Non-monetary Costs of Education with Substitution between Monetary and Non-monetary Costs

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Abstract

This note discusses an extension of our paper "Redistribution and Education Subsidies are Siamese Twins" where allow for non-pecuniary (effort) costs of education invested in human capital, besides, material costs. In contrast to the paper we allow for substitution between effort costs and resources invested in education. To do so, we analyze a two-input version of our model with non-linear income taxes and subsidies, similar to the two-input model with non-verifiable learning in the linear case. We derive optimal non-linear education subsidies and show that these are the product of earlier derivations in sections 3 and 5 of the paper.

1 Individual optimization

For notation we refer to the paper. The costs of y_n now directly enter the utility function rather than the budget constraint. Utility is given by

$$u(c_n, l_n, y_n) \equiv c_n - v(l_n, y_n) \tag{1}$$

with $v_l(.), v_y(.) > 0$, $v_{ll}(.), v_{yy}(.) > 0$, and $v_{ly}(.) \gtreqless 0$. We stick to the Diamond type of preferences without income effects but assume a sub-utility function $v(.)$ over labor supply l_n and efforts invested in education y_n.[1]

The budget constraint is modified as

$$c_n = nl_n\phi(\psi(x_n, y_n)) - T(nl_n\phi(x_n, y_n)) - p_x x_n + S(p_x x_n). \tag{2}$$

Whereas x_n is verifiable and can be subsidized, y_n is non verifiable and thus cannot be subsidized. The first-order conditions for labor supply l_n and investments in human capital x_n and y_n are[2]

$$v_l(l_n, y_n) = (1 - T'(.))n\phi(\psi(x_n, y_n)), \tag{3}$$

$$(1 - T'(z_n))nl_n\phi'\psi_x(x_n, y_n) = (1 - S'(p_x x_n))p_x, \tag{4}$$

$$(1 - T'(z_n))nl_n\phi'\psi_y(x_n, y_n) = v_y(l_n, y_n). \tag{5}$$

[1]The extension to substitution between material and immaterial benefits is straightforward.

[2]We assume that the first-order conditions are necessary and sufficient so that second-order conditions are satisfied.

2 Government

The incentive compatibility constraints are modified to allow for the extra term in the utility function:

$$u(c_n, l_n, y_n) \equiv u\left(c_n, \frac{z_n}{n\phi(x_n, y_n)}, y_n\right) = U(c_n, z_n, x_n, y_n, n). \tag{6}$$

We again have a differential equation for the utilities (after applying the envelope theorem)

$$\frac{dU_n}{dn} = -\frac{l_n}{n} u_l(c_n, l_n, y_n), \tag{7}$$

while the economy's resource constraint is modified to

$$\int_{\underline{n}}^{\overline{n}} (n\phi(x_n, y_n)l_n - p_x x_n - c_n - E)\, dF(n) = 0. \tag{8}$$

The corresponding Hamiltonian for maximizing social welfare is given by

$$\max_{\{x_n, l_n, u_n\}} \mathcal{H} = \Psi(u_n)f(n) + \theta_n \frac{l_n}{n} u_l(c_n, l_n, y_n) + \lambda\left(n\phi(\psi\left(x_n, y_n\right))l_n - p_x x_n - c_n - E\right) f(n) \tag{9}$$

3 Optimal education subsidies

The first-order condition for maximizing the Hamiltonian with respect to x_n is given by:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial x_n} &= \lambda\left(nl\phi'\psi_x(.) - p_x + nl_n\phi'\psi_y\left(.\right) \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}} - \left.\frac{dc_n}{dx_n}\right|_{\bar{u},\bar{l}}\right) f(n) + \\ & \theta_n \frac{l_n}{n}\left(u_{lc}(.) \left.\frac{dc_n}{dx_n}\right|_{\bar{u},\bar{l}} + u_{ly}(.) \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}}\right) = 0 \end{aligned} \tag{10}$$

