

# Self-selection bias in estimated wage premiums for earnings risk

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**Abstract** This note develops a simple occupational choice model to examine three types of selection biases that may occur in empirically estimating the premium for uncertain wages. Individuals may select themselves into risky (wage-uncertain) jobs because they have (1) lower risk aversion, or (2) lower income risks, or (3) higher individual ability. We show that (1) gives no bias, (2) biases the OLS estimate of the risk-premium in a wage regression upward, and (3) yields a bias that analytically may be positive or negative, but empirically is more likely to be negative if our occupational choice model is correct.

**Keywords** Wages · Earnings risk · Selectivity bias

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## 1 Introduction

A modest literature acknowledges the fact that individuals contemplating an investment in an education–occupation career trajectory are unable to make a perfect prediction of the associated wage because they face a distribution of wages rather than a single wage rate after they complete a given education. This literature seeks to test the hypothesis of compensation for the differences in variability of wages within occupations and/or educations, as the premium that has to be paid to overcome individuals' risk aversion, a hypothesis already discussed by [Smith \(1776/1976, p. 208\)](#). In these models, individual wages are regressed on measures of income risk, typically the residual variance of wages, within occupations or educations ([King 1974](#); [McGoldrick 1995](#); [McGoldrick and Robst 1996](#); [Feinberg 1981](#)). A series of recent papers confirms the basic finding that wages are higher in occupations/educations where the variance of wages is greater ([Hartog et al. 2003](#); [Hartog and Vijverberg 2007](#); [Diaz Serrano et al. 2003](#); [Diaz Serrano and Hartog 2006](#); [Christiansen and Nielsen 2002](#)).

To be more precise, the model is commonly estimated in two stages. The first stage is a standard Mincer earnings equation where wages are regressed on experience (linear and squared), demographic characteristics and regional variables as well as a fixed effect that captures common factors across wage workers grouped by education and/or occupation, i.e., groups that face the same wage risk. The residuals from this equation then yield group-specific measures of earnings uncertainty, typically the variance.<sup>1</sup> In the second stage, variance is added to the regression equation. Theory predicts that variance has a positive effect on wages, as potential students require compensation for risk. Estimates for seven countries so far all confirm this prediction, at high levels of significance. Roughly, the estimates indicate an increase in expected income by some 1 to 4% for an increase in risk (variance) of 10%.

The results in the papers cited above are commonly criticised for ignoring selectivity problems. Indeed, the simple two-stage model faces three problems that may bias the estimated regression coefficient as a measure of risk aversion. First, evidence suggests that risk attitudes differ among individuals ([Hartog et al. 2002](#); [Dohmen et al. 2006](#)). Unlike related literature that estimates a single coefficient of risk aversion, we will reflect on the implication of heterogeneous risk attitudes.

Second, it is quite possible that the earnings risk of an education differs among individuals. [Cunha et al. \(2005\)](#) claim that 60% of variability in returns to education is forecastable at the individual level and, hence, is related to individual heterogeneity, leaving 40% for risk. The reason could be that some individuals may by their nature be very targeted and organised, others may take life less as a controlled operation; and just as some individuals may be very accurate in manual activities, while others have larger dispersion in their performances, in the same way dispersion in mental activities may vary between individuals (see also [Hartog 2002](#)). We therefore believe that it is quite relevant to analyze the implications of selection on individual-specific risk on estimated risk premiums in wages.

<sup>1</sup> The literature also considers skew, but that is beyond the scope of this paper.

Third, individuals differ in ability: variation in a wide array of abilities among individuals is well documented.<sup>2</sup> If individuals know their abilities, but the researcher does not, correction for selectivity bias is in order. However, it may be that individuals are no better informed and can only assess the relevant distribution of earnings from observing workers already active in an occupation, without adjustment for ability.<sup>3</sup> Thus, one may argue over the extent to which individuals are informed about their own abilities. We will therefore analyse both the case where individuals are fully ignorant on their ability and the case where they are fully informed.

Under the assumption that there are no selection problems, the OLS estimator indeed correctly estimates the true economic price of risk. With selection of individuals in risky jobs, we can no longer be sure that this is the case. This note shows, using a simple occupational choice model, that the OLS estimate is *unbiased* under selection on risk aversion. OLS estimates are biased *upward* if selection on income risks is important. If both income risk and risk aversion differ among individuals, we cannot recover structural parameters from a single wage equation, but we can give a precise interpretation of the estimated coefficient. Under heterogeneity in unobserved ability, the bias in OLS estimates is ambiguous, and we revert to simulation with reasonable parameter values. The next three sections derive these results. The last section concludes.

## 2 Modeling the risk premium under selectivity

Individuals can choose between two types of jobs. There is a safe job without wage uncertainty where individual  $n$  earns  $w_n^s$ . Alternatively, there is a risky job where individual  $n$  earns  $\tilde{w}_n^r = w_n^r + \varepsilon_n$ , where  $w_n^r$  is the expected wage and  $\varepsilon_n$  is the random component, with mean zero and variance  $\sigma_{\varepsilon n}^2$ , independent from  $w_n^r$ . The expected wage is what the individual expects to earn on the basis of known skills and abilities; he can infer it from the wages of individuals with the same observable skills and abilities as he has. The random component reflects risks inherent in fluctuations in demand and supply conditions, unknown specifics of future employment, inherent randomness in individual performance and randomness due to aptitudes and abilities that the individual does not know when choosing a job (or, in the broader context, an education; see also [Levhari and Weiss 1974](#)).

