

Optimal Taxation of Risky Human Capital*

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Abstract

In a two-period life-cycle model with *ex ante* homogeneous households, earnings risk, and a general earnings function, we derive the optimal linear labor tax rate and optimal linear education subsidies. The optimal income tax trades off social insurance against incentives to work. Education subsidies are not used for social insurance, but they are only targeted at offsetting the distortions of the labor tax and internalizing a fiscal externality. Both optimal education subsidies and tax rates increase if labor and education are more complementary, because education subsidies indirectly lower labor tax distortions by stimulating labor supply. Optimal education subsidies (taxes) also correct non-tax distortions arising from missing insurance markets. Education subsidies internalize a positive (negative) fiscal externality if there is underinvestment (overinvestment) in education because of risk. Education policy unambiguously allows for more social insurance if education is a risky activity. However, if education hedges against labor-market risk, optimal tax rates could be lower than in the case without education subsidies.

Keywords: Education subsidies; human-capital investment; idiosyncratic risk; labor taxation risk properties of human capital

JEL classification: H21; I2; J2

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I. Introduction

Individuals face substantial labor-market risks during their working lives. They might become unemployed, sick, or disabled, or they might experience loss of skill as a result of old age, health problems, technological changes, and globalization. In principle, private insurance should be feasible, because individual idiosyncratic income risks can be pooled in the aggregate. However, in the case of human capital, private insurance markets tend to suffer from market failure, and private insurance is not available (or it is only available to a very limited extent) because of moral hazard, adverse selection, and various legal limitations in trading claims on human capital (Sinn, 1996).

Although all social insurance policies suffer from moral hazard problems, the government can overcome adverse selection and legal problems by providing mandatory social insurance against human-capital risk. Therefore, insuring human-capital risks is one of the key roles of modern welfare states. Indeed, virtually all social benefits (e.g., welfare, unemployment, sickness, disability, health, and old-age benefits) provide insurance against the loss of skill. Moreover, if individuals fail to acquire sufficient skills when young, they are liable to become dependent on social insurance benefits later in life. Thus, human-capital policies could be desirable to avoid dependency on the welfare state.

Despite the obvious policy relevance, it is rather surprising that only a limited number of papers have addressed the question of how social insurance should be organized when human capital is subject to non-insurable risks. Moreover, it is not clear whether education policy should be employed as a complementary policy to social insurance. In some earlier papers, the implications of human-capital risks for the design of optimal insurance and/or education policy have been analyzed (e.g., Eaton and Rosen, 1980a,b; Hamilton, 1987; Anderberg and Andersson, 2003; da Costa and Maestri, 2007; Anderberg, 2009; Grochulski and Piskorski, 2010). However, these authors do not explicitly derive answers to the following three questions.

First, is the optimal amount of social insurance higher or lower when education increases or reduces earnings risk? Anderberg and Andersson (2003, p. 1523) argue the following. "If human capital reduces earnings risk, encouraging education would seem to mitigate the insurance/redistribution problem." Hence, if education hedges against labor-market risk (increases labor-market risk), the government needs to rely less (more) on social insurance. However, this argument is not formally proven.

Second, should education policy correct underinvestment or overinvestment in human capital, or not? Human-capital investment is typically

inefficient, because risk-averse individuals reduce their exposure to income risk in the presence of missing insurance markets. Da Costa and Maestri (2007, p. 696) have suggested that education policy is optimal if education is a risk-increasing activity. "Optimal policies derived under these assumptions will then prescribe . . . educational subsidies to ameliorate the problem of underinvestment in human capital." In contrast, Anderberg and Andersson (2003, p. 1523) have argued that education policy is also needed, but now when education hedges against income risk. "The insight is thus that if education moderates wage uncertainty, a second-best policy should, rather unambiguously, encourage the formation of human capital (relative to the first-best), while if education exacerbates wage uncertainty the overall conclusion is ambiguous."

Third, does the availability of education policy optimally increase the amount of social insurance or not? Again, we would expect this to be true. For example, Bovenberg and Jacobs (2005) demonstrate in deterministic settings that optimal education policy typically lowers the cost of redistribution, and thus raises optimal tax rates.

In this paper, we demonstrate that the answers to these three questions are not trivial, and that they can be completely counterintuitive. Indeed, our analysis shows that all of the suggested answers to the questions raised above are either partially or completely incorrect. This is done by developing a model that integrates the previously studied approaches in order to characterize optimal linear tax and education policies in risky economies.

We utilize a two-period life-cycle model of human-capital investment, labor supply, and saving. *Ex ante* homogeneous households differ *ex post* because of the realization of idiosyncratic risk in their second-period income. Markets to insure earnings or human-capital risks are missing. Therefore, social insurance is welfare-enhancing, because we assume that there is no aggregate risk. Social insurance takes the form of a linear income tax. Full insurance is impossible, because of the endogeneity of labor supply, which causes a moral hazard problem. We extend the previous literature by employing a completely general earnings function, which depends on human-capital investment, labor supply, and a random variable that reflects the uncertain state of nature. This general earnings function allows for both the possibility that education is a risky activity that increases exposure to labor-market risk and the possibility that education reduces exposure to (i.e., hedges against) labor-market risk. The paper contributes in five major ways to the existing literature.

First, the study provides the answer to the first question raised above. We show that if educational investment increases (reduces) exposure to non-insurable income risks, the risk premium acts as a (pre-existing) implicit tax (subsidy) on human-capital investment. Therefore, missing insurance markets result in non-tax distortions, which generate fiscal externalities

that need to be taken into account in the design of tax and education policies. Income taxes exacerbate (mitigate) underinvestment (overinvestment) in human capital when education increases (decreases) earnings risk. Income taxes should optimally be lower (higher) as a result. Therefore, if education hedges against (increases) labor-market risk, it is incorrect to argue that optimal income taxes should be lower (higher) because individuals self-insure by overinvesting (underinvesting).

Second, this study answers the second question by demonstrating that education subsidies are not used for insurance. Indeed, when there is no social insurance, governments cannot improve upon the *laissez-faire* outcome by subsidizing education. Intuitively, education subsidies are state-independent and cannot insure income risks. Thus, subsidizing education upsets the optimal private response to market risk by distorting investment in human capital. As long as insurance markets are missing and social insurance is unavailable, it is incorrect to argue that governments should correct underinvestment or overinvestment in human capital with education subsidies.

