t+1}^K)(1 - \delta) - \tau_{t+1}^A] P_{t+1}$. And without stochastics and taxes or subsidies it is equal to: $\frac{R_{t+1}}{P_{t+1}} = \frac{1+i_t}{1+\pi_t} - (1 - \delta) \simeq r_t + \delta$, where the inflation rate is defined in terms of producer prices as $\frac{P_{t+1}}{P_t} \equiv 1 + \pi_t$. Hence, the real return on physical capital minus depreciation should be equal to the real return on bonds.

Optimal demand for safe assets implies the following intertemporal Euler equation for consumption:

$$\frac{U_C(C_t, G_t, \xi_t)}{(1 + \tau_t^C) P_t} = \beta(1 + \rho_t) \mathbb{E}_t \left[\frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^C) P_{t+1}} \right]. \quad (5)$$

Moreover, optimal demand for state-contingent bonds implies that the net stochastic discount factor equals:

$$\frac{Q_{t,t+1}}{1 + \rho_t} = \frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{U_C(C_t, G_t, \xi_t)} \frac{(1 + \tau_t^C) P_t}{(1 + \tau_{t+1}^C) P_{t+1}}, \quad \forall s_{t+1}. \quad (6)$$

Both these equations are completely standard in the New Keynesian model and pin down optimal demand for risk-free and state-contingent bonds. The reader is referred to Woodford (2003) for more extensive discussion.

Recall, the net nominal interest rate is $\rho_t \equiv (1 - \tau_t^K)i_{t-1} - \tau_t^A$. Therefore, a path of taxes on capital income τ_t^K (provided nominal interest rates are positive) or a path of taxes on wealth τ_t^A (for any nominal interest rate), can perfectly replicate any monetary policy consisting of a path of nominal interest rates i_{t-1} . Therefore, the government has one monetary instrument (the nominal interest rate) and three tax instruments (taxes on consumption, capital income and wealth) to control the intertemporal allocation of resources. Moreover, two of these instruments (taxes on consumption and wealth) are independent from the lower bound on the nominal interest rate.

Finally, the cut-off $\bar{\theta}_t$ for labor participation in each period is determined by:

$$\frac{V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \bar{\theta}_t}{U_C(C_t, G_t, \xi_t)} = \frac{(1 - \tau_t^L)w_t}{(1 + \tau_t^C)P_t}. \quad (7)$$

where $\tau_t^L \equiv \frac{T_t + b_t}{w_t}$ is the participation tax rate. Labor participation is distorted downwards if the government taxes labor income and provides unemployment benefits. Intuitively, an individual moving from unemployment into the labor market starts paying taxes and gives up an unemployment benefit. By reducing the net benefits of participation, the consumption tax also distorts labor participation downwards, since participation costs are untaxed utility costs, while the participation benefit, consumption, is taxed. For later reference, define period- t aggregate notional labor supply L_t as:

$$L_t = L \left(\frac{(1 - \tau_t^L)w_t}{(1 + \tau_t^C)P_t}, C_t, G_t, \xi_t \right) \equiv H(\bar{\theta}_t). \quad (8)$$

If wages are rigid, effective labor supply (equal to employment) can be below notional labor supply, thereby generating involuntary unemployment, see below.

By substituting the NACs into the household budget constraint, and defining total wealth as $W_{t+1} \equiv \bar{B}_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] + (1 - \sigma_t^K)P_tK_{t+1}$, we can derive the household budget constraint in terms of total wealth – see Appendix A:

$$W_{t+1} = (1 + \rho_{t-1})W_t + (w_t - T_t)H(\bar{\theta}_t) + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_tC_t. \quad (9)$$

3.2 Final goods firms

There is a perfectly competitive final goods sector producing output Y_t using a Dixit-Stiglitz aggregate over a continuum of $j \in [0, 1]$ different intermediate goods $Y_{j,t}$:

$$Y_t \equiv \left(\int_0^1 (Y_{j,t})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1, \quad (10)$$

where ε stands for the elasticity of intermediate good demands. Denote by $P_{j,t}$ the price of intermediate good j in period t . Final goods firms maximize profits by taking prices for each intermediate good as given. Optimal intermediate good demand for each variety is then given by:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y_t, \quad \forall j \in [0, 1]. \quad (11)$$

Moreover, the ideal price index of intermediate goods in period t is given by:

$$P_t \equiv \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \quad (12)$$

3.3 Intermediate good firms

A continuum $j \in [0, 1]$ of identical firms produces intermediate good $Y_{j,t}$ using labor $E_{j,t}$ and capital $K_{j,t}$ as inputs (E_t refers to employment). Each firm has an identical, constant-returns-to-scale production technology $F(\cdot)$, which features positive, but decreasing returns to each factor of production:

$$Y_{j,t} = A_t F(K_{j,t}, E_{j,t}), \quad F_K, F_E > 0, \quad F_{KK}, F_{EE}, -F_{KE} \leq 0, \quad A_t > 0. \quad (13)$$

A_t is a potentially stochastic time-varying technology parameter common to all firms j . Rental rates of capital are R_t . Firms take rental rates, wage rates, aggregate output, aggregate prices, and the demand curve in equation (11) as given. Nominal profits of each intermediate good producer are denoted by:

$$\Pi_{j,t} \equiv (1 + \sigma_t^Y) P_{j,t} Y_{j,t} - R_t K_{j,t} - (1 - \sigma_t^P) w_t E_{j,t}, \quad (14)$$

where σ_t^Y is a subsidy on output – to potentially off-set the monopoly distortion –, and σ_t^P denotes a wage subsidy (or payroll tax if it is negative). Profit maximization implies that firms equalize the marginal revenue product of capital and labor to the monopoly mark-up

$\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1}$ over marginal labor cost:

$$(1 + \sigma_t^Y)P_{j,t}A_tF_K(K_{j,t}, E_{j,t}) = \mathcal{M}R_t, \quad (15)$$

$$(1 + \sigma_t^Y)P_{j,t}A_tF_E(K_{j,t}, E_{j,t}) = \mathcal{M}(1 - \sigma_t^P)w_t. \quad (16)$$

Output subsidies or wage subsidies both increase factor demands.

Let the total cost of producing output $Y_{j,t}$ be denoted by $\Psi(Y_{j,t})$, which can be written as:

$$\Psi(Y_{j,t}) \equiv \frac{(1 + \sigma_t^Y)}{\mathcal{M}}P_{j,t}Y_{j,t}. \quad (17)$$

Under perfect competition ($\varepsilon \rightarrow \infty$, $\mathcal{M} = 1$), marginal cost of output $\psi(Y_{j,t}) \equiv \Psi'(Y_{j,t})$ would be constant and equal to $(1 + \sigma_t^Y)P_t$.

As usual in the New Keynesian framework, price rigidity is modeled by employing staggered Calvo (1983) pricing. During each period t a firm has probability $1 - \alpha$ to re-optimize its price at $P_{j,t}^*$, while with probability α the firm needs to keep prices fixed at the level of the previous period $P_{j,t-1}$. It's assumed that firms set prices before consumption taxes. This implies that the incidence of consumption taxes falls for 100 percent on consumers.

Profits in period $t + v$ of a firm having set price $P_{j,t}^*$ at t are then given by: $\Pi_{j,t+v} \equiv (1 + \sigma_t^Y)P_{j,t}^*Y_{j,t+v} - \Psi(Y_{j,t+v})$. All firms j that reoptimize prices in period t maximize the expected value of discounted real profits $\Pi_{j,t+v}/P_{t+v}$, subject to their demand function:

$$\max_{P_{j,t}^*} \mathbf{E}_t \left(\sum_{v=0}^{\infty} \alpha^v \frac{Q_{t,t+v}}{\prod_{s=t}^v (1 + \rho_s)} \left(\frac{(1 + \sigma_t^Y)P_{j,t}^*Y_{j,t+v}}{P_{t+v}} - \frac{\Psi(Y_{j,t+v})}{P_{t+v}} \right) \right), \quad \text{s.t. } Y_{j,t+v} = \left(\frac{P_{j,t}^*}{P_{t+v}} \right)^{-\varepsilon} Y_{t+v}. \quad (18)$$

The first-order condition for the optimal price in period t is then given by:

$$\sum_{v=0}^{\infty} \alpha^v \mathbf{E}_t \left[\frac{Q_{t,t+v}}{\prod_{s=t}^v (1 + \rho_s)} \frac{Y_{j,t+v}}{P_{t+v}} \left((1 + \sigma_t^Y)P_{j,t}^* - \frac{\varepsilon}{(\varepsilon - 1)} \psi(Y_{j,t}) \right) \right] = 0, \quad (19)$$

By using the ideal price index from eq. (12), and imposing symmetry across firms (i.e.,

$P_{j,t-1} = P_{t-1}$, $P_{j,t}^* = P_t^*$), the aggregate price level thus evolves over time according to:

$$P_t = [(1 - \alpha)P_t^{*1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}, \quad (20)$$

where the optimal price set in each period t is given by:

$$P_t^* = \frac{\varepsilon}{(\varepsilon - 1)} \frac{\sum_{v=0}^{\infty} \alpha^v \mathbb{E}_t \left[\frac{Q_{t,t+v} \frac{\psi(Y_{j,t})}{(1+\sigma_t^Y)} P_{t+v}^{\varepsilon-1} Y_{t+v}}{\prod_{s=t}^v (1+\rho_s)} \right]}{\sum_{v=0}^{\infty} \alpha^v \mathbb{E}_t \left[\frac{Q_{t,t+v} P_{t+v}^{\varepsilon-1} Y_{t+v}}{\prod_{s=t}^v (1+\rho_s)} \right]}. \quad (21)$$

Aggregate output and factor demands are driven below potential output due to staggered Calvo pricing. To find the firm aggregates, use the demand functions (13) and the production function (13), and impose symmetry to derive aggregate production:

$$Y_t = D_t A_t F(K_t, E_t), \quad (22)$$

where $D_t \equiv \left[\int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} dj \right]^{-1} \leq 1$ denotes price dispersion, which is a measure for the production distortion in the economy. If prices would be perfectly flexible, there would be no price dispersion ($D_t = 1$), and all production capacity is fully utilized. D_t can be rewritten as:

$$D_t^{-1} = \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} dj = \sum_{v=0}^{t+1} \omega_v \left(\frac{P_{t-v}^*}{P_t} \right)^{-\varepsilon} dj, \quad (23)$$

where $\omega_v \equiv \alpha^v(1 - \alpha)$ is share of firms that changed prices j periods before, and $\omega_{t+1} \equiv \alpha^{t+1}$ is the share of firms that never changed prices, charging price P_0^* .