There is a continuum of individuals indexed  $n \in [\underline{n}, \infty)$ ,  $\underline{n} > 0$ , who may differ in three dimensions: risk aversion  $\rho_n$ , variance of earnings in the risky job  $\sigma_{\varepsilon n}^2$  and expected wages in risky work  $w_n^r$ . These characteristics are assumed to be uncorrelated

<sup>2</sup> [Chen \(2008\)](#) and [Chen and Khan \(2007\)](#) correct for selectivity bias when estimating the risk associated with an education.

<sup>3</sup> [Dominitz and Manski \(1996\)](#) show that students in the U.S. are highly uncertain about the benefits gained from an education. [Wolter \(2000\)](#) reports the same result for Swiss students. [Hartog and Webbink \(2004\)](#) report that first-year students can predict mean salaries by education but they cannot predict their own starting salary after graduation, four years ahead: the correlation between an individual's prediction and the realisation is 0.06. The degree to which individuals are not informed about their abilities represents risk all the same.

with the random component  $\varepsilon_n$  of the wage at the risky job, which indeed is consistent with our characterization of  $\varepsilon_n$ .

Assuming that utility is a mean-variance function, selection by maximum expected utility implies<sup>4</sup> that we can write for the marginal individual (denoted by an asterisk)

$$w^{r*} - \frac{1}{2}\rho^* \sigma_\varepsilon^{*2} = w^{s*}. \quad (1)$$

If all individuals would be identical in ability, risk and risk attitude, we could drop the asterisk. A competitive market would establish a wage differential between the safe and the risky job equal to one half of the variance in risky wages weighted by the coefficient of risk aversion, an application of the standard model for risk compensation (e.g., [Gollier 2001](#), 19–20). This is essentially the model underlying the estimates of risk compensation referred to in the introduction. Under those conditions OLS yields an unbiased estimate of the coefficient of risk aversion (i.e., the price of risk). In this two-sector specification, it is estimated as twice the wage gap divided by the variance in the risky sector. The variance is an unbiased estimator of risk, as it is identical for everyone and observations are not blurred by selectivity. Allocation of individuals to sectors is arbitrary, as they yield identical expected utility.

Extension to the case where individuals only differ in risk attitude is straightforward. If individuals are identical in all dimensions except risk aversion risk aversion of the marginal individual  $\rho^*$  is defined by

$$\rho^* = \frac{w^r - w^s}{\frac{1}{2}\sigma_\varepsilon^2}. \quad (2)$$

All individuals with  $\rho_n > \rho^*$  will not choose the risky job and all individuals with  $\rho_n < \rho^*$  will. The expected market wage in the risky job should just compensate the marginal worker and will be equal to

$$w^r = w^s + \frac{1}{2}\rho^* \sigma_\varepsilon^2, \quad (3)$$

and the observed variance of wages in the risky job equals

$$V[\tilde{w}^r | \rho_n < \rho^*] = E\left[(\tilde{w}^r - E[\tilde{w}^r | \rho_n < \rho^*])^2 | \rho_n < \rho^*\right] = \sigma_\varepsilon^2, \quad (4)$$

as all individuals have the same risk. Hence, the OLS estimate for the risk-premium  $\frac{1}{2}\rho^*$  is consistent and unbiased.

We have specified the earnings function in terms of wages, whereas it is conventional in empirical work to specify a function for log wages. However, the only widely accepted theoretical underpinning, Mincer's human capital model, only applies for variables reflecting investment (years of schooling, experience). For other variables it

<sup>4</sup> This follows from a second-order Taylor approximation for a standard utility function, see [Varian \(1978\)](#) and [Hartog and Vijverberg \(2007\)](#).

is not at all obvious that the relationship with wages should be logarithmic. In practice, labour supply may react to absolute wage differences at the low end of the scale and to relative differences at the high end. Blue-collar and low pay service worker, often paid on an hourly basis, tend to measure, perceive and evaluate wage differences in euro's (or dollars). For workers at the high end, differences tend to be evaluated at a relative (percentage) scale. This suggests that somewhere along the line, the underlying relevant scale for labour supply behaviour switches from absolute to relative. With respect to risk attitude, there is a similar ambivalence. Absolute risk aversion leads to risk compensation in euro's, relative risk aversion leads to relative compensation. In the formal development of our argument, we specify an earnings function in wages. In the qualitative sense (positive, negative or ambiguous bias) our analysis also holds for a logarithmic wage function. In the Appendix, we formally derive a logarithmic model that is fully equivalent to our model in the main text; according to that model, all our conclusions hold provided we interpret the coefficient of risk aversion as relative rather than absolute

Throughout this paper, we discuss issues in terms of risk averse individuals. However, the model is perfectly general, and also covers risk loving. Career choices based on a taste for risk are certainly conceivable. In the empirical work, positive estimates for  $\rho$  are the rule and we take risk aversion to be the normal case.