Third, this paper shows that education subsidies are optimally employed in a policy that combines income tax and education subsidies. Subsidies on education are optimal only in combination with social insurance in order to mitigate the tax and non-tax distortions associated with social insurance. The primary role of education subsidies is to reduce the tax distortions on labor supply if education is complementary to labor. Then, education subsidies boost labor supply, and thereby indirectly offset the labor tax distortion on work effort (see also Jacobs and Bovenberg, 2011). The second role of education subsidies is to internalize the fiscal externality caused by underinvestment or overinvestment in education. Da Costa and Maestri (2007) are correct to argue that education should be subsidized if it is a risky activity – but only to the extent that subsidies are needed to internalize the fiscal externality, and not to directly tackle overinvestment or underinvestment (see the previous point).

Fourth, the paper answers the third question. When both tax and education policies are optimized, we demonstrate that the design of social insurance becomes independent from the question whether education is a risky investment or not. Therefore, ambiguities arise as to whether more or less social insurance is provided, compared to the optimal tax policy without education subsidies. This crucially depends on the risk properties of human capital and the complementarity of education and work. Consequently, optimal education policy does not automatically allow for more social insurance.

Fifth, the paper complements Anderberg (2009), in which non-linear tax and education policies are analyzed in comparable settings. We bolster Anderberg's findings by showing that the risk properties of human capital are critical in shaping human-capital policies under much weaker informational

assumptions as well. In particular, only aggregate labor incomes and educational investments need to be verifiable to the government for linear policy instruments to be employed. Moreover, our findings suggest that great care should be taken when drawing inferences from the wedges that are now commonly analyzed in the new dynamic public finance literature. We show that a wedge on education does not prove that education is optimally taxed at the optimal second-best allocation, which is decentralized through income taxes and education subsidies.

The remainder of the paper is structured as follows. In Section II, we provide a survey of the literature. Then, we introduce the model in Section III. In Section IV, we discuss optimal tax and education policies, and we provide a discussion and conclusions in Section V.

II. Earlier Body of Literature

Levhari and Weiss (1974) were the first to examine the effect of idiosyncratic risks on human-capital formation. Human-capital investment can both increase and decrease exposure to income risk, depending on the risk properties of the earnings function. Individuals will self-insure by underinvesting in human capital if this increases the exposure to labor-market risk, but will overinvest in human capital if this hedges against labor-market risk. Empirically, both possibilities appear to be relevant.¹

The formal analysis of social insurance with endogenous human-capital investment began with the seminal paper by Eaton and Rosen (1980b). They assume a multiplicative earnings function, where labor earnings are a linear product of labor supply, human capital, and a stochastic risk factor. Thus, investments in education raise the exposure to labor-market risk. Consequently, private investment in education is driven below the socially desirable level. A distortionary income tax is shown to be welfare-enhancing, because it redistributes income from favorable to unfavorable states of nature. The linear income tax is a partial substitute for missing insurance markets. Consequently, human-capital risks are partially insured, and human-capital investment increases.

Hamilton (1987) adopted the model of Eaton and Rosen (1980b) to analyze taxes on savings, besides income tax. Hamilton points out that there remains socially inefficient underinvestment in human capital, because the labor tax cannot eliminate all income risk due to moral hazard in labor supply. Hamilton (1987) shows that the taxation of savings reduces the

¹ Hartog (2005) reviews a substantial body of literature that empirically establishes risk compensation in wages. Palacios-Huerta (2003, 2006) shows that human capital is risky, on average. However, he also finds that the human-capital premium decreases as workers become better educated, suggesting as well that human-capital investments hedge against labor-market risk on the margin.

opportunity costs of human-capital accumulation, and that this is optimal under the (very) strong assumptions of (i) inelastic labor supply and (ii) either zero equilibrium savings or constant absolute risk aversion.

Grochulski and Piskorski (2010) generalized the findings of Hamilton (1987) to non-linear policy instruments, without imposing the strong restrictions of Hamilton, while maintaining an earnings function with multiplicative risk. At the same time, they did not allow for education policy, because education is assumed to be non-verifiable to the government. They show that labor supply carries a wedge (i.e., it is distorted) for insurance purposes. This is analogous to the optimality of a distortionary labor tax. Moreover, there is an intertemporal wedge in consumption choices, indicating a role for capital income taxation, for two reasons. First, the intertemporal wedge stimulates labor supply, and indirectly reduces the tax distortions on labor supply (see also Diamond and Mirrlees, 1978, 1986; Golosov *et al.*, 2003).² Secondly, by lowering the opportunity costs, intertemporal wedges provide incentives to invest in human capital. This is optimal because the labor tax discourages human-capital investments (see also Jacobs and Bovenberg, 2010, who obtained the same result even in the absence of risk).

Anderberg and Andersson (2003) were the first to simultaneously optimize linear tax and education policies. They use a stripped-down version of the Eaton and Rosen (1980b) model, while allowing for a more general earnings function, as in Levhari and Weiss (1974). Anderberg and Andersson (2003) assume that the government directly controls educational investment. Also, they obtain a trade-off between social insurance and distortions in labor supply. In addition, they find that the use of education policy generates a “revenue creation effect”, because labor supply and education are complementary activities, so that education policy can mitigate the tax distortions on labor supply. Moreover, education policy entails an “insurance effect”, depending on the risk properties of human capital. Their main message is that education should be overprovided relative to first-best rules if it is risk-decreasing, and that it should be underprovided if it is risk-increasing.

Da Costa and Maestri (2007) and Anderberg (2009) also build on the Eaton–Rosen–Hamilton model, but they now assume that human-capital investments can be verified by the government so that it can employ education policies, besides capital taxation and non-linear income taxation. In addition to deriving the desirability of wedges on labor supply and saving,

² Cremer and Gavhari (1995a,b) also demonstrate that the optimal (linear) commodity tax is non-uniform under both linear and non-linear income taxation, when one of the commodities is consumed before the realization of risk, and the other thereafter. If we interpret these commodities as consumption today and consumption tomorrow, the result is immediate. Hence, intertemporal wedges or capital income taxes are optimal.

da Costa and Maestri (2007) argue that education policy should ensure social efficiency in human-capital investment. Anderberg (2009) concludes that this result is erroneous. Aggregate human-capital investment should be optimally distorted in a way that depends on the shape of the earnings function similar to Anderberg and Andersson (2003).

III. The Model

We follow Levhari and Weiss (1974) by analyzing a two-period life-cycle model of human-capital investment, labor supply, and saving. There is a continuum of *ex ante* identical individuals, who differ *ex post* because of an idiosyncratic shock θ , which is drawn from a probability distribution $f(\theta)$. We assume $\theta \in \Theta \equiv [\underline{\theta}, \infty)$, where Θ denotes the set of values for θ and $\underline{\theta}$ denotes the lower bound on θ .