This equation can be simplified in a number of steps. First of all, we have $u_{cl} = 0$ due to the Diamond preferences. Second, we can differentiate the household budget constraint to arrive at: $\left.\frac{dc_n}{dx_n}\right|_{\bar{u},\bar{l}} = (1 - T'(z_n))nl_n\phi'(e_n)\psi_y(.) \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}}$. After substituting these results we find

$$\lambda\left(nl\phi'(e_n)\psi_x(.) - p_x + T'nl_n\phi'(e_n)\psi_y(.) \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}}\right) f(n) = -\theta_n \frac{l_n}{n}\left(u_{ly}(.) \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}}\right)$$

We can simplify the last expression further using the first-order conditions for household optimization with respect to x_n and y_n (see (4) and (5)) to eliminate $\phi'(e_n)\psi_x(.)$ and $\phi'(e_n)\psi_y(.)$, respectively. We also use $v_{ly} = -u_{ly}(.)$ from the definition of u_n:

$$\left(\frac{T' - S'}{1 - T'}\right)\frac{p_x}{v_y(.)} + \frac{T'}{(1 - T')} \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}} = \frac{\theta_n/\lambda}{f(n)n}\frac{l_n v_{ly}(.)}{v_y(.)} \left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}}$$

Next, we employ the first-order conditions for x_n (4) and y_n (5) to eliminate $\frac{p_x}{v_y(.)} = \frac{\psi_x(.)}{\psi_y(.)(1-S')}$, and multiply all terms with $\frac{x_n}{y_n}$ and simplify:

$$\left(\frac{T'-S'}{(1-T')(1-S')}\right)\frac{x_n}{y_n}\frac{\psi_x(.)}{\psi_y(.)} + \frac{T'}{(1-T')}\frac{x_n}{y_n}\left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}} = \frac{\theta_n/\lambda}{f(n)n}\frac{l_n v_{ly}(.)}{v_y(.)}\frac{x_n}{y_n}\left.\frac{dy_n}{dx_n}\right|_{\bar{u},\bar{l}}.$$

Hence, we obtain

$$\left(\frac{T'-S'}{(1-T')(1-S')}\right)\frac{\alpha_n}{1-\alpha_n} + \frac{T'}{(1-T')}\varepsilon_{yx} = \frac{\theta_n/\lambda}{f(n)n}\frac{l_n v_{ly}(.)}{v_y(.)}\varepsilon_{yx}, \tag{11}$$

where we defined $\varepsilon_{yx} \equiv \frac{x_n}{y_n}\frac{dy_n}{dx_n}\Big|_{\bar{u},\bar{l}}$ and used $\alpha_n \equiv \frac{\psi_x(.)x_n}{\psi(.)}$, the share of material inputs in education. Homogeneity of $\psi(.)$ implies that $1-\alpha_n \equiv \frac{\psi_y(.)y_n}{\psi}$.

We derive ε_{yx} as follows. Combining the first-order condition for labor supply l_n (3) and inputs y_n (5) invested in education, we find

$$\frac{\phi(\psi(x_n,y_n))}{l_n\phi'(e_n)\psi_y(x_n,y_n)} = \frac{v_l(l_n,y_n)}{v_y(l_n,y_n)}. \tag{12}$$

Loglinearizing (12) this, we establish

$$-\tilde{l}_n + \frac{\phi'\psi}{\phi}\tilde{\psi}_n - \frac{\phi''\psi}{\phi'}\tilde{\psi}_n - \tilde{\psi}_y = \delta_n\tilde{l}_n - \gamma_n\tilde{y}_n, \tag{13}$$

where $\delta_n \equiv \frac{v_{ll}l_n}{v_l} - \frac{v_{ly}l_n}{v_y}$ and $\gamma_n \equiv \frac{v_{yy}y_n}{v_y} - \frac{v_{ly}y_n}{v_l}$. Next, we use the properties of the homothetic production function for human capital $\phi(\psi(x_n,y_n)) = (\psi(x_n,y_n))^{\beta}$, where ψ is a linear homogenous function with elasticity of substitution $\sigma_n \equiv d\ln\left(\frac{x_n}{y_n}\right)/d\ln\left(\frac{\psi_y(.)}{\psi_x(.)}\right)$:

$$\sigma_n \equiv -\frac{\tilde{x}_n - \tilde{y}_n}{\tilde{\psi}_x - \tilde{\psi}_y} = \frac{\psi_x\psi_y}{\psi\psi_{xy}},$$