In the presence of price dispersion, aggregate factor demands can thus be rewritten as:

$$(1 + \sigma_t^Y) P_t D_t A_t F_K(K_t, E_t) = \mathcal{M} R_t, \quad (24)$$

$$(1 + \sigma_t^Y) P_t D_t A_t F_E(K_t, E_t) = \mathcal{M}(1 - \sigma_t^P) w_t. \quad (25)$$

From the latter equation follows the labor demand equation:

$$E_t = E \left(\frac{\mathcal{M}(1 - \sigma_t^P) w_t}{(1 + \sigma_t^Y) P_t D_t A_t}, K_t \right). \quad (26)$$

3.4 Fiscal policy

The government provides public goods G_t , levies taxes T_t on labor income of the employed, provides benefits b_t to the unemployed, levies taxes on consumption τ_t^C , levies taxes on capital income τ_t^K , levies wealth taxes τ_t^A , levies profit taxes τ_t^Π , subsidizes labor costs at rate σ_t^P , subsidizes investment costs at rate σ_t^K and subsidizes output at rate σ_t^Y . The government can issue risk-free bonds B_{t+1} and state-contingent bonds $B_{t,t+1}$. Hence, the government budget constraint (GBC) is given by:

$$\begin{aligned}
& B_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] \\
& = (1 + i_{t-1})(B_t + B_{t-1,t}) + P_t G_t + \sigma_t^Y P_t D_t A_t F(E_t) + \sigma_t^K (K_{t+1} - (1 - \delta)K_t) P_t \\
& \quad - (T_t - \sigma_t^P w_t) E_t + b_t(1 - E_t) - (\tau_t^K i_{t-1} + \tau_t^A)(B_t + B_{t-1,t}) \\
& \quad - \tau_t^K R_t K_t - \tau_t^A P_t K_t - \tau_t^C P_t C_t - \tau_t^\Pi \Pi_t.
\end{aligned} \tag{27}$$

3.5 Monetary policy

It's assumed that monetary policy is delegated to a central bank, which determines nominal interest rates. One can be completely agnostic about the central bank's objectives and presume that nominal interest rates follow some sequence $\{i_t\}_0^\infty$. The central bank may, for example, follow a Taylor rule for the nominal interest rate i_t :

$$i_t = \begin{cases} \Phi(\pi_t, \pi_t^*, Y_t, Y_t^n) & \text{if } \Phi(\cdot) > 0 \\ 0 & \text{if } \Phi(\cdot) \leq 0 \end{cases}, \quad \Phi_\pi, \Phi_Y > 0, \quad \Phi_{\pi^n}, \Phi_{Y_t^n} < 0. \tag{28}$$

π^* denotes the inflation target and Y_t^n denotes potential or natural output. The case where the economy hits the effective lower bound on nominal interest rates is explicitly allowed for. The effective lower bound is taken to be zero in this paper, as is common in the literature.¹⁰

3.6 General equilibrium

Nominal wages are completely rigid in each period and follow an exogenous process $\{\bar{w}_t\}_{t=0}^\infty$. For the purposes of this paper, wages are assumed to be driven above market-clearing levels for all periods t . This process of wages may gradually revert to the market-clearing wage over time or towards the wage rate consistent with the natural rate of unemployment. It is

¹⁰Since our model is assumed to be cashless, there is no fundamental reason for a lower bound on interest rates. Indeed, abolishing cash is seen as one of the main ways to circumvent the lower bound (Buiter and Rahbari, 2015; Rogoff, 2016). However, the model can be extended to allow for money in the utility function to rationalize the lower bound. Alternatively, our model can be viewed as the cashless limit of a model with money in the utility function. In either case, no insights would be obtained that would change our findings.

possible to provide many plausible micro-economic foundations for the underlying source of wage rigidity. For example, staggered wage setting, union behavior, minimum wages, search frictions, or efficiency wages. However, to determine optimal stabilization policy it is not of critical importance to know explicit process that generates wage rigidity, as long as the process of wage rigidity itself is not determined by stabilization policy. This can be defended, since stabilization policy is aimed at the short run and is unlikely to affect deep, structural labor-market distortions.

Given that wages are rigid and individuals are heterogeneous in their participation costs θ , an assumption needs to be made on rationing in the labor market. The *rationing schedule* specifies how unemployment is distributed over individuals with different participation costs, see also Gerritsen (2017), Gerritsen and Jacobs (2019) and Hummel and Jacobs (2019). It is assumed that *labor rationing is efficient*. This implies that the individuals with the highest cost of work θ will become unemployed first. If labor rationing is efficient, there exists a level of participation costs $\hat{\theta}_t \equiv H^{-1} \left(E \left(\frac{\mathcal{M}(1-\sigma_t^P)\bar{w}_t}{(1+\sigma_t^Y)P_t D_t A_t}, K_t \right) \right)$ below which individuals are employed and above which individuals are unemployed. Moreover, there is involuntary unemployment, since $L_t \equiv H(\bar{\theta}_t) > E_t \equiv H(\hat{\theta}_t)$ because $\bar{\theta}_t > \hat{\theta}_t$. Intuitively, notional labor supply is larger than labor demand if the wage is driven above the market-clearing level. Hence, all individuals with participation costs $\theta_t \in (-\infty, \hat{\theta}_t)$ are employed, individuals with participation costs $\theta_t \in (\hat{\theta}_t, \bar{\theta}_t)$ are involuntarily unemployed, and individuals with ability $\theta_t \in [\hat{\theta}_t, \infty)$ are voluntarily unemployed.

Due to the rationing in the labor market, the implicit tax rate $\tau_t^I \equiv \frac{\hat{w}_t - \bar{w}_t}{\bar{w}_t}$ on labor participation can be defined as the percentage difference between the rationed wage \bar{w}_t and the ‘virtual wage’ \hat{w}_t at which notional labor supply $L_t = H(\hat{\theta}_t)$ equals employment at wage \bar{w}_t in a perfectly competitive market, see Figure 1:

$$\hat{w}_t \equiv \frac{(1 + \tau_t^C)P_t}{(1 - \tau_t^L)} \frac{V' \left(\int_{-\infty}^{\hat{\theta}_t} \theta_t h(\theta_t) d\theta_t \right)}{U_C(C_t, G_t, \xi_t)} \hat{\theta}_t. \quad (29)$$

Consequently, using the implicit tax rate τ_t^I effective labor supply can be written as:

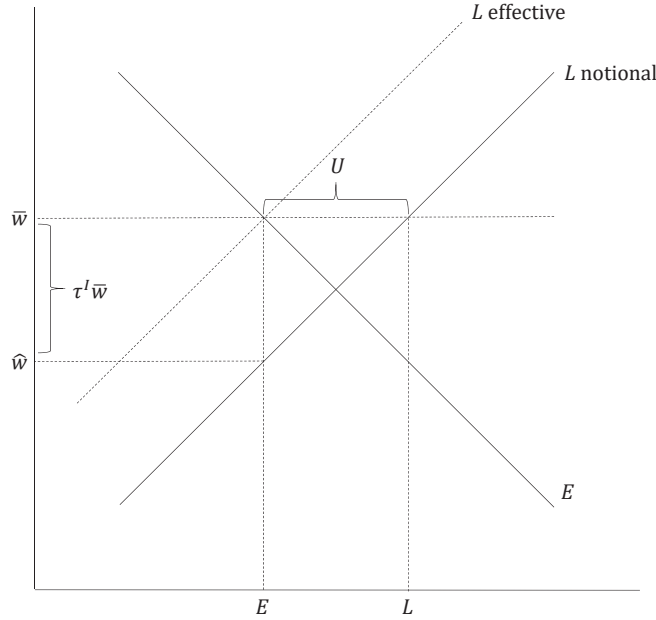
$$H(\hat{\theta}_t) \equiv H((1 - \tau_t^L - \tau_t^I)\bar{w}_t). \quad (30)$$

Total employment E_t and unemployment $U_t \equiv 1 - E_t$ in period t are thus given by:

$$E_t = L \left(\frac{(1 - \tau_t^L - \tau_t^I)\bar{w}_t}{(1 + \tau_t^C)P_t} \right), \quad U_t \equiv 1 - L \left(\frac{(1 - \tau_t^L - \tau_t^I)\bar{w}_t}{(1 + \tau_t^C)P_t} \right). \quad (31)$$

Involuntary unemployment results if the wage \bar{w}_t is set above the market-clearing level

Figure 1: Labor market rationing due to rigid wages



$(E_t < L \left(\frac{(1-\tau_t^L)\bar{w}_t}{(1+\tau_t^C)P_t} \right))$. If wages are flexible, the implicit tax is zero ($\tau_t^I = 0$), and there is full employment ($E_t = L \left(\frac{(1-\tau_t^L)w_t}{(1+\tau_t^C)P_t} \right)$). The implicit tax on labor τ_t^I thus captures the distortions created by involuntary unemployment, which is key to understand the concerns with involuntary unemployment. This distortion is absent in the standard New Keynesian model.