Households derive utility from consumption c_1 in period one and consumption c_2 in period two. Moreover, they derive disutility from labor supply l in the second period. There is no labor-leisure choice in the first period. Households maximize a von Neumann-Morgenstern expected-utility function, which is assumed to be separable between the subutility function of consumption in both periods and the disutility of work:

$$\mathcal{E}[u(c_1, c_2)] - v(l), \quad u_1, u_2, v_l > 0, \quad u_{11}, u_{22}, -v_{ll} < 0. \quad (1)$$

Here, \mathcal{E} denotes the expectation operator (i.e., $\mathcal{E}[X] \equiv \int_{\Theta} X df(\theta)$), and the subscripts refer to the argument of differentiation. The subutility function of consumption is increasing and concave, whereas the disutility function of labor supply is increasing and convex. Furthermore, we impose the Inada conditions on both subutility functions in order to avoid corner solutions.

In the first period, individuals have a unit time endowment, which is spent on investment in education (e) and work ($1 - e$). Consequently, individuals forego labor earnings while learning.³ The wage per unit of time worked in the first period is normalized to one. In addition, individuals have an exogenous income endowment ω . Apart from investing in education, individuals can borrow and lend in perfect capital markets at a constant real interest rate r . Total savings are denoted by a .⁴

Gross labor income in the second period is represented by a general earnings function, which depends on labor supply l and education e :

$$\Phi(\theta, l, e), \quad \Phi_e, \Phi_l > 0, \quad \Phi_{ee} < 0, \quad \Phi_{ll} \leq 0. \quad (2)$$

³ Without any loss of generality, we could also allow for direct costs of education, as long as all educational investments are verifiable and can be subsidized (cf. Bovenberg and Jacobs, 2005).

⁴ We assume that the lower bound $\underline{\theta}$ is sufficiently large such that second-period income is always high enough to prevent individuals defaulting on their loans. See Jacobs and Yang (2010) for the analysis of optimal taxation of human capital with imperfect capital markets.

Therefore, both income and the returns to education are risky. We assume that, for any given value of θ , the marginal returns to education are positive and decreasing. Similarly, the marginal returns to labor effort are positive and non-increasing. Furthermore, the random variable θ is assumed to exert a positive effect on income: $\Phi_\theta > 0$. In the remainder of the analysis, we focus on the two cases identified in the literature: (i) educational investment amplifies income risks ($\Phi_{\theta e} > 0$); (ii) educational investment hedges against income risks ($\Phi_{\theta e} < 0$).

Social insurance takes place through a linear tax system with a positive marginal tax rate t on labor earnings in both periods, and a lump-sum transfer T , which can be seen as a negative income tax or a basic income. Without loss of generality, the transfer is only given in the second period.⁵ Because foregone labor earnings are the only cost of education, all educational investments are tax-deductible. We introduce a flat-rate subsidy s on net foregone earnings (i.e., opportunity costs of education). This can be viewed as a subsidy per unit of time enrolled in education.⁶ The informational assumptions for employing linear instruments are that only aggregate incomes and education choices need to be verifiable to the government.

Consequently, the first-period and second-period budget constraints can be written as

$$c_1 = (1 - t)[1 - (1 - s)e] - a + \omega, \tag{3}$$

and

$$c_2 = (1 - t)\Phi(\theta, l, e) + Ra + T, \tag{4}$$

where $R \equiv 1 + r$ is the interest factor.

The timing structure of the model is as follows. The government sets the proportional tax rate t , the subsidy rate s , and the lump-sum transfer T before the choices of households and the revelation of the risk θ . Moreover, educational investment e , savings a , and labor supply l are simultaneously chosen before risk is realized.⁷ This implies that first-period consumption is pinned down by these choices. After the shock occurs, incomes are earned and second-period consumption takes place.

The household's unconstrained maximization problem can be obtained upon substitution of the household budget constraints into the utility

⁵ Because we assume perfect capital markets, individuals can always borrow against the transfer to finance first-period consumption.

⁶ We abstract from taxes on saving and refer to Hamilton (1987) and Schindler and Yang (2009) for the analysis of optimal capital taxes in a similar model.

⁷ It can be shown that a timing sequence in which labor supply is chosen after uncertainty has been resolved does not change any of the results qualitatively (cf. Cremer and Gavhari, 1995a; Anderberg and Andersson, 2003).

function:

$$\max_{\{e,l,a\}} U(e, l, a) \equiv \mathcal{E}[u((1-t)[1-(1-s)e] - a + \omega, (1-t)\Phi(\theta, l, e) + Ra + T)] - v(l). \tag{5}$$

The first-order conditions for this maximization problem are given by⁸

$$\mathcal{E}[u_2(\cdot)\Phi_e(\cdot)] = \mathcal{E}[u_1(\cdot)](1-s), \tag{6}$$

$$(1-t)\mathcal{E}[u_2(\cdot)\Phi_l(\cdot)] = v_l(l), \tag{7}$$

$$R\mathcal{E}[u_2(\cdot)] = \mathcal{E}[u_1(\cdot)]. \tag{8}$$

The first-order conditions for education (6) and labor supply (7) can be rewritten by employing the risk premia in education and labor supply:

$$\pi_i \equiv -\frac{\text{cov}[u_2(\cdot), \Phi_i(\cdot)]}{\mathcal{E}[u_2(\cdot)]\mathcal{E}[\Phi_i(\cdot)]}, \quad i = e, l. \tag{9}$$

Here, π_e is the negative of the normalized covariance between the marginal utility of consumption and the marginal return of human capital. A positive risk premium implies that education increases income risk, because $\pi_e > 0$ corresponds to $\Phi_{\theta e} > 0$. Instead, a negative risk premium $\pi_e < 0$ mirrors a risk-reducing effect of education, as a result of $\Phi_{\theta e} < 0$. Similarly, π_l is the negative of the normalized covariance between the marginal utility of consumption and the marginal return to labor, representing the risk premium in labor supply. Its interpretation is analogous to the risk premium in educational investment. Note that if individuals were risk-neutral, both risk premia would be zero. Similarly, risk premia are zero if the marginal returns to education or labor are not state-dependent (i.e., when there is no risk). Both risk premia are also zero when the risk factor θ enters the earnings function in an additively separable fashion ($\Phi_{\theta e} = \Phi_{\theta l} = 0$), because education and labor supply do not affect income risk in this case.