### 3 Selection on income risk

Now consider the case in which risk in the risky job,  $\sigma_{\varepsilon n}^2$ , differs between individuals. Ex ante, this is a risk; ex ante, individuals know the dispersion of the distribution relevant to them and base their occupational choice on it. Ex post, they will be paid according to performance that is random. Individuals are equal in all other respects, i.e.,  $w_n^s = w^s$ ,  $w_n^r = w^r$ , and  $\rho_n = \rho$ . In this case, the degree of income risk for the marginal individual equals

$$\sigma_{\varepsilon}^{*2} = \frac{w^r - w^s}{\frac{1}{2}\rho} \quad (5)$$

Individuals with  $\sigma_{\varepsilon n} > \sigma_{\varepsilon}^*$  will not take the risky job and workers with  $\sigma_{\varepsilon n} < \sigma_{\varepsilon}^*$  do. Wage realisations in the risky job will vary because of different risk. The variance of observed wages will reflect the mean variance of individuals who have selected into the risky job, which is lower than the risk for the marginal worker. The market will compensate the marginal worker in the risky job, at a price  $\frac{1}{2}\rho$ . Since the observed wage variance is an underestimate of the risk that the marginal worker faces, too much of the wage gap between the safe and the risky job is allocated to risk aversion—as (5) indicates, dividing the estimated wage gap by the underestimated variance yields an overestimate of risk aversion. The magnitude of the overestimation will depend on the underestimation of risk, that is, on the relation between marginal and average risk. This will depend on the distribution function of risk and the number of individuals selecting the risky job.

For example, consider the case where risk follows a normal distribution truncated at 0, i.e., where  $\sigma_\varepsilon \sim N^+(\mu, \theta^2)$ . Under this assumption, the bias increases with the level of threshold risk (i.e., marginal risk  $\sigma_\varepsilon^*$ ) and may increase or decrease with the dispersion  $\theta$  of risk across individuals. Defining  $v^* = (\sigma_\varepsilon^* - \mu)/\theta$  and  $v^0 = -\mu/\theta$ , and denoting the standard normal probability density and cumulative distribution functions with  $\phi()$  and  $\Phi()$  respectively, the mean observed risk equals,

$$E[\sigma_\varepsilon | \sigma_\varepsilon < \sigma_\varepsilon^*] = \mu - \theta \frac{\phi(v^*) - \phi(v^0)}{\Phi(v^*) - \Phi(v^0)} \equiv \mu - \theta\lambda. \tag{6}$$

Define the degree of underestimation (or bias) of risk as the difference between the threshold risk<sup>5</sup> and mean risk:

$$D = \sigma_\varepsilon^* - E[\sigma_\varepsilon | \sigma_\varepsilon < \sigma_\varepsilon^*] = \sigma_\varepsilon^* - \mu + \theta\lambda \tag{7}$$

Note that  $\lambda > (<)0$  as  $v^* > (<)-v^0$ , or  $\sigma_\varepsilon^* > (<) 2\mu$ . Because of this, the spread of risk among the population has an ambiguous impact on the bias. Furthermore, applying L'Hôpital's Rule shows that  $D$  goes to 0 when  $\sigma_\varepsilon^*$  converges to 0, the smallest value it could have. Moreover,

$$\begin{aligned} \frac{\partial D}{\partial \sigma_\varepsilon^*} &= 1 - v^*\lambda - \lambda^2 - \frac{(v^* + \lambda)\phi(v^0)}{\Phi(v^*) - \Phi(v^0)} \\ &= \frac{V[\sigma_\varepsilon | \sigma_\varepsilon < \sigma_\varepsilon^*]}{\theta^2} + \frac{E[\sigma_\varepsilon | \sigma_\varepsilon < \sigma_\varepsilon^*]}{\theta} \frac{\phi(v^0)}{\Phi(v^*) - \Phi(v^0)} > 0 \end{aligned} \tag{8}$$

The slope as represented on the first line is difficult to evaluate but it may be written as the scaled sum of the conditional variance of  $\sigma_\varepsilon$ , which is positive, and the conditional mean of  $\sigma_\varepsilon$ , which is positive as well. Thus,  $D$  rises from 0 as  $\sigma_\varepsilon^*$  grows: the higher the upper bound, the larger the difference between the bound and the conditional mean. In our application, this means that the underestimation of risk and hence, the overestimation of risk aversion, increases with an increase in the dispersion of risk and with an increase in marginal risk.<sup>6</sup>

If individuals *also* differ in risk aversion, the situation is more complicated. Now, there is no longer a single marginal worker, but a set of marginal workers, defined by the condition that the product of risk and risk aversion be equal to twice the wage gap:  $\frac{1}{2}(\rho\sigma_\varepsilon^2)^* = (w^r - w^s)$ . A marginal worker may have a large risk aversion and a low income risk, or a low risk aversion and a large income risk. Structural parameters of interest are now the parameters of the joint distribution of risk and risk aversion,

<sup>5</sup> We use threshold risk as the benchmark, as this is the relevant parameter in our case: the market would determine the price of risk  $\rho$  as the wage gap per unit of risk for the marginal worker.

<sup>6</sup> A truncated normal distribution allows for a nonnegligible number of jobs with near-zero risk, which may be realistic since zero-risk jobs are present as well. Alternatively, one might assume that  $\sigma_\varepsilon$  follows a lognormal distribution, without truncation and therefore in essence without job with near-zero risk. This leads to the same results, since  $\sigma_\varepsilon < \sigma_\varepsilon^*$  implies  $\ln \sigma_\varepsilon < \ln \sigma_\varepsilon^*$  and thus the relationship between  $D$  and  $\sigma_\varepsilon^*$  is a nonlinear but monotonic transformation of the case just examined here.

and they can never be estimated from a single wage regression. The best we can hope for is an estimate of the threshold contour, and a decomposition of this threshold in combinations of risk aversion and risk at the margin.