Capital market clearing implies that demand for assets by households equals the supply of assets by firms and the government:

$$W_{t+1} = \bar{B}_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] + (1 - \sigma_t^K)P_tK_{t+1}. \quad (32)$$

Goods market clearing implies that total production equals total private consumption, public goods and net investment – see Appendix B:

$$C_t + G_t + K_{t+1} - (1 - \delta)K_t = D_t A_t F(K_t, E_t). \quad (33)$$

D_t is the loss in output due to price dispersion, which acts as a tax on production.

4 Optimal stabilization policy

This section explores optimal stabilization policy in the model. It starts by deriving the decentralized allocation. Subsequently, it gives the first-best allocation. Then, the optimal policy implementation is derived.

4.1 Decentralized allocation

The next Proposition summarizes the decentralized allocation of the model.

Proposition 1 *For any given set of government instruments, the decentralized allocation in all periods t is characterized by:*

$$\frac{U_C(C_t, G_t, \xi_t)}{(1 + \tau_t^C)P_t} = \beta(1 + \rho_t)E_t \left[\frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^C)P_{t+1}} \right], \quad \rho_t \equiv (1 - \tau_t^K)i_{t-1} - \tau_t^A, \quad (34)$$

$$\frac{Q_{t,t+1}}{1 + \rho_t} = \frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{U_C(C_t, G_t, \xi_t)} \frac{(1 + \tau_t^C)P_t}{(1 + \tau_{t+1}^C)P_{t+1}}, \quad \forall s_{t+1}, \quad (35)$$

$$E_t[Q_{t,t+1}] = 1, \quad (36)$$

$$E_t \left[\frac{Q_{t,t+1}}{1 + \rho_t} \left[\frac{1 - \sigma_{t+1}^K}{1 - \sigma_t^K} (1 - \delta)P_{t+1} + \frac{(1 - \tau_{t+1}^K)R_{t+1} - \tau_{t+1}^A P_{t+1}}{1 - \sigma_t^K} \right] \right] = P_t, \quad (37)$$

$$\bar{\theta}_t = \frac{U_C(C_t, G_t, \xi_t)}{V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right)} \frac{(1 - \tau_t^L)w_t}{(1 + \tau_t^C)P_t}, \quad (38)$$

$$W_{t+1} = (1 + \rho_{t-1})W_t + (w_t - T_t)H(\hat{\theta}_t) + b_t(1 - H(\hat{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_t C_t, \quad (39)$$

$$(1 + \sigma_t^Y)D_t A_t F_K(K_t, E_t) = \mathcal{M} \frac{R_t}{P_t}, \quad \mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1}, \quad (40)$$

$$(1 + \sigma_t^Y)D_t A_t F_E(K_t, E_t) = \mathcal{M}(1 - \sigma_t^P) \frac{w_t}{P_t}, \quad (41)$$

$$P_t^* = \frac{\varepsilon}{(\varepsilon - 1)} \frac{\sum_{v=0}^{\infty} \alpha^v E_t \left[\frac{Q_{t,t+v}}{\prod_{s=t}^v (1 + \rho_s)} \frac{\psi(Y_{j,t})}{(1 + \sigma_t^Y)} P_{t+v}^{\varepsilon-1} Y_{t+v} \right]}{\sum_{v=0}^{\infty} \alpha^v E_t \left[\frac{Q_{t,t+v}}{\prod_{s=t}^v (1 + \rho_s)} P_{t+v}^{\varepsilon-1} Y_{t+v} \right]}, \quad (42)$$

$$D_t = \left[\int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} dj \right]^{-1} = \left[\sum_{v=0}^{t+1} \omega_v \left(\frac{P_{t-v}^*}{P_t} \right)^{-\varepsilon} dj \right]^{-1}, \quad (43)$$

$$P_{t+1} = \left[(1 - \alpha)P_{t+1}^{*1-\varepsilon} + \alpha P_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad \pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1, \quad (44)$$

$$\Pi_t = (1 + \sigma_t^Y) \left(\frac{\mathcal{M} - 1}{\mathcal{M}} \right) P_t D_t A_t F(K_t, E_t), \quad (45)$$

$$i_t = \bar{i}_t \text{ or } i_t = \begin{cases} \Phi(\pi_t, \pi_t^*, Y_t, Y_t^n) & \text{if } \Phi(\cdot) > 0 \\ 0 & \text{if } \Phi(\cdot) \leq 0 \end{cases}, \quad (46)$$

$$E_t = L\left(\frac{(1 - \tau_t^L - \tau_t^I)\bar{w}_t}{(1 + \tau_t^C)P_t}\right), \quad U_t \equiv 1 - L\left(\frac{(1 - \tau_t^L - \tau_t^I)\bar{w}_t}{(1 + \tau_t^C)P_t}\right), \quad \tau_t^I \equiv \frac{\hat{w}_t - \bar{w}_t}{\bar{w}_t}, \quad (47)$$

$$\hat{\theta}_t \equiv H^{-1}\left(E\left(\frac{\mathcal{M}(1 - \sigma_t^P)\bar{w}_t}{(1 + \sigma_t^Y)P_t D_t A_t}, K_t\right)\right), \quad \hat{w}_t \equiv \frac{(1 + \tau_t^C)P_t}{(1 - \tau_t^L)} \frac{V'\left(\int_{-\infty}^{\hat{\theta}_t} \theta_t h(\theta_t) d\theta_t\right)}{U_C(C_t, G_t, \xi_t)} \hat{\theta}_t \quad (48)$$

$$Y_t = D_t A_t F(K_t, E_t) = C_t + G_t + K_{t+1} - (1 - \delta)K_t, \quad (49)$$

$$W_{t+1} = \bar{B}_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] + (1 - \sigma_t^K)P_t K_{t+1}. \quad (50)$$

4.2 First-best allocation

To derive the optimal policy implementation, the next proposition derives the first-best allocation.

Proposition 2 *Let the natural rate of interest be denoted as r_t^n and the natural wage rate w_t^n , then the first-best allocation in all periods t is characterized by:*

$$\frac{U_C(C_t, G_t, \xi_t)}{E_t[U_C(C_{t+1}, G_{t+1}, \xi_{t+1})]} = \beta(1 + r_t^n), \quad (51)$$

$$Q_{t,t+1} = 1, \quad (52)$$

$$\frac{R_{t+1}}{P_{t+1}} = r_t^n + \delta, \quad (53)$$

$$\bar{\theta}_t = \frac{U_C(C_t, G_t, \xi_t)}{V'\left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t\right)} \frac{w_t^n}{P_t}, \quad (54)$$

$$W_{t+1} = (1 + r_t^n)W_t + w_t H(\bar{\theta}_t) - P_t C_t, \quad (55)$$

$$A_t F_E(K_t, E_t) = \frac{w_t^n}{P_t}, \quad (56)$$

$$A_t F_K(K_t, E_t) = \frac{R_t}{P_t}, \quad (57)$$

$$P_{t+1} = P_t^* = P_t, \quad \pi_t = 0, \quad D_t = 1, \quad (58)$$

$$\Pi_t = 0, \quad (59)$$

$$i_t = r_t^n = \rho_t, \quad (60)$$

$$E\left(\frac{w_t}{P_t A_t}, K_t\right) = L\left(\frac{w_t}{P_t}\right), \quad U_t \equiv 1 - L\left(\frac{w_t}{P_t}\right), \quad \tau_t^I = 0, \quad (61)$$

$$\hat{\theta}_t = \bar{\theta}_t = H^{-1}\left(E\left(\frac{w_t}{P_t A_t}, K_t\right)\right), \quad \hat{w}_t = w_t \quad (62)$$

$$Y_t = A_t F(K_t, E_t) = C_t + G_t + K_{t+1} - (1 - \delta)K_t, \quad (63)$$

$$W_{t+1} = \bar{B}_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] + P_t K_{t+1}. \quad (64)$$

$$\frac{U_G(C_t, G_t, \xi_t)}{U_C(C_t, G_t, \xi_t)} = 1. \quad (65)$$

Proof. See Appendix C. ■

The first-best allocation features constant marginal utilities of consumption across states (full insurance), constant prices, zero inflation and no price dispersion. Indeed, any deviation from zero inflation results in production distortions due to staggered price setting. Moreover, the consumption Euler equation should be such that consumption choices are based on the real natural rate of interest – the interest rate that prevails in the flexible price and wage equilibrium at full capacity utilization and full employment. The natural, real rate of interest should equal the real rental rate of capital minus the depreciation rate. In the absence of inflation both real and nominal interest rates are the same. Optimal labor participation would be determined only by the gross real wage $\frac{w_t}{P_t}$. Public goods provision is dictated only by classical motives, i.e., the Samuelson (1954)-rule. The marginal rate of substitution between public and private goods $\frac{U_G}{U_C}$ equals the unit marginal rate of transformation of public and private goods. Labor demand would be based on the real marginal cost of labor without any mark-ups for monopoly pricing. Wages would be perfectly flexible so that no involuntary unemployment results. Hence, implicit taxes on labor would be zero.

4.3 Implementation of first-best

The question is whether the government has sufficient instruments to implement the first-best allocation, and the next Proposition demonstrates that it has.