Using the definition of π_e , the first-order condition (6) can be written as

$$(1 - \pi_e)\mathcal{E}[\Phi_e(\theta, l, e)] = R(1 - s). \tag{10}$$

The risk-adjusted expected marginal return to education is equal to the marginal cost of education. Note that the tax system does not directly affect

⁸ In general, the second-order conditions are not automatically satisfied because of the interaction between learning and working, which generates non-linear budget sets. We assume that second-order conditions are always satisfied. This requires that the complementarity between education and labor is sufficiently weak (low Φ_{el}) and absolute risk-aversion is sufficiently large (see also Jacobs *et al.*, 2009).

investment in education, because all costs of education are tax deductible. However, taxation generally affects investment in education indirectly via labor supply. More labor supply raises the returns to human-capital investments as long as $\Phi_{el} > 0$. This is the case for all earnings functions discussed in the literature (see Jacobs and Bovenberg, 2011). Education subsidies naturally boost educational investments, because they reduce the marginal cost of human-capital investment.

If income is risky, the expected marginal return of education can be either higher or lower than marginal costs, depending on the sign of the risk premium π_e . If education increases exposure to labor-market risk, $\pi_e > 0$, then individuals command a positive risk premium on their educational investment. Hence, from a social point of view, risk-averse individuals invest too little in education. Thus, missing insurance markets for risk related to human-capital income create an implicit tax on human-capital investment. If income risk decreases with education, individuals command a negative risk premium on their educational investment, $\pi_e < 0$. In this case, risk-averse individuals invest too much in education in order to reduce their exposure to labor-market risk. Thus, missing insurance markets create an implicit subsidy on human-capital investments. If there is no income risk, condition (10) reduces to $\Phi_e(\cdot) = 1 - s$, which is the optimality condition for investment in human capital under certainty.

The first-order condition for labor supply (7) can be rewritten using π_l :

$$\frac{v_l(l)}{\mathcal{E}[u_2(\cdot)]} = (1 - t)(1 - \pi_l) \mathcal{E}[\Phi_l(\theta, l, e)]. \quad (11)$$

The marginal rate of substitution between consumption and labor must be equal to the risk-adjusted net wage. A higher tax rate reduces the incentives to supply labor. Note that if education raises the wage rate, incentives to supply labor are stronger when individuals are better educated. Thus, education and labor are complementary as long as $\Phi_{el} > 0$. If an increase in labor supply increases risk, $\pi_l > 0$, individuals supply less labor than is socially efficient. If labor supply decreases the exposure to risk ($\pi_l < 0$), the risk premium becomes negative, leading to socially inefficient precautionary labor supply. Again, the risk premium acts as an implicit tax (subsidy) on labor if labor supply increases (reduces) exposure to labor-market risk (i.e., if $\pi_l > 0$ ($\pi_l < 0$)).

Equation (8) is the stochastic Euler equation for consumption. The larger the interest rate, the stronger the incentives to save, and the more individuals allocate resources to the second period of the life-cycle.

IV. Optimal Tax and Education Policies

We assume a benevolent government with full commitment, which maximizes social welfare by optimally choosing linear tax and education policies. The intertemporal government budget constraint is given by

$$t\mathcal{E}[\Phi(\theta, l, e)] + tR(1 - e) = R(1 - t)se + T + G, \tag{12}$$

where G is an exogenous revenue requirement. Because income risk is idiosyncratic, tax revenue is deterministic according to the law of large numbers, and tax revenue equals its expected value. We abstract from any systematic risk.

Social welfare is the (*ex ante*) expected indirect utility $V(\cdot)$ of the representative household:

$$V(T, t, s) \equiv \mathcal{E}[u((1 - t)[1 - (1 - s)\hat{e}] - \hat{a} + \omega, (1 - t)\Phi(\theta, \hat{l}, \hat{e}) + R\hat{a} + T)] - v(\hat{l}), \tag{13}$$

where the hat symbols denote the optimized values for l , e , and a . For later reference, we apply Roy's lemma to find the derivatives of the indirect utility function:

$$\frac{\partial V(\cdot)}{\partial T} = \mathcal{E}[u_2(\cdot)],$$

$$\frac{\partial V(\cdot)}{\partial t} = -\mathcal{E}[u_2(\cdot)(\Phi(\cdot) + R[1 - (1 - s)\hat{e}])],$$

and

$$\frac{\partial V(\cdot)}{\partial s} = \mathcal{E}[u_2(\cdot)]R(1 - t)\hat{e}.$$

The Lagrangian for maximization of social welfare is given by

$$\max_{\{T, t, s\}} \mathcal{L} \equiv \mathcal{E}[V(T, t, s) + \eta[t\Phi(\theta, l, e) + tR(1 - e) - R(1 - t)se - T - G]], \tag{14}$$

where η denotes the Lagrange multiplier of the government budget constraint (12).

In order to characterize the optimal solutions for the optimal tax and subsidy rates, we introduce the following tax wedges on labor and education:

$$\Delta_l \equiv t\Phi_l(\cdot), \tag{15}$$

$$\Delta_e \equiv t[\Phi_e(\cdot) - R] - R(1 - t)s. \tag{16}$$

Here, Δ_l (Δ_e) measures the increase in tax revenue (measured in monetary units) if labor supply (education) is raised by one unit.

The first-order conditions for this maximization problem are given by

$$\frac{\partial \mathcal{L}}{\partial T} = \mathcal{E} \left[\frac{\partial V(\cdot)}{\partial T} - \eta + \eta \Delta_l \frac{\partial l}{\partial T} + \eta \Delta_e \frac{\partial e}{\partial T} \right] = 0, \quad (17)$$

$$\frac{\partial \mathcal{L}}{\partial t} = \mathcal{E} \left[\frac{\partial V(\cdot)}{\partial t} + \eta [\Phi(\cdot) + R(1 - e) + Rse] + \eta \Delta_l \frac{\partial l}{\partial t} + \eta \Delta_e \frac{\partial e}{\partial t} \right] = 0, \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial s} = \mathcal{E} \left[\frac{\partial V(\cdot)}{\partial s} - \eta R(1 - t)e + \eta \Delta_l \frac{\partial l}{\partial s} + \eta \Delta_e \frac{\partial e}{\partial s} \right] = 0. \quad (19)$$

In the remainder of this section, first we analyze optimal tax and education policies separately. Then, we derive the optimal structure of both tax and education policies simultaneously.

Optimal Lump-Sum Transfer

Using Roy's lemma, from equation (17), we obtain

$$\mathcal{E} \left[\frac{u_2}{\eta} + \Delta_l \frac{\partial l}{\partial T} + \Delta_e \frac{\partial e}{\partial T} \right] = 1. \quad (20)$$

Hence, the expected social marginal value of a unit increase in lump-sum income, including the income effects on the tax base, should be equal to its resource costs, which equal unity.