$$\alpha_n \equiv \frac{\psi_x x_n}{\psi},$$

$$\psi_n = \alpha_n\psi_x + (1-\alpha_n)\psi_y,$$

$$x_n\psi_{xx} = -y_n\psi_{xy},$$

$$y_n\psi_{yy} = -x_n\psi_{xy},$$

Therefore, we find $\tilde{\psi}_y = \frac{y\psi_{xx}}{\psi_y}\tilde{y}_n + \frac{x\psi_{yx}}{\psi_x}\tilde{x}_n = \frac{x\psi_{yx}}{\psi_x}(\tilde{x}_n - \tilde{y}_n) = \frac{\alpha_n}{\sigma_n}(\tilde{x}_n - \tilde{y}_n)$. Note that parameters α_n and σ_n depend on n, due to the utility function and the non-linearity of the subsidy schedule. This contrasts with section 2 of the paper with only monetary inputs and linear tax and subsidy instruments.

Substitution of these results into (13) yields:

$$\alpha_n\tilde{x}_n + (1-\alpha_n)\tilde{y}_n - \tilde{l}_n - \frac{\alpha_n}{\sigma_n}(\tilde{x}_n - \tilde{y}_n) = \delta_n\tilde{l} - \gamma_n\tilde{y}. \tag{14}$$

For the first-order condition for the optimal non-linear subsidy we need the elasticity of y_n with respect to x_n at constant labor supply, i.e., $\tilde{l}_n = 0$. Hence, from the equation (14), we find that the elasticity ε_{yl} is given by

$$\varepsilon_{yl}\big|_{\tilde{l}_n=0} \equiv \left.\frac{\tilde{y}_n}{\tilde{x}_n}\right|_{\tilde{l}_n=0} = \frac{\alpha_n(1-\sigma_n)}{1+\gamma_n\sigma_n-(1-\alpha_n)(1-\sigma_n)}. \tag{15}$$

Substitute $\varepsilon_{yx} \equiv \frac{\alpha_n(1-\sigma_n)}{1+\gamma_n\sigma_n-(1-\alpha_n)(1-\sigma_n)}$ in (11) to arrive at the the final expression for the optimal education subsidy

$$\frac{S'(p_x x_n)-T'(z_n)}{(1-T'(z_n))(1-S'(p_x x_n))} = \left(\frac{(1-\alpha_n)(1-\sigma_n)}{1+\sigma_n\gamma_n-(1-\alpha_n)(1-\sigma_n)}\right)\left(\frac{T'(.)}{1-T'(.)}-\frac{\theta_n/\lambda}{f(n)n}\frac{l_n v_{ly}(.)}{v_y(.)}\right). \tag{16}$$

where $\gamma_n \equiv \left(\frac{v_{yy}y_n}{v_y}-\frac{v_{ly}y_n}{v_l}\right) = \frac{\partial\ln(v_y/v_l)}{\partial l} \gtrless 0$.

Clearly, the expression for optimal subsidies (16) is just the product of the results in Section 3 of the paper and Section 5, see equations (81) and (41) in the paper, where the first term in brackets corresponds to (81) and the second term in brackets to (41). Indeed with zero substitution ($\sigma = 0$) we find that $\frac{(1-\alpha_n)(1-\sigma_n)}{1+\sigma_n\gamma_n-(1-\alpha_n)(1-\sigma_n)} = \frac{1}{\alpha_n} - 1 = \omega_n$. The only difference between (81) and the first term in brackets is the presence of $\sigma_n\gamma_n$. If preferences are separable, $v_{ly}(.) = 0$, and $\gamma_n = \frac{v_{yy}y_n}{v_y} > 0$, the absolute value of the optimal subsidy wedge on verifiable learning $S' - T'$ is smaller than with material costs of non-verifiable learning (*ceteris paribus*). The intuition is that subsidies on x_n become less effective to stimulate y_n because the marginal costs of educational effort rise with y_n. Only if $\gamma_n = \frac{v_{yy}y_n}{v_y} = 0$, i.e., if marginal costs of effort invested in education are constant ($v_{yy} = 0$), do we find the same term as in (81).