Formally, workers will take a risky job as long as  $E[\tilde{w}_n^r] > w_n^s + \frac{1}{2}\rho_n\sigma_{\varepsilon n}^2$ . Since  $E[\tilde{w}_n^r] = w_n^r$ ,  $w_n^r = w^r$  and  $w_n^s = w^s$ , we find that for the marginal worker  $w^r = w^s + \frac{1}{2}(\rho\sigma_\varepsilon^2)^*$ . Any worker for whom  $\rho_n\sigma_{\varepsilon n}^2 < 2(w^r - w^s) = (\rho\sigma_\varepsilon^2)^*$  will select into a risky job. For the mean of the observed wages, that is, the wages of those who select into risky jobs, we have:<sup>7</sup>

$$\begin{aligned} E[\tilde{w}^r | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*] &= w^r + E[\varepsilon | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*] \\ &= w^r + E_{\rho, \sigma_\varepsilon^2}^c [E_\varepsilon[\varepsilon | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*]] = w^r \end{aligned} \tag{9}$$

since  $\varepsilon$  is independent of  $\rho$  and  $\sigma_\varepsilon^2$ . This means that the estimated wage gap between the risky and the safe job is an unbiased estimate of the threshold for entering the risky job. But we cannot divide by a single estimate of risk to get a single estimate of marginal risk aversion. All combinations of risk and risk aversion that multiply to the threshold value are permissible. With heterogeneity in risk, the observed variance in the risky job is the mean value of risk for all those individuals who have chosen the risky job:

$$\begin{aligned} V(\tilde{w}^r | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*) &= E\left[\left(\tilde{w}^r - E[\tilde{w}^r | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*]\right)^2 | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*\right] \\ &= E[\varepsilon^2 | \rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*] = E_{\rho, \sigma_\varepsilon^2}^c [\sigma_\varepsilon^2] \end{aligned} \tag{10}$$

again since  $\varepsilon$  is independent of  $\rho$  and  $\sigma_\varepsilon^2$ , and where the last expectation is conditional on being in the range  $\rho\sigma_\varepsilon^2 < (\rho\sigma_\varepsilon^2)^*$ .

Equations (9) and (10) provide an interpretation of the regression coefficient: it is the highest degree of risk aversion among those in a risky job who face a level of risk equal to the average among all workers who hold risky jobs. One may debate whether this is interesting information, or whether one would rather uncover the mean risk aversion among the population or the mean risk aversion among risky job holders or the maximum admitted value of risk aversion at the population mean risk. But the fact of the matter is that the truly interesting parameters are those of the joint distribution of risk and risk aversion, and they can not be recovered from a single wage regression. To search for answers in this regard, a more elaborate model and a more extensive dataset are needed.

<sup>7</sup> A subscript on the  $E$  operator denotes the random variable over which the expectation is taken. The superscript  $c$  denotes a conditional expectation.

#### 4 Differences in ability

What is the impact of unobserved abilities on estimated risk premiums? As the introduction indicated that full information is not necessarily the correct assumption, let us start by assuming that individuals do not know their exact ability in the risky job but rather only the distribution from which their performance will be drawn. We will assume identical risk attitudes and equal ability in the safe job:  $\rho_n = \rho$ ,  $w_n^s = w^s$ . Moreover, risk is also identical:  $\sigma_{\varepsilon n}^2 = \sigma_\varepsilon^2$ , where as always  $\sigma_{\varepsilon n}^2$  refers to the variance of the random income component  $\varepsilon_n$ .

Assume an individual's expected wage in the risky job is given by the sum of a common wage component  $\omega^r$  and individual ability  $a_n$ , where  $a_n$  is a draw from a distribution with mean 0 and variance  $\sigma_a^2$ :

$$w_n^r \equiv \omega^r + a_n. \quad (11)$$

Thus, an individual's ability  $a_n$  matters only for the risky job.<sup>8</sup> As before, the observed wage is determined by  $\tilde{w}_n^r = w_n^r + \varepsilon_n$ . As noted, we will assume that all individuals face identical risk, reflected in the variance  $\sigma_\varepsilon^2$ . Now, clearly, individuals will only be observed in the risky job if they are compensated for the ability risk as well. As individuals are identical in every aspect except for unknown ability  $a_n$ , the expected risky wage equals  $E[w_n^r] = \omega^r$  for all  $n$ , which contains no reference to  $a_n$ . As a result, the equilibrium wage differential is determined by the equation

$$E[\tilde{w}^r] - w^s = \frac{1}{2}\rho(\sigma_a^2 + \sigma_\varepsilon^2), \quad (12)$$

assuming ability  $a_n$  and risk  $\varepsilon_n$  are uncorrelated. The variance in the safe job is zero, the variance in the risky job is  $\sigma_a^2 + \sigma_\varepsilon^2$ , and this is exactly what individuals demand compensation for: regressing wages on variances identifies  $\rho$ , the price of risk. Hence, the estimate for  $\rho$  is unbiased. In our reading,  $a_n$  represents the type of risk Smith was concerned about: "The probability that any particular person shall ever be qualified for the employment to which he is educated is very different in different occupations. ... In a profession where twenty fail for one that succeeds, that one ought to gain all that should have been gained by the unsuccessful twenty" (Smith 1776/1976, p. 208).