Optimal Taxation

In this subsection, we derive the optimal level of social insurance in the absence of education policy ($\bar{s} = 0$). We define the "insurance characteristic" ξ as the negative of the normalized covariance between gross income Φ and the private marginal value of income u_2 :

$$\xi \equiv - \frac{\text{cov}[\Phi, u_2]}{\mathcal{E}[\Phi] \mathcal{E}[u_2]} > 0. \quad (21)$$

The insurance characteristic ξ measures the (marginal) gain in social welfare of a larger income insurance. It is positive, because higher labor income is associated with lower marginal utility of consumption (i.e., $\text{cov}[\Phi, u_2] < 0$). Thus, a reduction of the variance in earnings by means of redistributive income taxes raises social welfare. Indeed, $\xi = 0$ if the government is not concerned about income insurance and if all individuals have the same marginal utility of income u_2 , or if household income Φ is deterministic, and there is no risk.

Using Roy's lemma and the risk-adjusted Slutsky equations, we can find the optimal tax rate at the optimal T from equation (18):

$$\frac{t}{1-t} = \frac{\xi}{\varepsilon_{lt} + \pi_e \varepsilon_{et}}. \quad (22)$$

Here,

$$\varepsilon_{lt} \equiv - \frac{\mathcal{E}[\Phi_l] l}{\mathcal{E}[\Phi]} \frac{\partial l^*}{\partial t} \frac{1-t}{l} > 0,$$

and

$$\varepsilon_{et} \equiv - \frac{\mathcal{E}[\Phi_e] e}{\mathcal{E}[\Phi]} \frac{\partial e^*}{\partial t} \frac{1-t}{e},$$

are the expected-utility compensated elasticities of labor supply and education, where an asterisk (*) denotes a compensated demand or supply function (see Appendix A). These elasticities are weighted by the expected earnings shares of labor and education in total earnings. The expression in equation (22) shows the trade-off between insurance and efficiency. The optimal tax on labor equates the marginal benefits of income insurance (ξ) with the marginal costs of providing it. The optimal tax rate increases when the government attaches a larger social value to income insurance, as measured by a higher ξ .

The marginal costs consist of two terms: (i) tax-induced distortions on labor supply $t/(1-t)\varepsilon_{lt}$; (ii) a fiscal externality $t/(1-t)\pi_e\varepsilon_{et}$, which stems from the missing insurance markets.⁹ The optimal tax decreases if the distortions in labor supply become more severe, as indicated by a higher elasticity ε_{lt} . Indeed, if labor supply (and educational investments) were completely inelastic ($\varepsilon_{lt} = \varepsilon_{et} = 0$), the optimal tax rate would be 100 percent ($t = 1$).

The optimal tax rate is also determined by the tax elasticity of investments in education, as can be seen from the presence of the term $\pi_e\varepsilon_{et}$. In particular, the income tax might exacerbate or mitigate the non-tax distortions arising from the missing insurance markets. If education increases the exposure to labor-market risk, the risk premium acts as if there is a pre-existing implicit tax on educational investment ($\pi_e > 0$). If educational investments hedge against labor-market risk, the risk premium acts as if there is a pre-existing implicit subsidy on educational investment ($\pi_e < 0$). Provided that investment in human capital falls with a higher tax rate ($\varepsilon_{et} > 0$),¹⁰ a higher income tax thus exacerbates (mitigates)

⁹ Following Heller and Starrett (1976), we interpret the (fiscal) impact of allocative distortions resulting from a missing market as an externality.

¹⁰ Although the tax system does not affect human-capital investments directly, it does so indirectly by lowering labor supply as long as labor and education are complementary in generating gross income (i.e., $\Phi_{el} > 0$).

underinvestment (overinvestment) in human capital if $\pi_e > 0$ ($\pi_e < 0$). The implicit tax (or subsidy) on education as a result of non-insurable income risks thereby creates a fiscal externality in the presence of positive income taxes.

This can be seen most clearly from the (expected) tax wedge on education $\mathcal{E}[\Delta_e]$, which measures the gain in tax revenue available for redistribution if human-capital investment increases by one unit. By applying the first-order equation for optimal human-capital investment (equation (10)), we can rewrite the expected net tax wedge on education as

$$\mathcal{E}[\Delta_e]_{s=0} = \frac{\pi_e}{1 - \pi_e} t. \quad (23)$$

Here, $\pi_e/(1 - \pi_e)$ represents the risk wedge on human-capital investment. If there is underinvestment ($\pi_e > 0$), the social marginal benefits of an additional unit invested in education are larger than the associated social marginal costs. Consequently, the cost of the tax deduction on the marginal costs of the investment is smaller than the tax revenue from the marginal benefits of the investment in education (i.e., $\mathcal{E}[\Delta_e]_{s=0} > 0$). Income taxation will exacerbate socially undesirable underinvestment by further reducing educational investments below first-best levels (if $\varepsilon_{et} > 0$), which decreases tax revenues. Consequently, optimal tax rates are set lower (*ceteris paribus*). In the case of overinvestment ($\pi_e < 0$), the opposite holds true. In particular, the public cost of the tax deduction on the marginal costs of the investment is larger than the marginal revenue generated by taxing the returns to education (i.e., $\mathcal{E}[\Delta_e]_{s=0} < 0$). Thus, social insurance reduces socially undesirable overinvestment in human capital, and it increases tax revenue (if $\varepsilon_{et} > 0$). Optimal tax rates are set higher as a result (*ceteris paribus*). If education has no effect on the exposure to risk, there is no risk premium on human-capital investment ($\pi_e = 0$). Thus, the implicit tax on education is zero, because all costs are deductible against the rate at which returns are taxed. Hence, the fiscal externality vanishes. In this case, the optimal tax is determined only by the labor supply elasticity.¹¹

Our results match those of Eaton and Rosen (1980b), if we assume that the earnings function exhibits multiplicative risk. This implies that education will always increase the exposure to risk, and that there will be underinvestment (i.e., $\pi_e > 0$). However, Eaton and Rosen (1980b, pp. 712–714) do not derive an explicit expression for the optimal tax rate. Here, we show that the optimal income tax is downward biased because of the negative fiscal externality ($\pi_e > 0$), which is a novel finding. If we assume that human-capital investment is exogenous ($\varepsilon_{et} = \pi_e = 0$), we

¹¹ The tax distortions in labor supply are still typically higher than in standard models without endogenous human-capital investment, as long as education and labor are complementary in earnings (e.g., Jacobs, 2005).

obtain the outcome of Eaton and Rosen (1980a): $t/(1-t) = \xi/\varepsilon_{lt}$. This equation captures the trade-off between insurance and labor supply distortions. In the following proposition, we summarize our findings from this subsection.