Proposition 3 *The first-best allocation can be implemented with fiscal instruments only for any central bank policy $\{i_t\}_0^\infty$ and for any pattern of nominal wages $\{\bar{w}_t\}_0^\infty$.*

Whether the effective lower bound on interest rates is binding or not, taxes on consumption, capital income and wealth should ensure intertemporal efficiency in consumption choices:

$$1 + r_{t+1}^n = \frac{(1 + \tau_t^C)}{(1 + \tau_{t+1}^C)} (1 + (1 - \tau_{t+1}^K) i_{t+1}^n - \tau_{t+1}^A), \quad (66)$$

The optimal investment tax credit ensures efficient investment in physical capital:

$$(\sigma_{t+1}^K - \sigma_t^K) + (\tau_{t+1}^K - \sigma_t^K) r_{t+1}^n + (\tau_{t+1}^K - \sigma_{t+1}^K) \delta + \tau_{t+1}^A = 0. \quad (67)$$

The optimal labor tax ensures efficiency in labor supply:

$$-\tau_t^L = \tau_t^C. \quad (68)$$

The labor subsidy ensures zero involuntary unemployment and zero implicit taxes on labor:

$$\sigma_t^P = \frac{\bar{w}_t - w_t^n}{\bar{w}_t}, \quad \tau_t^I = 0. \quad (69)$$

Public goods are provided according to the Samuelson rule:

$$\frac{U_G(C_t, G_t)}{U_C(C_t, G_t)} = 1. \quad (70)$$

The output subsidy ensures efficiency in aggregate supply:

$$\sigma_t^Y = \frac{1}{\varepsilon - 1}. \quad (71)$$

Proof. See Appendix D. ■

In a world with staggered pricing, wage rigidity, and a possibly binding lower bound on interest rates, the government has a sufficiently rich fiscal instrument set to implement the first-best allocation. Hence, the government can fight the consequences of any business-cycle deviation from natural output and employment. The optimal fiscal policy implementation ensures constant prices, zero inflation, and no price dispersion, as is standard in the New Keynesian model (Woodford, 2003; Galí, 2008). Furthermore, taxes on consumption, capital income and wealth should be consistent with intertemporal consumption efficiency in eq. (66).

Participation is optimally not taxed on a net basis as the optimal income tax in eq. (68) shows. There will be no distortion in labor supply. If consumption is taxed at positive rates, then labor income should be subsidized, and vice versa. Clearly, this result stems from the representative-agent setting and would change in heterogeneous-agent settings, where optimal participation taxes are positive (Diamond, 1980; Saez, 2002).

More importantly, subsidies on labor demand ensure that any distortion from rigid wages is perfectly eliminated so that no involuntary unemployment results, see eq. (69). Indeed, the optimal subsidy is equal to the percentage difference between the rationed wage and the natural wage rate at which there is no involuntary unemployment.

Public goods are provided according to the classical Samuelson rule as shown in eq. (70). Consequently, the sum of marginal benefits of the public good should be equal to its unit cost.

Finally, output subsidies on intermediate goods firms optimally off-set distortion from monopoly pricing, as can be seen from eq. (71). This result is well-known from the literature (Woodford, 2003; Galí, 2008). Since the focus of this paper is not on (structural) monopoly distortions in price setting, it will be assumed that these are always eliminated by means of the output subsidy, as in for example Galí (2008).

Surprisingly, from Proposition 3 follows that optimal fiscal policy in the New Keynesian model follows fundamentally *classical* policy prescriptions. Optimal policy ensures that all relevant economic distortions are removed with taxes and subsidies. The public finance approach to stabilize the business cycle thus gives different policy prescriptions than normally considered in standard macro-economic approaches. Optimal policy prescriptions are distinctly Pigouvian: optimal policy should remove wedges between private and social costs or benefits of economic decisions that originate from price and wage rigidities and the lower bound on interest rates (Pigou, 1920).¹¹

Moreover, the standard Keynesian argument for higher public spending to combat recessions and unemployment needs to be qualified. There is no sense in which over- or under-provision of public goods (or ‘digging holes’) can be desirable in the New Keynesian model to stabilize the business cycle. The government will not use public spending to stabilize the business cycle at all, since public goods provision always follows the Samuelson rule. Moreover, the government prefers to fight unemployment directly in the form of wage subsidies rather than public goods provision. Proposition 3 shows that the New Keynesian model is fundamentally ‘Pigouvian’ and ‘Samuelsonian’ and not ‘Keynesian’ in terms of policy implications.

This conclusion naturally depends on the set of available government instruments. The instrument considered here is sufficiently rich that multiple policy implementations ensure a first-best allocation. One can then ask the question: is the instrument set too rich? And, which realistic policy implementation is at least required to achieve the first-best allocation? One such minimalist implementation is given in the following Corollary.

Corollary 1 *A minimalist first-best policy implementation normalizes the tax rate on capital income to zero ($\tau_t^K = 0$), sets constant consumption taxes ($\tau_{t+1}^C = \tau_t^C$), and sets the wealth tax at $\tau_t^A = i_t - r_t^n$ for any i_t . The investment tax credit (σ_t^K), the optimal wage subsidy (σ_t^P) and the optimal participation tax (τ_t^L) follow from Proposition 3.*

This is very intuitive. The optimal wealth tax under flat consumption taxes equals minus the nominal interest rate minus the natural real rate of interest. Consequently, if the ELB

¹¹These findings are related to Correia, Nicolini, and Teles (2008), who also find that optimal fiscal policy can remove all distortions from price-rigidities. They allow for consumption and income taxes to eliminate any source of price rigidity.

is binding ($i_t = 0$), and the natural real rate of interest is substantially below zero, say -3 percent, then a wealth tax of 3 percent is able to overcome the ELB entirely. This role of the wealth tax in macro-economic stabilization policy barely been studied before in the macro-economics literature.¹²

Thus, an implementation with wealth taxes, investment credits, (constant) consumption and labor income taxes, and payroll taxes can implement the first-best allocation. Indeed, one does not require taxes on capital income, time-varying consumption taxes or even monetary policy to implement first-best. This instrument set is arguably not ‘too rich’, since all these fiscal policy instruments are commonly used by governments in advanced countries. We will get back to this in the concluding section.

Proposition 3 and Corollary 1 are importantly related to Correia et al. (2013). They exclude wealth taxes from the instrument set of the government. These authors show that a first-best allocation can also be achieved with an ‘unconventional fiscal policy’ consisting of an exponentially growing path of consumption taxes, joint with an exponentially growing path of labor subsidies, and investment tax credits on capital formation. There could be (political or time consistency) reasons why such a policy may be difficult to implement in practice. In contrast to Correia et al. (2013), this paper shows that exponential paths of consumption taxes and labor subsidies are no longer needed to implement the optimal allocation if wealth taxes are allowed for (whether the ELB is binding or not) or if taxes on capital income are allowed for (only if the ELB is not binding).

Our findings is also related to Mankiw and Weinzierl (2011) and Michaillat and Saez (2018), who analyze public spending and investment tax credits as second-best policy instruments in macro-models with demand shortfalls and a potentially binding lower bound. These authors find that deviations from Pigouvian and Samuelsonian principles can be desirable if the government has insufficient instruments to stabilize aggregate demand at first-best levels. In the New Keynesian model of this paper there is no such role for stabilization policy.

On a fundamental, information-theoretic level the New Keynesian model does not answer the question why the government would not be able to reach a first-best allocation. To do this, the government must be able to verify the following tax and subsidy bases: income, assets, consumption, investment costs and wage costs. In principle this information is all available to the government in modern economies. Hence, there must be other reasons – that the New Keynesian model does not address – why certain policy instruments are not

¹²To the best of the knowledge of the author, only Eggertsson (2011, p.87) mentions a wealth tax in passing by, but rules it out as a viable policy instrument. Moreover, in his model the ‘capital tax’ $\tilde{\tau}^K$ is a tax on the returns and the stock of wealth, so that net wealth equals $(1 - \tilde{\tau}_t^K)(1 + i_t)$ per unit of gross wealth. Hence, the ‘capital tax’ considered by Eggertsson is a combination of a tax on wealth and on interest income. This paper explicitly separates a tax on wealth and on capital income.

available to stabilize the business cycle.¹³ In the concluding section we return to this question and discuss political constraints, distributional issues and tax evasion.

4.4 Central banks are superfluous

The benchmark in the macro-economics literature is that stabilization policy is delegated to the central bank, whereas allocation and redistribution policies are delegated to the fiscal authorities. Under normal conditions, setting nominal interest rates via monetary policy is sufficient to implement the first-best allocation with constant prices, no inflation, and zero price dispersion, and thus avoid the adverse consequences of the business cycle, provided that staggered price setting is the only macro-economic distortion, i.e., the ‘divine coincidence’ applies (Blanchard and Galí, 2007).

Most macro-economic studies analyze settings in which the government has only one instrument to control the intertemporal allocation of consumption: the interest rate set by the central bank. However, standard fiscal policy instruments can control the intertemporal allocation of resources via differential consumption taxes, capital income taxes and wealth taxes, see eq. (66), which all determine the real *after-tax* interest rate. Therefore, the lines between monetary and fiscal policy are blurred and they cannot be treated in isolation. More importantly, the next Corollary shows that central banks are superfluous in the New Keynesian model.