If individuals know their ability but the researcher does not, we must address the selectivity process. We might consider the situation where ability differences take effect as *constant advantage*: both  $w_n^s$  and  $w_n^r$  vary across individuals but with constant productivity advantage between risky and safe jobs:  $w_n^r - w_n^s = c$ . Analytically, this is not a very interesting case, since, with productivity gap, risk aversion and risk fixed and identical for every individual, there is no mechanism to establish an equilibrium, since  $\rho\sigma_\varepsilon^2/2$  may always be less than or greater than  $c$ . Moreover, constant (or absolute)

<sup>8</sup> Technically, this unknown ability cannot possibly impact the productivity at a risk-free job. With unknown ability, the job would not be risk-free anymore. On the other hand, if  $a_n$  impacts the safe job, making it risky to the degree of  $\sigma_a^2$  makes no difference to the conclusion drawn from this case that  $\rho$  is estimated without bias, as is evident from Eq. (12), appropriately modified.



advantage is unlikely to hold in practice (Cunha et al. 2005; Willis and Rosen 1979). Hence, we will not elaborate this case.

More interesting and relevant is the case of *varying advantage* where  $w_n^s$  is identical for all individuals, and  $w_n^r$  reflects an advantage in the risky job that depends on ability: the productivity gap is individual-specific.<sup>9</sup> Let us assume that individuals only differ in their ability to earn incomes; everyone has equal risk aversion,  $\rho_n = \rho$ , and equal risk,  $\sigma_{\varepsilon n}^2 = \sigma_\varepsilon^2$ . Individuals again choose their sector based on expected wages, allowing for differences in risk. The marginal worker will be indifferent, and hence, the marginal worker in the risky job is defined by

$$w^* = w^s + \frac{1}{2}\rho\sigma_\varepsilon^2. \quad (13)$$

Individuals with expected wage  $w_n^r < w^*$  will take the safe job; individuals with  $w_n^r > w^*$  will take the risky job. There will not be a separate risk premium established by supply reactions, as in the standard case of compensating wage differentials. Rather, the risk premium will be covered by the ability rent: only individuals with an expected wage gain high enough to cover the required risk premium will opt for the risky job.

If we are able to control completely for all relevant ability, the parameters are all identified. This, of course, gives panel data estimation an advantage over estimation from cross-section data.<sup>10</sup> Residual variance identifies risk. Thus, after controlling for ability and hence accounting for  $w_n^r - w^s$ , a regression of wages on residual variance should find no effect, since the difference has expected value zero and is unrelated to its variance. Applying a selectivity correction would make no difference in this case, as all wage differences between the two jobs are accounted for by observed ability differences (i.e., the variables explaining the gap between  $w_n^r$  and  $w^s$ ). However, the price of risk  $\rho$  is identified from the selection equation (13).

In the case where we cannot control for ability and cannot control for the selection process, the observations on wages and variances combine heterogeneity and risk effects. For individuals in risky jobs, let  $w_n^r$  once again be defined by (11) and  $\tilde{w}_n^r = w_n^r + \varepsilon_n$ . A worker looks at the risky job as one that yields an expected wage of  $E[\tilde{w}_n^r | a_n] = \omega^r + a_n$  with a risk of  $V[\varepsilon_n] = \sigma_\varepsilon^2$ , which implies the assumption of independence between ability and random income components.<sup>11</sup> The marginal worker requires a compensation of  $\frac{1}{2}\rho\sigma_\varepsilon^2$  to overcome his exposure to income risk; in other words,  $\omega^r + a^* = w^s + \frac{1}{2}\rho\sigma_\varepsilon^2$ , where  $a^*$  is this marginal worker's ability. Workers

<sup>9</sup> A third case where  $w_n^r$ ,  $w_n^s$  and  $w_n^r - w_n^s$  all vary across  $n$  is analytically similar and does not need to be discussed separately.

<sup>10</sup> It is not obvious that panel data are the perfect solution in all cases. Panel data may allow for elimination of non-observed ability, but it is not so clear that panel data can also be exploited to eliminate individual specific preferences or earnings risk profiles, which may also bias the results as we have shown in the previous sections.

<sup>11</sup> This independence assumption is innocuous. If something in  $\varepsilon_n$  correlates with  $a_n$ , the value of  $E[\varepsilon_n | a_n]$  is actually a payoff to ability and so belongs to  $a_n$  already.

with a higher level of ability accept risky jobs. As a result, the average observed wage equals

$$E[\tilde{w}^r | a > a^*] = \omega^r + E[a | a > a^*]. \quad (14)$$

Note that the subscript  $n$  is dropped as this describes the mean across all who hold risky jobs. Similarly, the observed variance of wages in risky jobs is given by

$$V(\tilde{w}^r | a > a^*) = E\left[(\varepsilon + a - E[a | a > a^*])^2 | a > a^*\right] \quad (15)$$

which may be rewritten as

$$V[\tilde{w}^r | a > a^*] = V[\varepsilon | a > a^*] + V[a | a > a^*] + 2\text{cov}[a, \varepsilon | a > a^*] \quad (16)$$

The observed variance in wages is the sum of the variance of the random residual income component  $\varepsilon$ , the variance of ability  $a$  across individuals and the covariance between  $\varepsilon$  and  $a$ , all conditional on  $a$  being larger than  $a^*$ . As we assume independence between ability and risk, (16) simplifies:  $V(\varepsilon | a > a^*) = \sigma_\varepsilon^2$  and  $\text{cov}[a, \varepsilon | a > a^*] = 0$ .