Proposition 1. *The optimal income tax trades off social insurance against the incentives to work, and the internalization of the fiscal externality stemming from missing insurance markets. If education increases (reduces) exposure to labor-market risk, the income tax exacerbates (mitigates) the distortions of missing insurance markets on human-capital investment.*

Optimal Education Policy

In this subsection, we derive the optimal education policy for a given level of taxation \bar{t} . This provides us with the intuition for the optimal structure of taxes and subsidies when both policy instruments are simultaneously optimized. By using Roy's lemma and the Slutsky equations, we can rearrange the first-order condition for education subsidies (equation (19)) to find the optimal subsidy rate for a given \bar{t} at optimal T (see Appendix B):

$$\frac{s}{1-s} = \left(\frac{\varepsilon_{ls}/\varepsilon_{es}}{1-\pi_e} + \frac{\pi_e}{1-\pi_e} \right) \bar{t}. \tag{24}$$

Here,

$$\varepsilon_{ls} \equiv \frac{\mathcal{E}[\Phi_l(\cdot)]l}{\mathcal{E}[\Phi(\cdot)]} \frac{\partial l^*}{\partial s} \frac{1-s}{l},$$

and

$$\varepsilon_{es} \equiv \frac{\mathcal{E}[\Phi_e(\cdot)]e}{\mathcal{E}[\Phi(\cdot)]} \frac{\partial e^*}{\partial s} \frac{1-s}{e} > 0,$$

denote the expected-utility compensated elasticities of labor and education, respectively, with respect to the education subsidy. These elasticities are again weighted by the expected shares of labor and education in total earnings.

The insurance characteristic is absent in the expression for optimal education subsidies. In contrast to the optimal income tax (see previous subsection), there is no gain in using education subsidies for insurance. Education subsidies are not used at all ($s = 0$) when the income tax is zero ($\bar{t} = 0$). In this case, the only way households can reduce their exposure to risk is to self-insure: to overinvest or underinvest in education. This self-insurance is chosen optimally. Education subsidies do not yield additional welfare gains in the absence of income taxation ($\bar{t} = 0$), because education subsidies are state-independent. Therefore, education subsidies do not directly reduce the exposure to income risk. Consequently, in the

absence of tax-provided social insurance, the government cannot improve market outcomes by subsidizing education, as this policy would only upset the optimal private responses of individuals to income risk by distorting human-capital investment.

However, there is a role for education policy when the government organizes social insurance through an income tax system ($\bar{t} > 0$). At an exogenously given tax rate $\bar{t} > 0$, education subsidies correct the tax distortions of the income tax. There are two reasons why education subsidies are optimally employed.

First, labor taxation distorts labor supply. If $\varepsilon_{ls} > 0$, education and labor supply are complementary in generating income. Thus, by subsidizing education, the government can indirectly boost labor supply, and thereby reduce the tax distortions on labor supply. The higher the tax rate t , the larger the distortions on labor supply, and the larger the need will be to fight these labor–tax distortions with education subsidies. Similarly, if education and labor are substitutes, $\varepsilon_{ls} < 0$, education should be taxed so as to increase labor supply and to offset the tax wedge on labor (see also Jacobs and Bovenberg, 2011). In both cases, the government trades off fewer tax-induced distortions on labor supply against larger subsidy-induced distortions in educational investment. The more education responds to subsidies (larger ε_{es}), the larger the social cost of undesirable overinvestment will be, and the lower the optimal education subsidy.

Second, the education subsidy internalizes the fiscal externality, which is represented by the second term, $\pi_e/(1 - \pi_e)\bar{t}$. Note that this term equals the implicit tax wedge on education in equation (23), $\mathcal{E}[\Delta_e]_{s=0}$, where education subsidies are absent. Consequently, the education subsidy fully internalizes the fiscal externality arising from underinvestment or overinvestment in human capital in the presence of income taxes. The higher the exogenously given labor tax rate $\bar{t} > 0$, the larger the fiscal externality $\pi_e/(1 - \pi_e)$ because of the implicit tax on human capital. If education is risk-increasing ($\pi_e > 0$), education should be subsidized more in order to internalize the fiscal externality. If education has a risk-mitigating effect ($\pi_e < 0$), there is an implicit subsidy on human capital, which is *ceteris paribus* offset by an explicit tax on education.

By combining these two arguments, it becomes apparent that optimal education subsidies are unambiguously positive when education and labor supply are complementary ($\varepsilon_{ls} > 0$) and when there is underinvestment in education ($\pi_e > 0$). In this case, education subsidies help both to reduce tax distortions in labor supply and to internalize the fiscal externality of underinvestment in education. However, if education hedges against labor-market risks ($\pi_e < 0$), the two arguments pull in opposite directions as long as education and labor remain complementary ($\varepsilon_{ls} > 0$). Therefore, the sign of the education subsidy cannot be unambiguously determined. In

particular, education should be taxed if $\varepsilon_{ls}/\varepsilon_{es} < -\pi_e$. In this case, socially undesirable overinvestment in education is relatively large compared to the complementarity of education with labor supply.

These findings are related to those of Bovenberg and Jacobs (2005) and Jacobs and Bovenberg (2011), who have analyzed optimal redistribution and education policy with *ex ante* differing individuals and no income risk. On the one hand, these authors have also demonstrated that education subsidies boost labor supply, and thereby help to offset tax distortions from social insurance. On the other hand, education subsidies generate inequality, because of the ability bias in education. The latter effect is absent in our model, because everyone is identical *ex ante*.¹² We summarize the findings of this subsection in the following proposition.

Proposition 2. *Education subsidies do not provide income insurance, and they are only used for efficiency reasons. First, education subsidies boost labor supply when education and work effort are complementary. Education subsidies are higher if labor and education are more complementary. Second, education subsidies or taxes are used to internalize the fiscal externality. Optimal education subsidies are higher (lower) if there is more underinvestment (overinvestment) in human capital.*

Combining Optimal Tax and Education Policies

By combining the expressions for the optimal tax and education policies (equations (A4) and (B8) from Appendices A and B), we obtain the optimal tax rate \hat{t} and education subsidies \hat{s} if the government simultaneously optimizes income taxes and education subsidies:

$$\frac{\hat{t}}{1 - \hat{t}} = \frac{\xi}{\varepsilon_{lt} - (\varepsilon_{ls}/\varepsilon_{es})\varepsilon_{et}}, \quad (25)$$

$$\frac{\hat{s}}{1 - \hat{s}} = \left(\frac{\varepsilon_{ls}/\varepsilon_{es}}{1 - \pi_e} + \frac{\pi_e}{1 - \pi_e} \right) \hat{t}. \quad (26)$$

Note that all statements in Section IV about the optimal education subsidy for a given tax policy carry over to the case in which tax and education policies are simultaneously optimized. For this reason, here we do not discuss any further the expression in equation (26), and we refer to the previous subsection.