Corollary 2 *If the fiscal instrument set of the government is sufficiently rich, then monetary policy is superfluous.*

Eq. (66) shows that multiple policy configurations are available that implement the first-best allocation. If nominal interest rates are positive and given, then a rising path of consumption taxes, a tax on capital income or a wealth tax can all lower the net-of-tax real return to saving (and vice versa). Therefore, anything the central bank does, can in principle be off-set by suitable fiscal policy actions. This remains true if the lower bound is binding. What is always necessary is that all distortions of fiscal policies on capital accumulation are neutralized by investment tax credits. Hence, if the fiscal instrument set of

¹³This is similar to the Ramsey (1927) analysis of optimal commodity taxation. Lump-sum taxes are ruled out from the instrument set of the government, often as a short-cut for an unspecified distributional problem. However, the Ramsey (1927) model itself does not provide any information-theoretic foundation for excluding lump-sum taxes, as Atkinson and Stiglitz (1976) have shown. In particular, Atkinson and Stiglitz (1976) take the distributional problem explicitly into account and show that if the government would not be interested in redistribution, and only in revenue raising as in Ramsey (1927), then it would set all commodity taxes to zero if it has access to non-individualized lump-sum taxes.

governments is sufficiently large, central banks can be made superfluous with suitable fiscal policy implementations.

Corollary 2 thus sheds a different light on the importance and relevance of the effective lower bound (ELB) on nominal interest rates in most macro-economic analyses. The ELB imposes a constraint on monetary policy. However, whether the ELB constrains the set of attainable allocations or not is a policy choice, not a technological constraint. In particular, it is a policy choice not to use fiscal instruments to implement negative real net returns to savings.

As before, one may question whether fiscal policy is really able to undo any action of the central bank. This is once again questioning the validity of the instrument set of the government. There may well be constraints on fiscal policy such that monetary policy is no longer superfluous. However, as before, the New Keynesian model itself cannot answer the question which constraints are the relevant ones. All necessary information is in principle available to the government to implement first-best with fiscal policy instruments alone.

5 Robustness

5.1 Public investment

Bouakez, Guillard, and Roulleau-Pasdeloup (2019) analyze public investment. They argue that public investment should be employed to stabilize the business cycle if the lower bound is binding. To verify whether this result continues in the current setting, one can allow for public capital K_t^G , besides private capital K_t . In particular, the production function and the economy's resource constraint can be modified to allow for public capital:

$$Y_{j,t} = A_t F(K_{j,t}, E_{j,t}, K_t^G), \quad F_K, F_E, F_{K^G} > 0, \quad F_{KK}, F_{EE}, F_{K^G K^G}, -F_{KE}, -F_{K K^G} \leq 0, \quad A_t > 0, \quad (72)$$

$$C_t + G_t + K_{t+1} - (1 - \delta)K_t + K_{t+1}^G - (1 - \delta)K_t^G = D_t A_t F(K_t, K_t^G, E_t), \quad (73)$$

where it is assumed that private and public capital are (weakly) complementary and public capital has the same depreciation rate of private capital δ . The next Proposition shows that optimal public investment is efficient if fiscal policy optimally stabilizes the business cycle.

Proposition 4 *Optimal fiscal policy follows the rules that are described in Proposition (3)*

and public investment all periods t is efficient and characterized by:

$$A_t F_{K^G}(K_t, E_t, K_t^G) = r_t^n + \delta. \quad (74)$$

Proof. See Appendix E. ■

This result does not come as a surprise. Since the market outcome without public investment and optimal fiscal policy is efficient, public investment cannot improve on the market outcome. Classical principles should therefore also guide optimal public investment. The results in Bouakez, Guillard, and Roulleau-Pasdeloup (2019) are therefore due to constraints on government instrument set so that a inefficiencies due to the lower bound cannot be removed in the absence of public investment.

6 Optimal fiscal policy implementation in the Eurozone during the Great Recession

Suppose that Eurozone countries implemented the optimal first-best policy as described in Proposition 3 during the Great Recession. How would the policy have looked like? In principle, no sophisticated DSGE analysis is needed to find the answer, since our model gives the optimal policy in terms of empirically measurable sufficient statistics. This section thus presents some back-of-the-envelope calculation of the intertemporal and labor wedges that monetary and fiscal policy should aim to stabilize.

The ECB has set interest rates to around zero since 2014. So we take the nominal interest rate to be constrained by a lower bound of zero. This is a downward biased estimate, since the ECB was initially reluctant to lower interest rates sufficiently and even raised it in 2011 after it thought the worst was over. If interest rates are bound at zero, the tax rate on capital income τ^K can take any arbitrary value. It is assumed that consumption taxes τ^C and participation taxes τ^L remain constant over time. Public goods G are assumed to be provided according to the Samuelson rule and the output subsidy σ^Y ensures efficient aggregate supply. Profit taxes τ^Π can take an arbitrary value. We assume that there was no wealth tax before the crisis, since it generally is non-existent in most Eurozone countries and if it exists it is typically very low.¹⁴ Under these conditions, the optimal wealth tax should be equal to: $\tau^A = -r^n$. The optimal payroll subsidy should ensure full employment

¹⁴OECD (2018) reports that in 2017 only 2 Eurozone countries have a wealth tax: France and Spain. The Netherlands should be added to this list, since it misleadingly labels its wealth tax as a capital income tax on fictitious returns. Outside the Eurozone only 2 OECD countries have a wealth tax: Norway and Switzerland. Top wealth tax rates vary from about 0.5–2.5 percent. Revenues are typically low: ranging from 0.2% of GDP in Spain to 1.0% of GDP in Switzerland.

after the rise in employment: $\sigma^P = \frac{\bar{w}-w^n}{\bar{w}}$. If the investment subsidy is constant over time, it should be equal to: $\sigma^K = \frac{\tau^A}{r^n+\delta}$. Hence, we require estimates of the natural rate r^n , the wedge on labor $\frac{\bar{w}-w^n}{\bar{w}}$ and the depreciation rate on capital δ to calculate the optimal wealth tax, the optimal payroll subsidy and the optimal investment subsidy.

By using applied DSGE models for the Eurozone, Hristov (2016) estimates that the natural rate of interest in the Eurozone during the Great Recession lies somewhere between -2% and -4% , depending on how credit frictions affect the real economy. Moreover, the real rates were lower in the period 2009–2013. Constâncio (2015) reports natural real interest rates for the Eurozone during period 2009–2016 lie between 0% and -2% according to market expectations, VAR-model estimates, DSGE-model estimates and so-called ‘inflation stabilizing’ interest rates, which stabilize inflation at the ECB target of 2% . Again, real rates were more negative in the earlier years. Using the Laubach and Williams (2003) methodology, Holston, Laubach, and Williams (2017, 2019) estimate that the real interest rate for the Eurozone declined during the Great recession and reached a trough of around -0.5% in 2013 and in August 2019 it stands at around -0.4% .

Based on this evidence one can reasonably argue that the real natural interest rate in the eurozone in 2010 was around -2% . Consequently, a wealth tax of $\tau^A = 2\%$ would have been sufficient to overcome the zero lower bound on nominal interest rates. If ECB interest rates would be positive, optimal wealth taxes would have been even higher to ensure a negative natural rate. Piketty (2014) estimated capital-output ratios for some advanced countries, including Germany, France, the UK and the US. Typically, the capital-output ratio for most advanced countries is around 5. Consequently, if all wealth would be subject to the wealth tax, then this would have raised approximately 10% of GDP in tax revenue.

A labor wedge is estimated using a standard labor demand function. The ECB (2019) reports that Eurozone unemployment increased from around 7% in 2008 to around 12% of the workforce in 2013. However, the Eurozone was booming in 2008, implying that the unemployment rate was below the natural rate. Indeed, the OECD estimates that the structural unemployment in the Eurozone in 2010 was around 9% (OECD, 2019). Therefore, a decrease in employment of 3% -points ($= 12\% - 9\%$) is taken to measure the labor wedge. This is an upward biased estimate since, fiscal austerity has driven unemployment upwards. At this moment it is not possible to correct the bias.

Labor demand elasticities can be used to infer the increase in the implicit tax on labor associated with the increase in unemployment. A labor demand elasticity of 0.25 is a common estimate in the literature (Lichter, Peichl, and Siegloch, 2015). A 3% -point decrease in employment is thus associated with a 12% increase in the real wage. Consequently, the implicit tax associated with the a 3% -point increase in employment has been 12% . Therefore,

if governments wanted to prevent the rise in unemployment, it had to reduce wage costs with $\sigma^P = 12\%$. If the labor share is 70% of GDP, a common estimate for many countries, this implies a wage subsidy of $0.7 \times 12\% = 8.4\%$ of GDP.

Finally, the increase in the wealth tax requires an investment tax credit on new capital formation to off-set distortions on capital formation. The depreciation rate δ is usually set at 2% in standard macro-models, see for example Krusell and Smith (2015). However, this does not make sense if real natural rates are -2.5% since, real rental rates of physical capital would then be negative in the absence of taxes and subsidies. Therefore, we take δ to be much higher at 8%, potentially to also account for an equity risk-premium. With a depreciation rate of 8%, the optimal subsidy equals $\sigma^K = 45\%$. Investment in the Eurozone in 2010 is around 21% GDP (IMF, 2019). Hence the optimal investment subsidy equals 9.5% of GDP. This number would be even higher if depreciation rates were assumed to be lower than 8%.

These figures demonstrate how ‘aggressive’ fiscal policy needs to be to overcome the zero lower bound on interest rates, to eliminate involuntary unemployment and to make sure the correct investment incentives remain in place. Indeed, the orders of magnitude of each of the policies are much larger than anything normally considered in everyday economic policy making. Moreover, the revenue from wealth taxes of 10% GDP is by far not sufficient to finance the payroll subsidy of 8.4% of GDP and the investment tax credits of 9.5% of GDP. Indeed, a fiscal expansion of 8% of GDP would be impossible according to budgetary rules of the Maastricht Treaty, given that no Eurozone country has had a budgetary surplus of more than 5% of GDP.