Thus, let us pull the threads together. The value of  $\rho$  relates to the parameters of the model as

$$\rho = \frac{\omega^r + a^* - w^s}{\sigma_\varepsilon^2/2} \quad (17)$$

but its estimate must rely on sample information represented by the equation

$$\hat{\rho} = \frac{E[\tilde{w}^r | a > a^*] - w^s}{V[\tilde{w}^r | a > a^*]/2} = \frac{\omega^r + E[a | a > a^*] - w^s}{(\sigma_\varepsilon^2 + V(a | a > a^*))/2} \quad (18)$$

Both the numerator and the denominator in (18) are larger than those in (17). As a result the bias in the estimate of  $\rho$  becomes indeterminate.

The direction of the bias depends of course on the magnitudes of  $E[a | a > a^*] - a^*$  and  $V(a | a > a^*)$  relative to the other parameters of the model. It is possible that for plausible parameter values and plausible distributions of  $a$  and  $\varepsilon$  the bias still tends to point in one particular direction. To get an indication which way the bias will go, let us simulate this model, based on a guess of reasonable parameter values.

As the model presented in this paper is additive in ability and risk, it is quite natural to assume additive, normal distributions for ability and risk:  $a \sim N(0, \sigma_a^2)$  and  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ , and thus  $V[\tilde{w}] = \sigma_a^2 + \sigma_\varepsilon^2$ . Of course, the earnings function literature uses log earnings and lognormal distributions, hence employing a multiplicative structure of ability and risk, whereas our model has an additive structure. Even so, it is still desirable to simulate it with a skewed distribution such as the lognormal as well. Since in the lognormal distribution both mean and variance move with the parameter that measures spread, we make an adjustment to maintain an expected value of zero.

We start from  $\tilde{w} = \omega^r + a + \varepsilon$ . As to distributions we assume  $a^\# \sim \log N(0, \sigma_{\ln a}^2)$  and  $\varepsilon^\# \sim \log N(0, \sigma_{\ln \varepsilon}^2)$ , and we transform  $a = a^\# - E[a^\#]$  and  $\varepsilon = \varepsilon^\# - E[\varepsilon^\#]$ .<sup>12</sup> The variance then equals

$$V[\tilde{w}] = e^{\sigma_{\ln a}^2} (e^{\sigma_{\ln a}^2} - 1) + e^{\sigma_{\ln \varepsilon}^2} (e^{\sigma_{\ln \varepsilon}^2} - 1) \tag{19}$$

For parameter values, we start from  $V[\tilde{w}] = 43.4059$ , which holds for a lognormal distribution with  $E[w] = 15$  and standard error of the regression equal to 0.42. This dispersion is taken from the wage regressions with data from the Current Population Survey in [Hartog and Vijverberg \(2007\)](#); the mean just matches quite nicely with our simulations. We let  $\sigma_{\ln \varepsilon}$  vary between 1.15 and 1.35, implying  $V[\varepsilon]$  ranges from 10.33 to 32.10. Because  $V[\tilde{w}]$  is constrained at 43.4059,  $\sigma_{\ln a}$  and  $\sigma_{\ln \varepsilon}$  relate negatively, allowing this simulation exercise to consider the effect of changes in the relative weight of heterogeneity and risk. The wage in the safe job  $w^s$  is fixed at 10, but through  $\omega^r$  the expected wage in the risky job varies, to allow for variation in the proportion of individuals choosing the risky job.

The simulation generates 10,000 workers who choose between the safe and the risky job.  $\rho$  is estimated as in (18); thus,  $\hat{\rho} = 2 \left( \hat{E}[\tilde{w}^r | a > a^*] - w^s \right) / \hat{V}[\tilde{w}^r | a > a^*]$  where  $a^* = w^s + 0.5\rho\sigma_\varepsilon^2 - \omega^r$ . In other words,  $\hat{\rho}$  equals the ratio of two times the difference in the average wage on risky jobs and the wage on safe jobs over the variance among risky jobs. The observed wage of the risky job contains  $a$  and  $\varepsilon$ . In fact, all of the distribution of  $\varepsilon$  is represented, but only the right tail of the distribution of  $a$  is allowed to contribute to the observed risky wage distribution.

Results are given in Table 1. For the normal distribution, we always find workers in both jobs. For the lognormal distribution, the ability distribution has a short lower tail and a long upper tail, so with large  $\omega^r$ , everyone prefers the risky job and we will ignore such cases. For the normal distribution case,  $\rho$  is always overestimated. An increase in  $\omega^r$  has a non-monotonic effect on  $\hat{\rho}$ . Basically, there is a U-shaped pattern, although it is not always fully visible: it depends on the ability/risk ratio. If ability variation increases and risk decreases, the positive bias grows larger if  $\rho = 0.4$ , and moves non-monotonically if  $\rho = 0.6$ .

For the lognormal distribution,  $\hat{\rho}$  is always underestimated. If  $\omega^r$  increases,  $\hat{\rho}$  increases, and hence, the bias decreases. If ability variation increases and risk decreases,  $\hat{\rho}$  moves non-monotonically (U-shaped), although again, the parameter combination may allow only part of the U to show up.