Our results bolster the findings of Anderberg (2009) that the risk properties of human capital are crucial for the design of optimal human-capital

¹² This is also the reason why education subsidies are not used for insurance.

policies. While Anderberg (2009) considers a general set of information-rich non-linear policy instruments, our analysis shows that the risk properties of human capital are also key for optimal human-capital policies under linear policy instruments, which are less informationally demanding. Optimal education policies will not ensure aggregate efficiency in human-capital investment, because not all income risk will be fully diversified. Moreover, our analysis points out that the fiscal externalities associated with missing insurance markets are crucial for the design of educational policy.

The optimal tax rate \hat{t} is no longer directly affected by the risk wedge, because (compared to equation (22)) the risk premium in education π_e ceases to enter the optimal tax formula. Hence, the income tax no longer exacerbates underinvestment if $\pi_e > 0$, and it no longer mitigates overinvestment if $\pi_e < 0$. The expression for the optimal income tax confirms that the education subsidy perfectly internalizes the fiscal externality arising from underinvestment or overinvestment in human capital. Education subsidies are a more efficient instrument with which to internalize the fiscal externality than income taxes, because income taxes also distort labor supply. This finding mirrors the results on optimal taxation in the presence of externalities by Sandmo (1975, p. 92, p. 95). He shows that externalities should optimally be internalized by only correcting the price of the commodity, which causes the externality, in an additive way (“additive property”). In our case, this commodity is education. We also find that the correction term enters additively into expression (26) for optimal education subsidies.

Therefore, the interpretation of the optimal tax rate changes slightly, because it is now exclusively used for insurance purposes. Naturally, the optimal tax rate still increases in the marginal benefits of insurance (ξ) and decreases in higher tax-induced distortions in labor supply (ε_{lt}). However, the new optimal tax expression (25) reveals that the optimal income tax increases if education and labor supply become more complementary, as indicated by $\varepsilon_{ls}/\varepsilon_{es}$ (see also the expression for the optimal education subsidy). Education subsidies boost labor supply if $\varepsilon_{ls} > 0$, and thereby they help to offset the tax distortions on labor effort. Consequently, income taxes increase (*ceteris paribus*). If education responds very elastically to education subsidies, then ε_{es} is large and optimal tax rates are lower, because subsidies are more distortionary and they exacerbate overinvestment in education.

The optimal use of education policy does not unambiguously increase optimal income tax rates, for given demand for redistribution ξ , and assuming that the elasticities remain the same. This can be inferred from comparing the optimal tax policy joint with optimal education subsidies, in equation (25), with the optimal tax rate in equation (22), where education policy is absent ($s = 0$). Intuitively, we would expect the optimal tax rate to be higher if the government has more instruments. This conclusion is

not necessarily valid in the current second-best setting with multiple distortions. The intuition is only confirmed for the case where $\pi_e > 0$. Without education subsidies, income taxation exacerbates the non-tax distortions from missing insurance markets, which causes a negative fiscal externality. Thus, a lower tax rate is optimal. With optimal education policy internalizing the fiscal externality, the optimal income tax is unambiguously higher (even if $\varepsilon_{ls} = 0$). However, in the case of overinvestment in human capital as a result of missing insurance markets ($\pi_e < 0$), the income tax features a positive fiscal externality by mitigating non-tax distortions in human-capital investment. However, when education subsidies, or even education taxes (e.g., if $\varepsilon_{ls} = 0$), are available, there is no longer a role for the income tax to correct for overinvestment in human capital. As a result, optimal income taxes might well be lower. We summarize our findings in the following proposition.

Proposition 3. *If labor and education are more complementary, both the optimal tax rate and optimal education subsidies increase. If the risk premium on education rises, there will also be a rise in optimal education subsidies. If education increases earnings risk, education policy allows for more social insurance compared to tax policy alone. If education hedges against labor-market risk, then optimal tax rates with education policy could be lower than the case without education policy, if the complementarity between education and labor is sufficiently weak.*

Our findings are importantly related to those of Hamilton (1987), Anderberg and Andersson (2003), and Anderberg (2009). Hamilton (1987) extends the findings by Eaton and Rosen (1980b) and analyzes capital taxes as an indirect education subsidy. Hamilton (1987) is right in pointing out that there remains underinvestment in education when income taxes are optimally set. Consequently, a capital tax could be welfare-enhancing, because a capital tax is an indirect education subsidy. Because we assume that education is verifiable, we can allow for direct education subsidies. We have shown that the role for education policy is to internalize the fiscal externality associated with underinvestment. Therefore, we are able to show that the use of education subsidies is always welfare-enhancing. Hamilton (1987) needs strong assumptions (constant absolute risk aversion and inelastic labor supply) to show that his education policy is desirable, because – in contrast to education subsidies – capital taxes also distort savings.

Of the above-mentioned studies, the analysis by Anderberg and Andersson (2003) is closest to ours. The major difference is that they assume that the government can impose a mandatory level of education centrally. Consequently, there is no fiscal externality in human-capital

investment, which explains the absence of the risk premium in their optimal tax formula (in their equation (11)). Education policy then has an insurance effect, because it replaces the self-insurance of households in a decentralized setting. Anderberg and Andersson (2003, p. 1523) state that “the insight is thus that if education moderates wage uncertainty, a second-best policy should, rather unambiguously, encourage the formation of human capital (relative to the first-best), while if education exacerbates wage uncertainty the overall conclusion is ambiguous.” Although this statement is correct, it would be misleading to conclude that education subsidies (or taxes) would constitute an optimal policy when education decisions are made at the decentralized level. Indeed, if households choose educational investment themselves, there is no insurance effect of educational policy. More importantly, the novel finding of our paper is that – in the presence of income taxation – there will be socially “excessive” underinvestment (overinvestment) by households, compared to the constrained second-best optimal amount of underinvestment (overinvestment). *Ceteris paribus*, this calls for a policy that encourages (discourages) educational investment. Even under linear policy instruments, it can be misleading to obtain policy recommendations by looking at the optimal wedges on individual choices, and by comparing these with the first-best choice rules. The policy implementation in our setting is the polar opposite of what the wedges on education seem to suggest. As has also been stressed by Golosov *et al.* (2003, 2006), there is generally no clear-cut correspondence between tax wedges and tax rates that would implement optimal second-best allocations. We believe that this could also be an important issue for the recent papers in the new dynamic public finance tradition (e.g., da Costa and Maestri, 2007; Anderberg, 2009). In light of this discussion, we rephrase our results in the following corollary.