7 Conclusions

This paper analyzed optimal fiscal policy in the New Keynesian model with three main distortions: price rigidities, the lower bound on the nominal interest of the central bank and involuntary unemployment. It is shown that a sufficiently rich fiscal policy implementation is able to support a first-best allocation. In particular, the government optimally employs a wealth tax to overcome the lower bound, an investment tax credit to avoid intertemporal distortions in investment, and a wage subsidy to combat involuntary unemployment. Importantly, optimal fiscal policy has a distinctly Pigouvian flavor: eliminate wedges between the social costs and benefits in the capital and labor market. Moreover, the New Keynesian model is not Keynesian, since provision of public goods follows classical principles and are not provided to stabilize the business cycle. Central banks are superfluous in the New Keynesian model as any action of the central bank can be undone by the fiscal authority.

A Derivation no-arbitrage conditions and household budget constraint in terms of wealth

The Lagrangian for individual optimization is given by using $E_t[Q_{t,t+1}B_{t,t+1}] = \sum_{s_{t+1}}[Q_{t,t+1}B_{t,t+1}]\pi(s_{t+1})$:

$$\begin{aligned}
\mathcal{L} \equiv & \sum_{t=0}^{\infty} \sum_{s_t} \beta^t \left[U(C_t, G_t, \xi_t) - V \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \right] \pi(s_t) \\
& + \sum_{s_t} \lambda_t^s [(1 + \rho_{t-1})(\bar{B}_t + B_{t-1,t}) + (1 - \sigma_t^K)(1 - \delta)P_t K_t] \pi(s_t) \\
& + \sum_{s_t} \lambda_t^s [(w_t - T_t)H(\bar{\theta}_t) + (1 - \tau_t^K)R_t K_t - \tau_t^A P_t K_t] \pi(s_t) \\
& + \sum_{s_t} \lambda_t^s [b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_t C_t] \pi(s_t) \\
& - \sum_{s_t} \lambda_t^s \left[\bar{B}_{t+1} + \sum_{s_{t+1}} Q_{t,t+1} B_{t,t+1} \pi(s_{t+1}) + (1 - \sigma_t^K)P_t K_{t+1} \right] \pi(s_t),
\end{aligned} \tag{75}$$

where λ_t^s denotes the marginal utility of income in period t and state s_t . The first-order conditions (FOCs) for utility maximization in each state of the world s_t are given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 : \beta^t U_C(C_t, G_t, \xi_t) = \lambda_t(1 + \tau_t^C)P_t, \tag{76}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\theta}_t} = 0 : \beta^t V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \bar{\theta}_t h(\bar{\theta}_t) = \lambda_t(w_t - T_t - b_t)h(\bar{\theta}_t), \tag{77}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t,t+1}} = 0 : (1 + \rho_t)\lambda_{t+1}^s \pi(s_{t+1}) = \lambda_t Q_{t,t+1} \pi(s_{t+1}), \tag{78}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{B}_{t+1}} = 0 : (1 + \rho_t) \sum_{s_{t+1}} \lambda_{t+1} \pi(s_{t+1}) = \lambda_t, \tag{79}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 : \sum_{s_{t+1}} \lambda_{t+1} [(1 - \sigma_{t+1}^K)(1 - \delta)P_{t+1} + (1 - \tau_{t+1}^K)R_{t+1} - \tau_{t+1}^A P_{t+1}] \pi(s_{t+1}) = \lambda_t(1 - \sigma_t^K)P_t. \tag{80}$$

Note that we already took out the expectations of variables that are realized at period t , where $\lambda_t \equiv \sum_{s_t} \lambda_t^s \pi(s_t)$ is the non-stochastic multiplier on the realized date- t budget constraint.

From the FOC for C_t in eq. (76) and the FOC for θ_t in (77) follows:

$$\frac{V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \bar{\theta}_t}{U_C(C_t, G_t, \xi_t)} = \frac{(1 - \tau_t^L) w_t}{(1 + \tau_t^C) P_t}, \quad (81)$$

Combine the FOC for C_t in eq. (76) with the FOC for $B_{t,t+1}$ in eq. (78) to find the stochastic discount factor:

$$\frac{Q_{t,t+1}}{1 + \rho_t} = \frac{\lambda_{t+1}^s}{\lambda_t} = \frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{U_C(C_t, G_t, \xi_t)} \frac{(1 + \tau_t^C) P_t}{(1 + \tau_{t+1}^C) P_{t+1}}, \quad \forall s_{t+1}. \quad (82)$$

Moreover, take the expectations over $Q_{t,t+1} \lambda_t^s = (1 + \rho_t) \lambda_{t+1}^s$ and note that λ_t and ρ_t are not stochastic as of t to find:

$$\lambda_t \sum_{s_{t+1}} Q_{t,t+1} \pi(s_{t+1}) = (1 + \rho_t) \sum_{s_{t+1}} \lambda_{t+1}^s \pi(s_{t+1}), \quad (83)$$

$$\lambda_t \mathbf{E}_t[Q_{t,t+1}] = (1 + \rho_t) \mathbf{E}_t[\lambda_{t+1}^s]. \quad (84)$$

Rewrite the FOC for \bar{B}_{t+1} in eq. (79) using eq. (83) to find:

$$\lambda_t = (1 + \rho_t) \sum_{s_{t+1}} \lambda_{t+1}^s \pi(s_{t+1}) = \lambda_t \sum_{s_{t+1}} Q_{t,t+1} \pi(s_{t+1}), \quad (85)$$

$$\frac{\lambda_t}{1 + \rho_t} = \mathbf{E}_t[\lambda_{t+1}^s] = \frac{\lambda_t \mathbf{E}_t[Q_{t,t+1}]}{1 + \rho_t}. \quad (86)$$

Consequently, we derive the no-arbitrage condition between risk-free bonds and state-contingent bonds:

$$\mathbf{E}_t[Q_{t,t+1}] = 1. \quad (87)$$

The stochastic Euler equation follows from taking expectations on both sides of the

stochastic discount factor (82):

$$\frac{\lambda_t Q_{t,t+1}}{1 + \rho_t} = \lambda_{t+1}^s = \lambda_t \frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{U_C(C_t, G_t, \xi_t)} \frac{(1 + \tau_t^C) P_t}{(1 + \tau_{t+1}^C) P_{t+1}}, \quad \forall s_{t+1}. \quad (88)$$

$$\frac{\lambda_t \mathbb{E}_t[Q_{t,t+1}]}{1 + \rho_t} = \lambda_t \mathbb{E}_t \left[\frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{U_C(C_t, G_t, \xi_t)} \frac{(1 + \tau_t^C) P_t}{(1 + \tau_{t+1}^C) P_{t+1}} \right] = \mathbb{E}_t[\lambda_{t+1}^s]. \quad (89)$$

$$\mathbb{E}_t \left[\frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^C) P_{t+1}} \right] = \frac{U_C(C_t, G_t, \xi_t)}{(1 + \tau_t^C) P_t} \frac{1}{1 + \rho_t}. \quad (90)$$

We derive optimal capital investment of the household. Use the stochastic discount factor $Q_{t,t+1} \lambda_t^s = (1 + \rho_t) \lambda_{t+1}^s$ so that we find the no-arbitrage condition from the FOC for K in eq. (80) by substituting $Q_{t,t+1} \lambda_t^s = (1 + \rho_t) \lambda_{t+1}^s$:

$$\sum_{s_{t+1}} \frac{Q_{t,t+1}}{1 + \rho_t} \left[\frac{1 - \sigma_{t+1}^K}{1 - \sigma_t^K} (1 - \delta) P_{t+1} + \frac{(1 - \tau_{t+1}^K) R_{t+1} - \tau_{t+1}^A P_{t+1}}{1 - \sigma_t^K} \right] \pi(s_{t+1}) = P_t. \quad (91)$$

Define total (expected) assets in time $t + 1$ as of time t as $W_{t+1} \equiv \bar{B}_{t+1} + \mathbb{E}_t[Q_{t,t+1} B_{t,t+1}] + (1 - \sigma_t^K) P_t K_{t+1}$. Then the household budget constraint at time t (after state history s^t) can be written as:

$$\begin{aligned} \bar{B}_{t+1} + \mathbb{E}_t[Q_{t,t+1} B_{t,t+1}] + (1 - \sigma_t^K) P_t K_{t+1} &= W_{t+1} \\ &= (1 + \rho_{t-1})(\bar{B}_t + B_{t-1,t}) + (1 - \sigma_t^K)(1 - \delta) P_t K_t \\ &\quad + (w_t - T_t) H(\bar{\theta}_t) + (1 - \tau_t^K) R_t K_t - \tau_t^A P_t K_t \\ &\quad + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi) \Pi_t - (1 + \tau_t^C) P_t C_t, \quad \forall t, \end{aligned} \quad (92)$$

Lag the definition of W_{t+1} with one period and multiply with $1 + \rho_{t-1}$ to find:

$$(1 + \rho_{t-1}) W_t = (1 + \rho_{t-1}) \bar{B}_t + (1 + \rho_{t-1}) \mathbb{E}_{t-1}[Q_{t-1,t} B_{t-1,t}] + (1 + \rho_{t-1})(1 - \sigma_{t-1}^K) P_{t-1} K_t \quad (93)$$