The differently signed biases in the two cases can be understood from the underlying distributions. By design,  $\sigma_\varepsilon^2$  is similar between the normal and lognormal specifications, but the right tail of the distribution of  $a$  that selects risky jobs is shaped differently, much thicker. For both normal and lognormal, when  $a^*$  rises, the conditional mean of  $a$  rises (obviously), but while the conditional variance of the normal  $a$  falls, the conditional variance of the lognormal  $a$  rises.<sup>13</sup> Thus, the conditional variance of

<sup>12</sup> Of course,  $E[a^\#] = \exp(0.5\sigma_{\ln a}^2)$ .

<sup>13</sup> This is all visible from our calculations, but will not be reproduced here.

**Table 1** Simulated estimates of  $\rho$  for varying  $\omega^r$ ,  $\sigma_\varepsilon$  and  $\sigma_a$  when ability is heterogeneous

A. Normal									
	$\sigma_\varepsilon$ :	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00
$\omega^r$	$\sigma_a$ :	6.10	5.87	5.58	5.23	4.81	4.29	3.63	2.70
A1: $\rho = 0.4$									
12		0.616	0.602	0.586	0.565	0.541	0.513	0.482	0.451
13		0.612	0.601	0.586	0.568	0.545	0.519	0.488	0.454
14		0.609	0.600	0.588	0.571	0.550	0.525	0.494	0.458
15		0.608	0.600	0.590	0.575	0.556	0.532	0.502	0.464
16		0.609	0.602	0.593	0.580	0.562	0.540	0.510	0.472
17		0.612	0.606	0.598	0.586	0.570	0.548	0.520	0.482
18		0.618	0.612	0.605	0.594	0.579	0.559	0.531	0.494
19		0.627	0.621	0.614	0.604	0.590	0.571	0.546	0.509
A2: $\rho = 0.6$									
12		0.681	0.692	0.698	0.699	0.692	0.680	0.664	1.947
13		0.673	0.684	0.693	0.696	0.692	0.681	0.663	0.732
14		0.666	0.678	0.688	0.694	0.692	0.682	0.665	0.669
15		0.660	0.672	0.684	0.691	0.691	0.684	0.667	0.651
16		0.656	0.669	0.681	0.689	0.691	0.686	0.670	0.648
17		0.655	0.667	0.679	0.688	0.692	0.688	0.674	0.650
18		0.656	0.667	0.679	0.688	0.693	0.691	0.678	0.653
19		0.661	0.671	0.681	0.689	0.695	0.694	0.683	0.658
B. Lognormal									
	$\sigma_{\ln \varepsilon}$ :	1.15	1.20	1.25	1.30	1.35			
	$\sigma_{\ln a}$ :	1.36	1.34	1.31	1.26	1.17			
	$\sigma_\varepsilon$ :	3.21	3.69	4.24	4.89	5.67			
$\omega^r$	$\sigma_a$ :	5.75	5.46	5.04	4.41	3.36			
B1: $\rho = 0.4$									
12		0.141	0.149	0.163	0.190	0.259			
13		0.159	0.167	0.179	0.205	0.275			
14		0.190	0.192	0.201	0.224	0.291			
15		a	0.235	0.232	0.249	0.311			
16		a	a	a	0.281	0.334			
17		a	a	a	0.329	0.357			
18		a	a	a	0.376	0.385			
19		a	a	a	0.424	0.429			
B2: $\rho = 0.6$									
12		0.143	0.152	0.167	0.196	0.280			
13		0.160	0.167	0.181	0.209	0.292			
14		0.185	0.188	0.199	0.226	0.307			
15		0.226	0.218	0.222	0.246	0.326			
16		a	0.268	0.255	0.27	0.348			
17		a	a	0.306	0.303	0.372			
18		a	a	a	0.346	0.401			
19		a	a	a	0.411	0.434			

<sup>a</sup> All labor force participants would choose for the risky job. The average wage on the safe jobs would not be determined and  $\rho$  would not be estimated

$\tilde{w}^r$  falls with a rise in  $a^*$  in the normal case, whereas the conditional variance of  $\tilde{w}^r$  rises with  $a^*$  in the lognormal case. Moreover, for a very negative  $a^*$ , there is hardly any selection in the lognormal case, since all want the risky job. Thus, the conditional mean of  $a$  is close to 0. In the normal case, there is always selection, and always a conditional mean of  $a$  that is greater than 0.

We can only conclude that the predicted bias is very sensitive to variation in the specification of the model. Apparently, the overestimation of both the numerator and the denominator precludes the emergence of simple patterns. One distributional assumption leads to positive bias, another to negative bias and the basic pattern of sensitivity to parameter values appears to be reflected in a U-shaped profile. Thus, ambiguity remains within the set of parameter values we used for our simulations.<sup>14</sup> However, as empirical wage distributions resemble lognormality rather than normality, we may conclude that underestimation of risk aversion is most likely.