Corollary 1. *From a positive (negative) tax wedge on education compared to the first-best rule, the conclusion cannot be drawn that education should be subsidized (taxed) if human-capital investment is made at the decentralized level, and if the government only has indirect control over individual choices via subsidies and taxes.*

V. Conclusions

In this paper, we have analyzed optimal social insurance and education policy. The optimal income tax strikes a balance between the benefits of social insurance and the distortions in labor supply. The optimal income tax is higher if education and work are more complementary, because the government can indirectly offset labor–tax distortions by subsidizing

education. Optimal education subsidies unambiguously increase if education and labor are more complementary. In this case, subsidies on education are a more attractive instrument with which to fight tax distortions on labor supply. The optimal income tax is not determined by the risk properties of human capital – but optimal education policies are. Education subsidies are not used to offset underinvestment or overinvestment in human capital in the absence of taxation, because this would upset the optimal private response to market risks. However, the non-insurable risk as a result of missing insurance markets gives rise to a fiscal externality from income taxation. Boosting education yields higher (lower) tax revenues if there is underinvestment (overinvestment) in human capital. Subsidizing (taxing) education is optimal in order to internalize the fiscal externality originating from the missing insurance markets, *ceteris paribus*. Hence, if education is a risky activity, there is a strong role for subsidizing education on a net basis in order to offset the distortions of social insurance on human-capital investments and labor supply. Social insurance will then increase. However, if education hedges against labor-market risk, the case for education subsidies is weakened, and social insurance can even be reduced compared to the outcome in the absence of education subsidies. Whether education subsidies or education taxes should be employed is an empirical question that can only be answered by ascertaining the risk properties of human capital.

Appendix A: Optimal Taxation

We simplify the first-order condition for the tax rate t (equation (18)) by substituting Roy's lemma and the Slutsky equations (the derivation for the Slutsky equations is available upon request):

$$\frac{\partial e}{\partial t} = \frac{\partial e^*}{\partial t} - ((1 - \xi)\mathcal{E}[\Phi] + R[1 - (1 - s)e])\frac{\partial e}{\partial T}, \quad (\text{A1})$$

$$\frac{\partial l}{\partial t} = \frac{\partial l^*}{\partial t} - ((1 - \xi)\mathcal{E}[\Phi] + R[1 - (1 - s)e])\frac{\partial l}{\partial T}. \quad (\text{A2})$$

Here, the asterisk denotes compensated demand or supply functions. After using Steiner's rule, the definition of ξ from equation (21), and the first-order condition for T from equation (20), we find

$$\xi \mathcal{E}[\Phi(\cdot)] = -\mathcal{E}[\Delta_e] \frac{\partial e^*}{\partial t} - \mathcal{E}[\Delta_l] \frac{\partial l^*}{\partial t}. \quad (\text{A3})$$

We substitute Δ_e and Δ_l from equations (15) and (16), as well as $\mathcal{E}[\Phi_e] = [R(1-s)/(1-\pi_e)]$ from equation (10), and we rearrange to obtain

$$\xi = \frac{t}{1-t} \varepsilon_{lt} + \left(\frac{\pi_e [s + t(1-s)] - s}{(1-s)(1-t)} \right) \varepsilon_{et}. \tag{A4}$$

Here,

$$\varepsilon_{et} \equiv - \frac{\mathcal{E}[\Phi_e(\cdot)]e}{\mathcal{E}[\Phi(\cdot)]} \frac{\partial e^*}{\partial t} \frac{1-t}{e},$$

and

$$\varepsilon_{lt} \equiv - \frac{\mathcal{E}[\Phi_l(\cdot)]l}{\mathcal{E}[\Phi(\cdot)]} \frac{\partial l^*}{\partial t} \frac{1-t}{l},$$

equal the (negative) income-weighted expected-utility compensated elasticities of education and labor, respectively, with respect to the tax rate. The elasticities are weighted by the expected shares of education and labor in total earnings. Finally, applying $\bar{s} = 0$, and rearranging and collecting terms, we obtain the expression in the text.

Appendix B: Optimal Education Policy

We simplify the first-order condition for education by substituting Roy’s lemma and the Slutsky equations (the derivation for the Slutsky equations is available upon request):

$$\frac{\partial e}{\partial s} = \frac{\partial e^*}{\partial s} + R(1-t)e \frac{\partial e}{\partial T}; \tag{B1}$$

$$\frac{\partial l}{\partial s} = \frac{\partial l^*}{\partial s} + R(1-t)e \frac{\partial l}{\partial T}. \tag{B2}$$

Thus, we find

$$\mathcal{E} \left[R(1-t)e \left(\frac{u_2}{\eta} + \Delta_l \frac{\partial l}{\partial T} + \Delta_e \frac{\partial e}{\partial T} - 1 \right) \right] = -\mathcal{E}[\Delta_e] \frac{\partial e^*}{\partial s} - \mathcal{E}[\Delta_l] \frac{\partial l^*}{\partial s}. \tag{B3}$$

Because e is not stochastic, we have

$$\mathcal{E} \left[R(1-t)e \left(\frac{u_2}{\eta} + \Delta_l \frac{\partial l}{\partial T} + \Delta_e \frac{\partial e}{\partial T} - 1 \right) \right] = 0$$

from the first-order condition for T in equation (20). Substituting Δ_e and Δ_l from equations (15) and (16), as well as using the first-order condition for learning $\mathcal{E}[\Phi_e] = [R(1-s)/(1-\pi_e)]$ from equation (10), we obtain

$$t\varepsilon_{ls} + \left(\frac{\pi_e [s + t(1-s)] - s}{1-s} \right) \varepsilon_{es} = 0. \tag{B4}$$

Here, we have defined the subsidy elasticities analogously to the tax elasticities as the income-weighted expected-utility compensated elasticities:

$$\varepsilon_{es} \equiv \frac{\mathcal{E}[\Phi_e(\cdot)]e}{\mathcal{E}[\Phi(\cdot)]} \frac{\partial e^*}{\partial s} \frac{1-s}{e},$$

$$\varepsilon_{ls} \equiv \frac{\mathcal{E}[\Phi_l(\cdot)]l}{\mathcal{E}[\Phi(\cdot)]} \frac{\partial l^*}{\partial s} \frac{1-s}{l}.$$

Rewriting yields the expression in the text.

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