Note that W_t is realized wealth conditional on history s_t . Hence, state contingent bonds have only paid out $(1 + \rho_{t-1}) B_{t-1,t}$ if state s_t occurred and zero otherwise. This implies that we can rewrite the last term for state s_t as:

$$\mathbb{E}_{t-1}[Q_{t-1,t} B_{t-1,t}] = \sum_{s_{t-1}} Q_{t-1,t} B_{t-1,t} \pi(s_{t-1}) = B_{t-1,t}. \quad (94)$$

So that the lagged realization of W_{t+1} can be rewritten as:

$$(1 + \rho_{t-1})W_t = (1 + \rho_{t-1})(\bar{B}_t + B_{t-1,t}) + (1 + \rho_{t-1})(1 - \sigma_{t-1}^K)P_{t-1}K_t. \quad (95)$$

Substitute this in the HBC to find:

$$\begin{aligned} W_{t+1} &= (1 + \rho_{t-1})W_t + (1 - \tau_t^K)R_tK_t - \tau_t^A P_t K_t \\ &\quad - (1 + \rho_{t-1})(1 - \sigma_{t-1}^K)P_{t-1}K_t + (1 - \sigma_t^K)(1 - \delta)P_t K_t \\ &\quad + (w_t - T_t)H(\bar{\theta}_t) + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_t C_t, \quad \forall t, \end{aligned} \quad (96)$$

Rewrite by collecting all terms involving K_t :

$$\begin{aligned} W_{t+1} &= (1 + \rho_{t-1})W_t \\ &\quad + (1 - \sigma_{t-1}^K)(1 + \rho_{t-1})K_t \left[\frac{1}{(1 + \rho_{t-1})} \left(\frac{1 - \sigma_t^K}{1 - \sigma_{t-1}^K} (1 - \delta)P_t + \frac{(1 - \tau_t^K)R_t - \tau_t^A P_t}{(1 - \sigma_{t-1}^K)} \right) - P_{t-1} \right] \\ &\quad + (w_t - T_t)H(\bar{\theta}_t) + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_t C_t, \quad \forall t, \end{aligned} \quad (97)$$

Use the realized NAC for investment in capital lagged with one period:

$$\frac{1}{1 + \rho_{t-1}} \left[\frac{1 - \sigma_t^K}{1 - \sigma_{t-1}^K} (1 - \delta)P_t + \frac{(1 - \tau_t^K)R_t - \tau_t^A P_t}{1 - \sigma_{t-1}^K} \right] = P_{t-1}. \quad (98)$$

Substitute this in the household budget constraint to find the HBC:

$$W_{t+1} = (1 + \rho_{t-1})W_t + (w_t - T_t)H(\bar{\theta}_t) + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_t C_t, \quad \forall t. \quad (99)$$

B Derivation resource constraint

The government budget constraint (GBC) is given by:

$$\begin{aligned} &B_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] \\ &= (1 + i_{t-1})(B_t + B_{t-1,t}) + P_t G_t + \sigma_t^Y P_t D_t A_t F(E_t) + \sigma_t^K (K_{t+1} - (1 - \delta)K_t)P_t \\ &\quad - (T_t - \sigma_t^P w_t)E_t + b_t(1 - E_t) - (\tau_t^K i_{t-1} + \tau_t^A)(B_t + B_{t-1,t}) \\ &\quad - \tau_t^K R_t K_t - \tau_t^A P_t K_t - \tau_t^C P_t C_t - \tau_t^\Pi \Pi_t. \end{aligned} \quad (100)$$

The household budget constraint (HBC) is:

$$\begin{aligned}
& \bar{B}_{t+1} + E_t[Q_{t,t+1}B_{t,t+1}] + (1 - \sigma_t^K)P_tK_{t+1} \\
& = (1 + \rho_{t-1})(\bar{B}_t + B_{t-1,t}) + (1 - \sigma_t^K)(1 - \delta)P_tK_t \\
& + (w_t - T_t)H(\bar{\theta}_t) + (1 - \tau_t^K)R_tK_t - \tau_t^A P_tK_t \\
& + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_tC_t.
\end{aligned} \tag{101}$$

And equilibrium profits are:

$$\Pi_t = (1 + \sigma_t^Y)D_tP_tY_t - R_tK_t - (1 - \sigma_t^P)w_tE_t. \tag{102}$$

Substitute the GBC in the HBC and rearrange terms:

$$\begin{aligned}
& (1 + i_{t-1})(B_t + B_{t-1,t}) + P_tG_t + \sigma_t^Y P_tD_tA_tF(E_t) + \sigma_t^K(K_{t+1} - (1 - \delta)K_t)P_t \\
& - (T_t - \sigma_t^P w_t)E_t + b_t(1 - E_t) \\
& - (\tau_t^K i_{t-1} + \tau_t^A)(B_t + B_{t-1,t}) - \tau_t^K R_tK_t - \tau_t^A P_tK_t - \tau_t^C P_tC_t - \tau_t^\Pi \Pi_t \\
& + (1 - \sigma_t^K)P_tK_{t+1} \\
& = (1 + \rho_{t-1})(\bar{B}_t + B_{t-1,t}) + (1 - \sigma_t^K)(1 - \delta)P_tK_t \\
& + (w_t - T_t)H(\bar{\theta}_t) + (1 - \tau_t^K)R_tK_t - \tau_t^A P_tK_t \\
& + b_t(1 - H(\bar{\theta}_t)) + (1 - \tau_t^\Pi)\Pi_t - (1 + \tau_t^C)P_tC_t,
\end{aligned} \tag{103}$$

$$P_tG_t + \sigma_t^Y P_tD_tA_tF(E_t) + \sigma_t^P w_tE_t + P_tK_{t+1} = (1 - \delta)P_tK_t + w_tE_t + R_tK_t + \Pi_t - P_tC_t. \tag{104}$$

Substitute profits, rearrange and divide by P_t :

$$\begin{aligned}
& P_tG_t + \sigma_t^Y P_tD_tA_tF(E_t) + \sigma_t^P w_tE_t + P_tK_{t+1} \\
& = (1 - \delta)P_tK_t + w_tE_t + R_tK_t + (1 + \sigma_t^Y)P_tD_tY_t - R_tK_t - (1 - \sigma_t^P)w_tE_t - P_tC_t,
\end{aligned} \tag{105}$$

Hence, the resource constraint is given by:

$$C_t + G_t + K_{t+1} - (1 - \delta)K_t = D_tY_t. \tag{106}$$

C First-best allocation

The first-best allocation maximizes the utility of the household (1) subject to the economy's resource constraint (33), the production technologies (10), and (13), and the market-clearing conditions. This gives the following Lagrangian for maximizing social welfare with associated multipliers $\mu_{s,t}^R$, $\mu_t^{s,Y}$, μ_t^{s,Y_j} , $\mu_t^{s,K}$, and $\mu_t^{s,L}$ in each state s at time t :

$$\begin{aligned} \mathcal{L} \equiv & \sum_{t=0}^{\infty} \sum_{s_t} \left[\beta^t \left[U(C_t, G_t, \xi_t) - V \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \right] \right. \\ & + \mu_t^{s,R} (Y_t - C_t - G_t - K_{t+1} + (1 - \delta)K_t) + \mu_t^{s,Y} \left(\left(\int_0^1 (Y_{j,t})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} - Y_t \right) \\ & \left. + \mu_t^{s,Y_j} (A_t F(K_{j,t}, E_{j,t}) - Y_{j,t}) + \mu_t^{s,K} \left(K_t - \int_0^1 K_{j,t} dj \right) + \mu_t^{s,L} \left(H(\bar{\theta}_t) - \int_0^1 E_j dj \right) \right] \pi(s_t). \end{aligned} \quad (107)$$

The first-order conditions are both necessary and sufficient:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 : \beta^t U_C = \mu_t^{s,R}, \quad (108)$$

$$\frac{\partial \mathcal{L}}{\partial G_t} = 0 : \beta^t U_G = \mu_t^{s,R}, \quad (109)$$

$$\frac{\partial \mathcal{L}}{\partial Y_t} = 0 : \mu_t^{s,R} = \mu_t^{s,Y}, \quad (110)$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 : \mu_t^{s,R} = \mu_{t+1}^{s,R} (1 - \delta) + \mu_{t+1}^{s,K}, \quad (111)$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\theta}_t} = 0 : \beta^t V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \bar{\theta}_t = \mu_t^{s,L}, \quad (112)$$

$$\frac{\partial \mathcal{L}}{\partial Y_{j,t}} = 0 : \mu_t^{s,Y} \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{1}{\varepsilon-1}} Y_{j,t}^{-\frac{1}{\varepsilon}} = \mu_t^{s,Y_j} \iff \mu_t^{s,Y} \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{1}{\varepsilon-1}} = \mu_t^{s,Y_j} Y_{j,t}^{\frac{1}{\varepsilon}}, \quad (113)$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t}} = 0 : \mu_{s,t}^{s,Y_j} A_t F_K(K_{j,t}, E_{j,t}) = \mu_t^{s,K} \iff A_t F_K(K_{j,t}, E_{j,t}) = \frac{\mu_t^{s,K}}{\mu_t^{s,Y_j}}, \quad (114)$$

$$\frac{\partial \mathcal{L}}{\partial E_{j,t}} = 0 : \mu_t^{s,Y_j} A_t F_E(K_{j,t}, E_{j,t}) = \mu_t^{s,L} : A_t F_E(K_{j,t}, E_{j,t}) = \frac{\mu_{s,t}^{s,L}}{\mu_t^{s,Y_j}}. \quad (115)$$