Empirical evidence on the relevance of ability bias is not completely conclusive either but tends to point in the direction of underestimation. In a Danish panel, [Diaz Serrano et al. \(2003\)](#), distinguishing residual variance between individuals (permanent shocks) and within individual incomes over time (transitory shocks), find significant wage compensation for both components, thus indicating that the results from cross-section OLS estimates cannot be wholly due to upward bias. The elasticity of wage compensation, while still very low, increases by a third in the panel estimates relative to annual cross-sections on the same data. [Bajdechi \(2005\)](#), using NLSY data, finds significant compensation for transitory shocks and insignificant compensation for permanent shocks. With risk measured by IQ decile, the compensation for transitory shocks is unaffected, while compensation for permanent shocks increases and becomes significant. [Berkhout et al. \(2005\)](#) use secondary school grades to condition on ability, and find that for higher vocational graduates the risk compensation coefficient slightly increases, while for university graduates it falls substantially.

We conclude that if ability is unknown to individuals the estimated risk attitude is unbiased, as unknown ability is simply a component of risk. If individuals do know their ability and we have a case of unobserved heterogeneity, analytically the bias can go either way, but empirically we consider underestimation more likely.

## 5 Conclusion

Risk compensation in wages has been estimated from OLS regression of wages on residual variance in education/occupation. Selection biases may result in biased estimates for the true price of risk because the true price is determined by the marginal individual deciding to switch to more risky jobs. If individuals only differ in risk attitude, we have a textbook case where the simple two-stage OLS estimation obtains an unbiased estimate of marginal risk aversion. If only risk differs between individuals, risk aversion is overestimated. If only ability differs between individuals, assumptions

<sup>14</sup> We have considered allowing for correlation between ability and risk. But apart from the fact there is no obvious reason to expect this, we conjecture that this will not resolve the basic ambiguity. Nor will we dwell on the possibility that risk aversion simultaneously varies among individuals, as Sect. 4 has indicated that this will lead into very complicated and untractable situations.

on information are vital. With individuals ignorant on their abilities, as in the fifth of Smith's famous principal circumstances for wage differentials, abilities are a source of risk and risk aversion is estimated without bias. If individuals themselves know their abilities, we have a textbook case of selectivity bias from unobserved heterogeneity. The equation for the bias shows an ambiguity in sign that is not resolved by simulation. This also implies that attempts to correct for this bias must be aware of the consequences of alternative distributional specifications. On empirical grounds, we think that underestimation is more likely than overestimation, but as always, this conjecture rests on the realism of the simple selection model we analyzed.

### Appendix: A lognormal specification

We can derive analytically that utility is a mean-variance utility function—as in the main text—if wage income is log-normally distributed and utility displays constant relative risk aversion, see also Weiss (1972). Assume that all income is consumed, and there is no other asset or non-labor income as in the text. Utility is then given by:

$$u(w) = \frac{w^{1-\rho}}{1-\rho} = \frac{w^\beta}{\beta},$$

where  $\rho \equiv 1 - \beta$  is the coefficient of relative risk aversion.  $w$  is log-normally distributed with mean  $\mu$  and standard deviation  $\sigma$ :  $\ln w \sim N(\mu, \sigma^2)$ . Taking logarithms and expectations  $E[\cdot]$  of the utility function yields:

$$E[\ln[u(w)]] = \beta E[\ln(w)] - \ln(\beta) = \beta\mu - \ln(\beta).$$

Using the properties of the normal distribution, we can express the variance  $V[\cdot]$  of utility as

$$V[u(w)] = \beta^2 \sigma^2.$$

Therefore, we can derive that  $\ln[u(w)]$  is log-normally distributed with mean  $\beta\mu - \ln(\beta)$  and variance  $\beta^2 \sigma^2$ :  $\ln[u(w)] \sim N(\beta\mu - \ln(\beta), \beta\sigma)$ . Since  $E[\ln z] = \exp[\mu_z + \frac{1}{2}\sigma_z^2]$  for any log-normally distributed variable  $z$ , expected utility can be expressed as:

$$E[u(w)] = \frac{1}{\beta} \exp\left[\beta\mu + \frac{1}{2}\beta^2\sigma^2\right].$$

After rewriting we obtain:

$$\begin{aligned} E[u(w)] &= \frac{1}{\beta} \exp\left[\beta\left(\mu + \frac{1}{2}\sigma^2\right)\right] \exp\left[\frac{1}{2}\beta(\beta-1)\sigma^2\right] \\ &= \frac{1}{\beta} \bar{w}^\beta \exp\left[\frac{1}{2}\beta(\beta-1)\sigma^2\right], \end{aligned}$$

where  $\bar{w}$  is the mean wage. Taking logarithms gives a utility function having only the mean and variance as its arguments:

$$\ln E[u(x)] = \ln(1/\beta) + \beta \ln \bar{w} - \frac{1}{2}\beta(1 - \beta)\sigma^2.$$

After taking an affine transformation, we can therefore express utility as a mean-variance function  $v(\mu, \sigma^2)$ :

$$v(\mu, \sigma^2) = \ln \bar{w} - \frac{1}{2}\rho\sigma^2,$$

where  $v(\mu, \sigma^2) \equiv \ln[Eu(w)]/(1 - \rho) - c$  and  $c \equiv \ln(1/(1 - \rho))/\rho$ .

From the last equation we see that expected utility is only a function of log income and the variance of log income. Therefore, our model in the main text carries over to the case for log-normally distributed wages, provided that non-labor income is zero and utility features constant relative risk aversion. Note that after estimation of the wage equation on log wages,  $\rho$  should now correctly be interpreted as the coefficient of relative risk aversion and not as the coefficient of absolute risk aversion.

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