To simplify these expressions, first it is proven first that the optimal allocation is symmetric. This also implies that there is no price dispersion and inflation is zero. Use the production

function for final goods in eq. (10) to derive

$$Y_t^{\frac{1}{\varepsilon}} = \left(\int_0^1 (Y_{j,t})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{1}{\varepsilon-1}}. \quad (116)$$

Hence, eq. (113) can be written as

$$\mu_t^{s,Y} \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{1}{\varepsilon-1}} = \mu_t^{s,Y_j} Y_{j,t}^{\frac{1}{\varepsilon}}, \quad (117)$$

$$\mu_t^{s,Y} Y_t^{\frac{1}{\varepsilon}} = \mu_t^{s,Y_j} Y_{j,t}^{\frac{1}{\varepsilon}}. \quad (118)$$

Therefore, for all j it is found that $Y_{j,t} = Y_t$. This also implies that $\mu_t^{s,Y} = \mu_t^{s,Y_j}$. Moreover, if all intermediate good producers produce the same quantities Y_j , then their capital $K_{j,t}$ and labor demands $E_{j,t}$ are all equal, i.e., $K_{j,t} = K_t$ and $E_{j,t} = E_t$, which follows from eqs. (114) and (115). Consequently, it can be derived that

$$A_t F_K(K_t, E_t) = \frac{\mu_t^{s,K}}{\mu_t^{s,Y}}, \quad A_t F_L(K_t, E_t) = \frac{\mu_t^{s,L}}{\mu_t^{s,Y}}. \quad (119)$$

Note furthermore that if output is the same for all intermediaries, all prices for all intermediate goods must be the same: $P_{j,t} = P_t$. Moreover, $D_t = 1$ in that case. Hence, prices remain constant ($P_{t+1} = P_t$) and inflation is zero ($\pi_{t+1} = 0$). The central bank should thus set $i_t = r_t^n$.

Second, from the capital accumulation equation follows that there is full consumption smoothing over time and across states. To see why, substitute eq. (110) in eq. (119) to derive that

$$A_{t+1} F_K(K_{j,t+1}, E_{j,t+1}) = r_t^n + \delta = \frac{\mu_{t+1}^{s,K}}{\mu_{t+1}^{s,Y}} = \frac{\mu_{t+1}^{s,K}}{\mu_{t+1}^{s,R}}. \quad (120)$$

Moreover, eq. (111) can be rewritten as:

$$\mu_t^{s,R} = \mu_{t+1}^{s,R} (1 - \delta) + \mu_{t+1}^{s,R} \frac{\mu_{t+1}^{s,K}}{\mu_{t+1}^{s,R}} = \mu_{t+1}^{s,R} (1 - \delta) + \mu_{t+1}^{s,R} (r_t^n + \delta). \quad (121)$$

Divide by $\mu_{t+1}^{s,R}$ to find:

$$\frac{\mu_t^{s,R}}{\mu_{t+1}^{s,R}} = 1 + r_t^n, \quad (122)$$

Note that this equation implies that the marginal utility of consumption is equalized across states in period $t + 1$, since $\mu_t^{s,R}$ and $1 + r_t^n$ are non-stochastic as of t . Hence, substituting eq. (108) yields:

$$\mu_{t+1}^{s,R} = \beta^t U_C(C_{t+1}, G_{t+1}, \xi_{t+1}) = \frac{\mu_t^R}{1 + r_t^n}. \quad (123)$$

Moreover, from this result follows that the price of state-contingent bonds equals 1 for all states:

$$Q_{t,t+1} = (1 + r_t^n) \frac{\beta U_C(C_{t+1}, G_{t+1}, \xi_{t+1})}{U_C(C_t, G_t, \xi_t)} = (1 + r_t^n) \frac{\mu_{t+1}^{s,R}}{\mu_t^R} = 1. \quad (124)$$

Hence, the expected price of state-contingent bonds also equals 1:

$$\mathbb{E}_t[Q_{t,t+1}] = 1. \quad (125)$$

The last results imply that state-contingent bonds are superfluous at the first-best allocation. Intuitively, there is no state-contingent risk at the optimal second-best allocation. Hence, state-contingent bonds become equivalent to risk-free bonds.

Rewriting eq. (108), gives the intertemporal Euler equation:

$$\frac{\mu_t^R}{\mathbb{E}_t[\mu_{t+1}^{s,R}]} = \frac{U_C(C_t, G_t, \xi_t)}{\mathbb{E}_t[U_C(C_{t+1}, G_{t+1}, \xi_{t+1})]} = \beta(1 + r_t^n). \quad (126)$$

Substitute eq. (115) in eq. (112) impose symmetry, and substitute eqs. (110) and (108):

$$\beta^t V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \bar{\theta}_t = \beta^t U_C(C_t, G_t, \xi_t) A_t F_E(K_{j,t}, E_{j,t})$$

$$\frac{V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right)}{U_C(C_t, G_t, \xi_t)} \bar{\theta}_t = A_t F_E(K_t, E_{j,t}) = \frac{w_t^n}{P_t}. \quad (127)$$

Define the natural interest rate as $r_t^n \equiv A_{t+1} F_K(K_{t+1}, E_{t+1}) - \delta$. Moreover, define the real rental and wage rates as $\frac{R_t}{P_t} = A_t F_K(K_t, E_t)$, and $\frac{w_t^n}{P_t} = A_t F_E(K_t, E_t)$. The latter imply that

$\Pi_t = 0$ due to constant returns to scale in production.

Finally, the Samuelson rule for public goods applies:

$$\frac{U_G}{U_C} = 1. \quad (128)$$

D Implementation first-best

The decentralized allocation is summarized in Proposition 1. The optimal first-best allocation is given in Proposition 2. From these Propositions follows that the following prices implement the first-best allocation as a decentralized allocation:

$$P_t = P_t^* = P_{t-1}, \quad \pi_t = 0, \quad D_t = 1, \quad (129)$$

$$1 + r_{t+1}^n = \frac{(1 + \tau_t^C)}{(1 + \tau_{t+1}^C)} (1 + (1 - \tau_{t+1}^K) i_t - \tau_{t+1}^A), \quad (130)$$

$$r_{t+1}^n + \delta = \frac{(1 - \sigma_t^K)(1 + \rho_{t+1}) - (1 - \sigma_{t+1}^K)(1 - \delta) + \tau_{t+1}^A}{(1 - \tau_{t+1}^K)}, \quad (131)$$

$$\hat{w}_t = \frac{(1 + \tau_t^C) P_t}{(1 - \tau_t^L)} \frac{V' \left(\int_{-\infty}^{\hat{\theta}_t} \theta_t h(\theta_t) d\theta_t \right)}{U_C(C_t, G_t)} \hat{\theta}_t = P_t \frac{V' \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right)}{U_C(C_t, G_t)} \bar{\theta}_t = \bar{w}_t \quad (132)$$

$$\tau_t^C = -\tau_t^L, \quad (133)$$

$$\tau_t^I = \frac{\hat{w}_t - \bar{w}_t}{\bar{w}_t} = 0, \quad (134)$$

$$\sigma_t^Y = \mathcal{M} - 1 = \frac{1}{\varepsilon - 1}, \quad (135)$$

$$\sigma_t^P = \frac{\bar{w}_t - w_t^n}{\bar{w}_t}. \quad (136)$$

Substitute these prices in Proposition 1 to find Proposition 3.

E Optimal public investment

We modify the Lagrangian from Appendix C to include public investment:

$$\begin{aligned} \mathcal{L} \equiv & \sum_{t=0}^{\infty} \sum_{\mathcal{S}_t} \left[\beta^t \left[U(C_t, G_t, \xi_t) - V \left(\int_{-\infty}^{\bar{\theta}_t} \theta_t h(\theta_t) d\theta_t \right) \right] \right. \\ & \left. + \mu_t^{s,R} (Y_t - C_t - G_t - K_{t+1} - K_{t+1}^G + (1 - \delta)K_t + (1 - \delta)K_t^G) \right] \end{aligned} \quad (137)$$

$$\begin{aligned}
& + \mu_t^{s,Y} \left(\left(\int_0^1 (Y_{j,t})^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - Y_t \right) + \mu_t^{s,Y_j} (A_t F(K_{j,t}, E_{j,t}, K_t^G) - Y_{j,t}) \\
& + \mu_t^{s,K} \left(K_t - \int_0^1 K_{j,t} dj \right) + \mu_t^{s,L} \left(H(\bar{\theta}_t) - \int_0^1 E_j dj \right) \Big] \pi(s_t).
\end{aligned}$$

The first-order conditions for C_t , G_t , Y_t , K_{t+1} , $\bar{\theta}_t$, $Y_{j,t}$, $K_{j,t}$ and $E_{j,t}$ remain the same, except that the production function is modified to include public investment. Hence, using the same policy implementation as in Proposition (3) we can obtain the first-best allocation for given public investment K_{t+1}^G . The first-order condition for optimal investment in public capital K_{t+1}^G is given by:

$$\mu_t^{s,R} = \mu_{t+1}^{s,R} (1 - \delta) + \mu_t^{s,Y_j} A_t F_{K^G}(K_t, E_t, K_t^G). \quad (138)$$

Rewriting and using $\mu_t^{s,Y_j} = \mu_t^{s,Y} = \mu_t^{s,R}$ from Appendix C yields that the marginal product of public capital should be equal to that of private capital:

$$A_t F_{K^G}(K_t, E_t, K_t^G) = A_t F_K(K_t, E_t, K_t^G). \quad (139)$$

Since the marginal product of capital equals $A_t F_K(K_t, E_t, K_t^G) = r_t^n + \delta$ the result in the Proposition follows.

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